POLYCET - 2025

MATHEMATICS

STUDY MATERIAL

Contents

1 REAL NUMBERS

SYNOPSIS

1. **The Fundamental Theorem of Arithmetic:** Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.
2. If p is a prime number and p divides a^2, then p divides a, where a is a positive integer.
3. The sum or difference of a rational and an irrational number is irrational
4. The product and quotient of a non-zero rational and irrational number is irrational
5. There are infinitely many positive prime numbers.
6. Every positive integer different from 1 can be expressed as product of non negative powers of 2 and an odd number
7. If p is a positive prime, then $\sqrt{p}$ is an irrational number. For eg $\sqrt{2}, \sqrt{3}, \sqrt{5}, \ldots$ etc. are irrational numbers.
8. For any two positive integers a and b, HCF (a, b) × LCM $(a, b) = a \times b$.
9. HCF $(p,q,r) \times$ LCM $(p,q,r) \neq p \times q \times r$, where p, q, r are positive integers
10. LCM $(p,q,r) = \dfrac{p.q.r \text{ HCF } (p,q,r)}{\text{HCF}(p,q,r).\text{HCF}(q,r).\text{HCF}(p,r)}$
11. HCF $(p,q,r) = \dfrac{p.q.r \text{ LCM } (p,q,r)}{\text{LCM}(p,q,r).\text{LCM}(q,r).\text{LCM}(p,r)}$

SOLVED PROBLEMS

Question 1: Which of the following is an irrational number

(A) $\sqrt{4}$ (B) $\sqrt{5}$ (C) $\frac{3}{2}$ (D) $\frac{4}{3}$

Solution: (B) Since, $\sqrt{p}$ is irrational when p is prime.

Question 2: The product of prime factors of 765 is

(A) $3^2 \times 5^2 \times 17$ (B) $3^2 \times 5^3 \times 13$ (C) $3^3 \times 5 \times 17$ (D) $3^2 \times 5 \times 17$

Solution: (B) We have, $765 = 3 \times 3 \times 5 \times 17 = 3^2 \times 5 \times 17$

Question 3: If the LCM of 12 and 42 is 10 m +4, then the value of 'm' is

(A) 2 (B) 4 (C) 6 (D) 8

Solution: (D) We have, L.C.M of 12and 42 is $84 = 10(8) + 4$.

$\therefore m = 8$.

MULTIPLE CHOICE QUESTIONS

1. Which of the following is an irrational number

(A) $\sqrt{4}$ (B) $\sqrt{3}$ (C) $\frac{5}{2}$ (D) $\frac{2}{3}$

2. Which of the following is a rational number

(A) $2-\sqrt{3}$ (B) $\sqrt{2}+\sqrt{3}$ (C) $\sqrt{4}-\sqrt{25}$ (D) $\sqrt{5}-\sqrt{9}$

3. The rational number lie in between $\sqrt{2}$ and $\sqrt{3}$ is

(A) $\frac{3}{2}$ (B) $\frac{5}{2}$ (C) $\frac{1}{2}$ (D) 1

4. Which of the following rational number does not lie in between $\frac{1}{2}$ and 1

(A) $\frac{3}{5}$ (B) $\frac{7}{10}$ (C) $\frac{3}{4}$ (D) $\frac{6}{5}$

5. Which of the following is not a factor of rational number 5005

(A) 11 (B) 7 (C) 5 (D) 3

6. The product of prime factors of 3825 is

(A) $3^2\times5^2\times17$ (B) $3^2\times5^3\times13$ (C) $3^3\times5^2\times17$ (D) $3^3\times5^3\times7$

7. If $8232=2^3\times3\times7^n$ then the value of n is

(A) 1 (B) 2 (C) 3 (D) 4

8. If $156=2^2\times3\times k$ then the value k is

(A) 5 (B) 7 (C) 13 (D) 11

9. The H.C.F of $2^3\times3^2\times5$ and $2^2\times3^3\times5^2$ is

(A) $2^2\times3^3\times5^2$ (B) $2^2\times3^2\times5$ (C) $2^3\times3^2\times5$ (D) $2\times3\times5$

