MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. If $x|x-3|+3|x-2|+1=0$, then the number of real solution is

(1) 2 (2) 4

(3) 1 (4) 6

Answer (3)

Sol. $x|x-3|+3|x-2|+1=0$

Case-I

$x \le 2$

$x(3-x)+3(2-x)+1=0$

$3x-x^2+6-3x+1=0$

$x^2=7$

$x=\pm\sqrt{7}$ $\quad x=-\sqrt{7}$

Case-II

$2<x\le 3$

$x(3-x)+3(x-2)+1=0$

$3x-x^2+3x-6+1=0$

$x^2-6x+5=0$

$(x-5)(x-1)=0$

$\Rightarrow x=5,1$ (no solution)

Case-III

$x\ge 3$

$x(x-3)+3(x-2)+1=0$

$x^2-3x+3x-6+1=0$

$x^2=5$

$\Rightarrow x=\pm\sqrt{5}$ (no solution)

Only 1 solution i.e., $-\sqrt{7}$

2. If two vectors $\vec{a}$ and $\vec{b}$ satisfies $\dfrac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|}=\sqrt{2}+1$, then the value of $\dfrac{|\vec{a}+\vec{b}|^2}{|\vec{a}-\vec{b}|^2}$ is equal to

(1) $1+\sqrt{2}$ (2) $2+4\sqrt{2}$

(3) $1+2\sqrt{2}$ (4) $3+2\sqrt{2}$

Answer (4)

Sol. $\dfrac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|}=\left(\sqrt{2}+1\right)$

$\dfrac{2|\vec{a}+\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|}=\left(\sqrt{2}+2\right)$

$\dfrac{2|\vec{a}+\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|}=\sqrt{2}$

$\Rightarrow \dfrac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|}=\dfrac{\sqrt{2}+2}{\sqrt{2}}=\left(\sqrt{2}+1\right)$

$\Rightarrow \left|\dfrac{\vec{a}+\vec{b}}{\vec{a}-\vec{b}}\right|^2=\left(\sqrt{2}+1\right)^2=2+1+2\sqrt{2}$

$=3+2\sqrt{2}$

3. If a box contains 19 unbiased coins and 1 biased coin with both faces head. If a coin is randomly chosen out of this box and tossed. If the head appears, then the probability that the biased coin was selected

(1) $\dfrac{19}{21}$ (2) $\dfrac{1}{3}$

(3) $\dfrac{1}{5}$ (4) $\dfrac{1}{6}$

Answer (1)

Sol. $P\left(\frac{u}{H}\right)=\frac{P(u)\,P\left(\frac{H}{u}\right)}{P(u)\,P\left(\frac{H}{u}\right)+P(\bar{u})\,P\left(\frac{H}{\bar{u}}\right)}$

$$=\frac{\frac{1}{20}\times 1}{\frac{1}{20}\times 1+\frac{19}{20}\times\frac{1}{2}}=\frac{2}{2+19}=\frac{2}{21}$$

4. Let $y = y(x)$ and $(1 + x^2)y' - 2xy = (x^4 + 2x^2 + 1)\cos x$.

If $y(0) = 1$, then $\int_{-3}^{3} y(x)dx$ equals to

(1) 20 (2) 15

(3) 24 (4) 10

Answer (3)

Sol. $(1+x^2)\frac{dy}{dx}-2xy=(x^4+2x^2+1)\cos x$

$$\frac{dy}{dx}-\left(\frac{2x}{1+x^2}\right)y=\frac{(x^2+1)\cos x}{(x^2+1)}$$

$$\text{IF} = e^{-\int\frac{2x}{1+x^2}dx}=\frac{1}{1+x^2}$$

$$\frac{y}{1+x^2}=\int\cos x\,dx$$

$$\frac{y}{1+x^2}=\int\sin x+c$$

$\because\ y(0)=1$

$\Rightarrow\ 1=c$

$\therefore\ y=(1+\sin x)(1+x^2)$

$$\int_{-3}^{3} y(x)dx\ \int_{-3}^{3}(1+\sin x)(1+x^2)dx$$

$$=\int_{0}^{3}2(1+x^2)dx$$

$$=\left.2x+\frac{2x^3}{3}\right]_0^3$$

$= 6 + 18 = 24$

5. The sum of series $^2C_1 \cdot (1 \times 2) + {}^3C_2 \cdot (2 \times 3) + {}^4C_3 \cdot (3 \times 4) + \ldots + {}^{19}C_{18} \cdot (18 \times 19)$ is S, then $\frac{S}{295}$ is equal to