10. The H.C.F of $120,150$ and 210 is k^2-6, then the value of k is

(A) 5 (B) 7 (C) 13 (D) 11

11. The H.C.F of $17,23$ and 29 is

(A) 1 (B) 23 (C) 17 (D) $17\times23\times29$

12. The L.C.M of $2^3\times3\times5$ and $2^2\times5\times7$ is

(A) 1680 (B) 420 (C) 280 (D) 840

13. The product of two numbers is 1600 and their H.C.F is 5 then L.C.M is

(A) 8000 (B) 1595 (C) 320 (D) 1605

14. The L.C.M of two numbers is 216 and their H.C.F is 36, one number is 72 then second number is

(A) 108 (B) 180 (C) 156 (D) 144

15. π is a/an

(A) irrational number (B) rational number
(C) whole number (D) natural number

16. p prime number then $\sqrt{p}$

(A) irrational number (B) rational number (C) whole number (D) natural number

17. If for all values of a,b where a,b are natural numbers $\frac{a^2+b^2}{2ab}$ is

(A) irrational number (B) rational number (C) whole number (D) prime number

18. If 16380 can be expressed as p then the value of $2^2\times5\times7\times p^2\times13$ is

(A) 1 (B) 3 (C) 11 (D) 17

19. The irrational number lie in between 4 and 5 is

(A) $\sqrt{4}$ (B) $\sqrt{20}$ (C) $\sqrt{25}$ (D) $\sqrt{\frac{5}{4}}$

20. If x, y are prime numbers, then H.C.F of $x^3\,y^2$ and $x^2\,y^3$ is

(A) 1 (B) xy (C) $x^2\,y^2$ (D) $x^3\,y^3$

21. The H.C.F of $(306,657)=9$ then L.C.M of $(306,657)$ is

(A) 22338 (B) 22883 (C) 22838 (D) 22888

22. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

(A) 28 min (B) 24 min (C) 36 min (D) 16 min

23. Which of the following is true?

(A) HCF $(p,q,r)\times$ LCM $(p,q,r) \neq p\times q\times r$

(B) HCF $(p,q,r)+$ LCM $(p,q,r) = p\times q\times r$

(C) HCF $(p,q,r)-$ LCM $(p,q,r) = p\times q\times r$

(D) HCF $(p,q,r)\times$ LCM $(p,q,r) = p\times q\times r$

24. Which of the following is true?

(A) For any two positive integers a and b, HCF $(a, b) \neq$ LCM (a, b)

(B) For any two positive integers a and b, LCM (a, b) = HCF $(a, b) \times a \times b$.

(C) For any two positive integers a and b, HCF (a, b) = LCM $(a, b) \times a \times b$.

(D) For any two positive integers a and b, HCF $(a, b) \times$ LCM $(a, b) = a \times b$.

25. The product of two irrational numbers is

(A) irrational number (B) rational number

(C) some times irrational number and some times rational number (D) None

26. The reciprocal of two irrational numbers is

(A) irrational number (B) rational number

(C) some times irrational number and some times rational number (D) None

27. A prime number *p* divides a^2 where *a* is a positive integer, then

(A) p divides a (B) p does not divide a (C) p is equal to a (D) All of the above

28. Which of the following is correct?

(A) The sum or difference of a rational and an irrational number need not be irrational.

(B) The sum or difference of a rational and an irrational number is always irrational.

(C) The sum or difference of a rational and an irrational number is rational.

(D) None of the above.

29. A Physical Education Teacher wishes to distribute 60 balls and 135 bats equally among a number of boys. The greatest number receiving the gift in this way are

(A) 810 (B) 76 (C) 19 (D) 15

30. The H.C.F of $(96, 404) = 4$ then L.C.M of $(96, 404)$ is

(A) 6699 (B) 9966 (C) 9669 (D) 9696

ANSWERS

1	B	2	C	3	A	4	D	5	D	6	A	7	C	8	C	9	B	10	A
11	A	12	D	13	C	14	A	15	B	16	A	17	B	18	B	19	B	20	C
21	A	22	C	23	A	24	D	25	C	26	A	27	A	28	B	29	D	30	D