(1) 104 (2) 107

(3) 103 (4) 114

Answer (4)

Sol. $T_r = \left({}^{r+1}C_r\right)\cdot r(r+1)\ \forall r\in\{1, 2, 3, \ldots, 18\}$

$$T_r=\frac{(r+1)!}{r!}(r)(r+1)\ = r\,(r+1)^2$$

$$S=\sum_{r=1}^{18}T_r=\sum_{r=1}^{18}r(r+1)[r+2-1]$$

$$=\sum_{r=1}^{18}r(r+1)(r+2)-\sum_{r=1}^{18}r(r+1)$$

$$=\sum_{r=1}^{18}\frac{r(r+1)(r+2)\left[r+3-(r-1)\right]}{4}-\sum_{r=1}^{18}\frac{r(r+1)\left[(r+2)-(r-1)\right]}{3}$$

$$=\frac{18\times 19\times 20\times 21}{4}-\frac{18\times 19\times 20}{3}$$

$$=18\times 19\left[105-\frac{20}{3}\right]\ =\ =\frac{18\times 19}{3}[295]\ = 33630$$

$$\frac{S}{295}=6\times 19=114$$

6. If a triangle is formed using lines $y = x + 1$, $y = 4x - 8$, $y = mx + c$. If the orthocentre of this triangle is $(3, -1)$, then $m - c$ is equal to

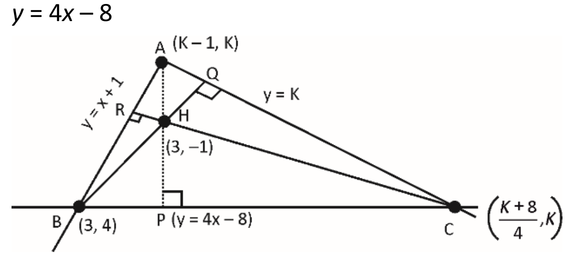
(1) 0 (2) 1

(3) –1 (4) 2

Answer (1)

Sol. $y = x + 1$

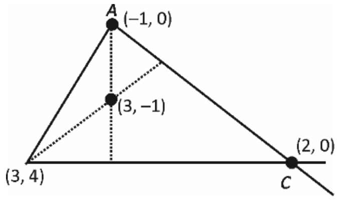
$y = 4x - 8$



$CR\perp y=x+1$

$\Rightarrow CR: x+y-2=0$

$$\frac{K+8}{4}+K-2=0 \Rightarrow K=0$$



Line AC : $y = 0$

7. Let the foot of perpendicular from $P(5, 1, -3)$ on the line $L_1 : x-1 = y-2 = z$ and $L_2 : x-2 = y = z-1$ is Q and R, respectively. The area of triangle P and R is equal to

(1) $\frac{7}{2}$ (2) $\frac{7}{\sqrt{2}}$

(3) $\frac{7\sqrt{3}}{2}$ (4) 7

Answer (3)

Sol. $P(5, 1, -3)$

$L_1 : x-1 = y-2 = z = \lambda$

$L_2 : x-2 = y = z-1 = \mu$

Any point of L_1 is $Q(\lambda+1, \lambda+2, \lambda)$

Any point of L_2 is $R(\mu+2, \mu, \mu+1)$

Now $PQ < \lambda-4, \lambda+1, \lambda+3 > \cdot < 1, 1, 1 > = 0$

$\lambda-4+\lambda+1+\lambda+3=0$

$3\lambda = 0$

$\Rightarrow \lambda = 0$

$\therefore Q(1, 2, 0)$

Also, $PR < \mu-3, \mu-1, \mu+4 > \cdot < 1, 1, 1 > = 0$

$\mu-3+\mu-1+\mu+4=0$

$\Rightarrow \mu = 0$

$R(2, 0, 1)$

$$\text{Area } = \frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = \left|\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -1 & -3\\ 3 & 1 & -4\end{vmatrix}\right|$$

$$=\frac{1}{2}\left|7\hat{i}+7\hat{j}+7\hat{k}\right| = \frac{1}{2}\times 7\times\sqrt{3} = \frac{7\sqrt{3}}{2}$$

8. Let α and β be roots of the equation

$$\left[(t+2)^{\frac{1}{7}}-1\right]x^2+\left[(t+2^{\frac{1}{6}}-1\right]x+\left((t+2)^{\frac{1}{21}}-1\right)=0$$

If $\lim\limits_{t\to-1}\alpha = a$ and $\lim\limits_{t\to-1}\beta = b$ then 72 $(a + b)^2$ is equal to

(1) 49 (2) 98

(3) 36 (4) 75

Answer (2)

Sol. Notice that

$$\alpha+\beta = -\frac{(t+2)^{\frac{1}{6}}-1}{(t+2)^{\frac{1}{7}}-1}$$

$$\alpha\beta = \frac{(t+2)^{\frac{1}{21}}-1}{(t+2)^{\frac{1}{7}}-1}$$

$$\lim_{t\to-1}(\alpha+\beta) = \frac{-\frac{1}{6}}{\frac{1}{7}} = \frac{-7}{6} = a+b$$

$$\lim_{t\to-1}(\alpha\beta) = \frac{\frac{1}{21}}{\frac{1}{7}} = \frac{7}{21} = \frac{1}{3} = ab$$

$\Rightarrow \quad (a+b)^2 = \frac{41}{36} \Rightarrow 72\ (a+b)^2 = 98$

9. In an experiment, a random variable X can take values 0, 1, 2, 3. If $P(X = 0) = P(X = 1)$, $P(X = 2) = P(X = 3)$ and $E(X^2) = 2E(X)$, then the value of $P(X = 0)$ is

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$

(3) $\frac{1}{2}$ (4) $\frac{3}{8}$

Answer (4)

Sol. $P(X = 0) = P(x = 1) = a$ and $P(X = 2) = P(X = 3) = b$

$2a + 2b = 1$

$\Rightarrow \quad a + b = \frac{1}{2}$

$E(X^2) = 2E(X)$

$= a\times 0^2 + a\times 1^2 + bX\times 2^2 + b\times 3^2$

$= 2(a\times 0 + a\times 1 + b\times 2 + b\times 3)$

$\Rightarrow \quad a = 3b$

$4b = \frac{1}{2} \;\Rightarrow\; b = \frac{1}{8}$

$\therefore\; a = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

$P(X = 0) = a = \frac{3}{8}$

10. Two sets A and B are defined as $A = \{(\alpha, \beta) : |\alpha - 1| \le 5, |\beta - 5| \le 6$ and $\alpha, \beta \in R\}$ and $B = \{(\alpha, \beta) : 9(\alpha - 2)^2 + 16(\beta - 6)^2 \le 144, \alpha, \beta \in R\}$ then

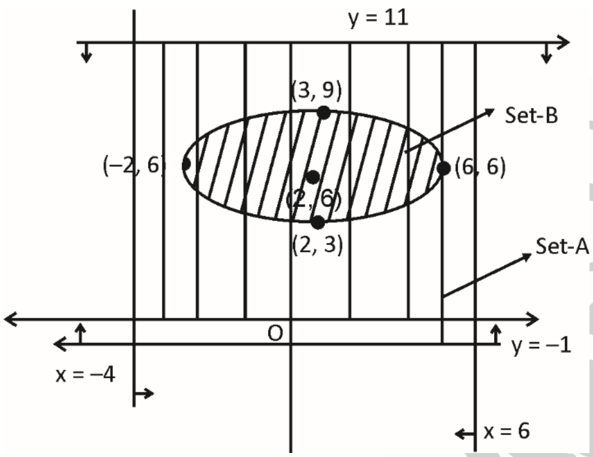
(1) $A \subset B$ (2) $B \subset A$

(3) $A = B$ (4) None of these

Answer (2)

Sol. $\because\; B \equiv \frac{(\alpha-2)^2}{16} + \frac{(\beta-6)^2}{9} \le 1$

As $A \equiv \alpha \in [-4, 6], \beta \in [-1, 11]$



$\therefore\quad B \subset A$

11. Evaluate

$$\int \left(\frac{1}{x} + \frac{1}{x^3}\right) \sqrt[23]{\frac{3}{x^{24}} + \frac{1}{x^{26}}}\, dx$$

(1) $\frac{23}{72}\left(\frac{3}{x} + \frac{1}{x^3}\right)^{\frac{24}{23}} + C$ (2) $\frac{-23}{72}\left(\frac{3}{x} + \frac{1}{x^3}\right)^{\frac{24}{23}} + C$

(3) $\frac{23}{72}\left(\frac{3}{x} - \frac{2}{x^3}\right)^{\frac{23}{22}} + C$ (4) $\frac{-23}{72}\left(\frac{3}{x} - \frac{1}{x^3}\right)^{\frac{23}{22}} + C$

Answer (2)

Sol. $\int \left(\frac{1}{x^2} + \frac{1}{x^4}\right) \sqrt[23]{\frac{3}{x} + \frac{1}{x^3}}\, dx$

$\frac{3}{x} + \frac{1}{x^3} = t^{23}$

$\frac{-3}{x^2} - \frac{3}{x^4} dx = 23t^{22} dt$

$= -\frac{23}{3} \int t \cdot t^{22} dt = -\frac{23}{3} \frac{t^{24}}{24} + C$

$= -\frac{23}{3 \times 24} \left(\frac{3}{x} + \frac{1}{x^3}\right)^{\frac{24}{23}} + C$

12. Number of solution(s) of the equation $(\cos 2\theta) \cdot \left(\cos \frac{\theta}{2}\right) + \cos \frac{5\theta}{2} = 2\cos^3\left(\frac{5\theta}{2}\right)$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is equal to

(1) 6 (2) 7

(3) 4 (4) 2

Answer (2)

Sol. $2(\cos 2\theta) \cdot \left(\cos \frac{\theta}{2}\right) + 2\cos \frac{5\theta}{2} = 4\cos^3\left(\frac{5\theta}{2}\right)$

$\Rightarrow \quad \cos\left(\frac{5\theta}{2}\right) + \cos \frac{3\theta}{2} + 2\cos\left(\frac{5\theta}{2}\right)$

$= \left(\cos \frac{15\theta}{2} + 3\cos \frac{5\theta}{2}\right)$

$\Rightarrow \quad \cos\left(\frac{3\theta}{2}\right) + \cos\left(\frac{15\theta}{2}\right)$

$\Rightarrow \quad \cos\left(\frac{3\theta}{2}\right) - \cos \frac{15\theta}{2} = 0$

$\Rightarrow \quad 2\sin\left(\frac{9\theta}{2}\right) \sin\left(\frac{6\theta}{2}\right) = 0, 3\theta = 2A\pi$

$\therefore \quad \frac{9\theta}{2} = \eta\pi \;\to\; \theta = \frac{2\eta\pi}{9}$

$\Rightarrow \quad \theta = \frac{2\eta\pi}{3}$

$\therefore \quad \theta = -\frac{4\pi}{9}, -\frac{3\pi}{9}, -\frac{2\pi}{9}, 0, \frac{2\pi}{9}, \frac{3\pi}{9}, \frac{4\pi}{9}$

13. If e_1 is the eccentricity of ellipse $\frac{x^2}{b^2}+\frac{y^2}{25}=1,\ |b|<5$ and e_2 is the eccentricity of hyperbola $\frac{x^2}{16}-\frac{y^2}{b^2}=1$ and $e_1 \cdot e_2 = 1$. Then the length of latus rectum of ellipse which passes through foci of the ellipse and the hyperbola and having centre at origin and axes along the coordinate axes is

(1) $\frac{16}{5}$ (2) $\frac{32}{5}$

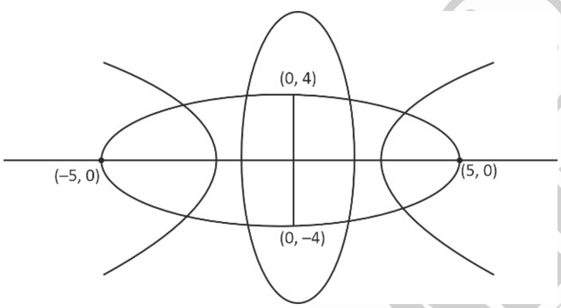
(3) $\frac{8}{5}$ (4) $\frac{64}{5}$

Answer (2)

Sol. $e_1=\sqrt{1-\frac{b^2}{25}}$

$e_1=\sqrt{1+\frac{b^2}{16}} \Rightarrow e_1e_2=1$

$\Rightarrow \left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right)=1 \Rightarrow b^2=9, b^2\neq 0$

$\Rightarrow$ 

$\Rightarrow$ $a = 5.\ b = 4 \Rightarrow \frac{2b^2}{a}=\frac{2(16)}{5}=\frac{32}{5}$

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. $\text{Re}\left(\frac{2z+i}{z+i}\right)+\text{Re}\left(\frac{2\bar{z}-i}{\bar{z}-i}\right)=2$ is a circle of radius r and centre (a, b), then $\frac{15ab}{r^2}$ is equal to

Answer (0)

Sol. $2\text{Re}\left(\frac{2z+i}{z+i}\right)=2$

$\text{Re}\left(\frac{2(x+iy)+i}{x+i(y+i)}\right)=1$

$\Rightarrow \text{Re}\left(\frac{(2x+i(2y+1))(x-i(y+1))}{x^2+(y+1)^2}\right)=1$

$2x^2+(2y+1)(y+1)=x^2+(y+1)^2$

$\Rightarrow x^2+y^2+y=0$

Centre $\left(0,-\frac{1}{2}\right), r=\frac{1}{2}$

$\frac{15ab}{r^2}=0$

22. Let $f(x)=\frac{x-5}{x^2-3x+2}$ if range of $f(x)$ is $(-\infty, \alpha) \cup (\beta, \infty)$ then $\alpha^2+\beta^2$ equals to

Answer (194)

Sol. $f(x)=\frac{x-5}{x^2-3x+2}$

$y(x^2-3x+2)=x-5$

$yx^2-x(3y+1)+2y+5=0$

$D>0$

$(3y+1)^2-4y(2y+5)>0$

$9y^2+1+6y-8y^2-20y>0$

$y^2-14y+1>\alpha-\beta$

$y\in(-\infty,\alpha)\cup(\beta,\infty)$

Now $\alpha+\beta=14$ $\quad\quad$ $\alpha\beta=1$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(14)^2-2=\boxed{194}$

23. If $f(\theta)=\dfrac{\tan(\tan\theta)-\tan(\sin\theta)}{\tan\theta-\sin\theta}$ is continuous at $\theta = 0$, then the value of $f(\theta)$ at $\theta = 0$ is equal to

Answer (1)

Sol. $\lim\limits_{\theta\to 0}=\dfrac{\tan(\tan\theta)-\tan(\sin\theta)}{\tan\theta-\sin\theta}$

$$=\frac{\left(\tan\theta+\frac{\tan^3\theta}{3}+\frac{2}{15}(\tan\theta)^5+\ldots\right)-\left(\sin\theta+\frac{(\sin\theta)^3}{3}+\ldots\right)}{\tan\theta-\sin\theta}$$

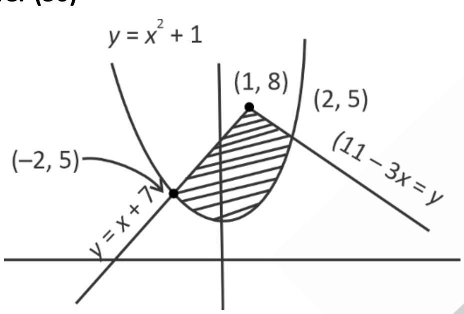
$$=\frac{(\tan\theta-\sin\theta)+\frac{1}{3}(\tan\theta-\sin\theta)(\tan^2\theta+\tan\theta.\sin\theta+\sin^2\theta)+\ldots}{\tan\theta-\sin\theta}$$

$= 1$

24. If A is the area of the region given by $x^2+1\le y\le \min(11-3x,x+7)$, then the value of $\dfrac{A}{3}$ is equal to (in square units)

Answer (50)

Sol.



$$A=\int_{-2}^{1}((x+7)-(x^2+1))dx+\int_{1}^{2}((1-3x)-(x^2+1))dx$$

$=\dfrac{50}{3}$

$\therefore\ 3A = 50$

25. If $a_1, a_2, a_3, \ldots, a_n$ are in AP, then find the value of a_n if it is given that $a_1 + a_2 + a_3 + \ldots + a_n = 700$, and $a_6 = 7$, $S_7 = 7$.

Answer (64)

Sol. $a + 5d = 7$

$\dfrac{7}{2}[2a+6d]=7$

$a + 3d = 1$

$a + 5d = 7$

$d = 3,\ a = -8$

$\dfrac{n}{2}[-16+(n-1)3]=700$

$\dfrac{n}{2}[(3n-19)]=700$

$3n^2 - 19n - 1400 = 0$

$\Rightarrow\ n = 25$

$\therefore\ a_n = a_{25} = -8 + 24 \times 3 = 64$

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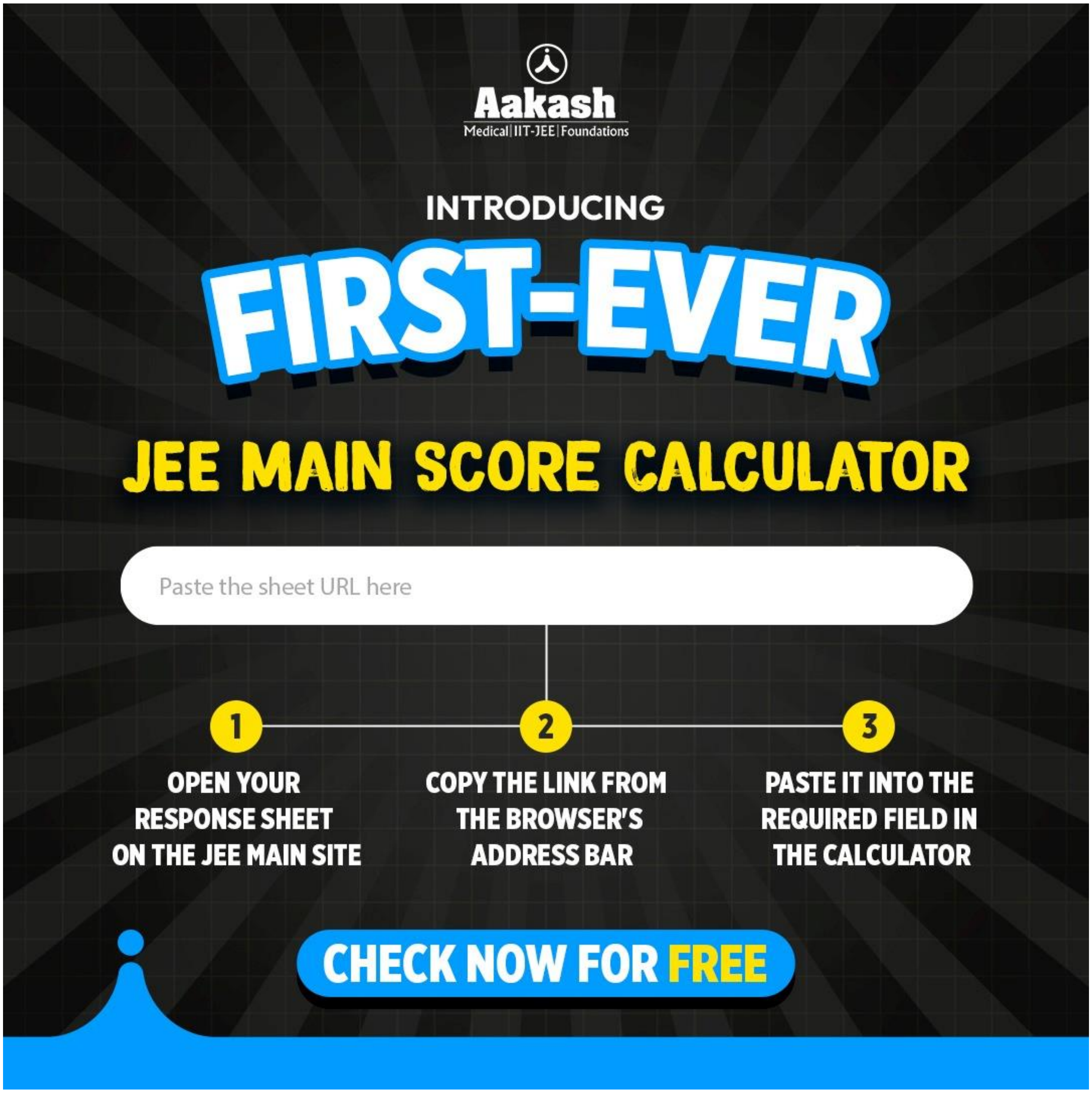

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