2. POLYNOMIALS

1. Polynomials of degrees 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
2. A quadratic polynomial in x with real coefficients is of the form ax^2+bx+c, where a, b, c are real numbers with $a \neq 0$
3. The zeroes of a polynomial $p(x)$ are precisely the x-coordinates of the points, where the graph of $y=p(x)$ intersects the x - axis.
4. A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.
5. If α and β are the zeros of the quadratic polynomial ax^2+bx+c, then $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$.
6. If α,β and γ are the zeros of the cubic polynomial ax^3+bx^2+cx+d, then $\alpha+\beta+\gamma=-\frac{b}{a}$, $\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}$ and $\alpha\beta\gamma=\frac{d}{a}$.
7. The quadratic polynomial whose roots are α and β is $x^2-(\alpha+\beta)x+\alpha\beta$
8. The cubic polynomial whose roots are α,β and γ is $x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma$

SOLVED PROBLEMS

Question 1: If one of the zeroes of the polynomial $(k-1)x^2+kx+1$ is -3, then the value of is k

(A) $\frac{4}{3}$ (B) $-\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

Solution: (A)

Let the given polynomial be $p(x)=(k-1)x^2+kx+1$

one of the zeroes of the polynomial $p(x)=(k-1)x^2+kx+1$ is -3, then $p(-3)=0$

$\Rightarrow (k-1)(-3)^2+k(-3)+1=0$

$\Rightarrow (k-1)9-3k+1=0$

$\Rightarrow 9k-9-3k+1=0$

$\Rightarrow 6k-8=0$

$\Rightarrow k=\frac{8}{6}$

$\Rightarrow k=\frac{4}{3}$

Question 2: A quadratic polynomial, whose zeroes are -3 and 4, is

(A) x^2-x+12 (B) x^2+x+12 (C) x^2-x-12 (D) $2x^2+2x-24$

Solution: (C)

Let ax^2+bx+c be a required polynomial whose zeroes are -3 and 4.

Then, sum of zeroes $= -3+4=1$

$i.e., -\frac{b}{a} = \frac{-(-1)}{1}$ … (i)

And product of zeroes $= -3\times 4 = -12$

$i.e., \frac{c}{a} = \frac{-12}{1}$ … (ii)

$\therefore a=1, b=-1, c=-12$

Therefore required polynomial is $x^2 - x - 12$

Question 3: If the zeroes of the quadratic polynomial $x^2+(a+1)x+b$ are 2 and -3, then

(A) $a=-7, b=-1$ (B) $a=5, b=-1$ (C) $a=2, b=-6$ (D) $a=0, b=-6$

Solution: (D) Let the given polynomial be $p(x)=x^2+(a+1)x+b$

Given that, 2 and -3 are the zeroes of the quadratic polynomial $p(x)$.

$p(2)=0$ and $p(-3)=0$

Now, $\Rightarrow (2)^2+(a+1)(2)+b=0$ $p(2)=0$

$\Rightarrow 4+2a+2+b=0$

$\Rightarrow 2a+b=-6$ …(i)

Also $\Rightarrow (-3)^2+(a+1)(-3)+b=0$ $p(-3)=0$

$\Rightarrow 9-3a-3+b=0$

$\Rightarrow -3a+b=-6$ …(ii)

By solving equations …(i) and …(ii), we get $a=0, b=-6$

Therefore, required values are $a=0$ and $b=-6$.

Question 4: The number of polynomials having zeroes as -2 and 5 is

(A) 1 (B) 2 (C) 3 (D) more than 3

Solution: (D) Let $p(x)=ax^2+bx+c$ be the required polynomial whose zeroes are -2 and 5.

Then, sum of zeroes $= -2+5=3$

$i.e., -\frac{b}{a} = \frac{-(-3)}{1}$ …(i)

And product of zeroes $= -2\times 5 = -10$

$i.e., \frac{c}{a} = \frac{-10}{1}$ …(ii)

$\therefore a=1, b=-3, c=-10$

$\therefore p(x)=x^2-3x-10$

But, we know that if we multiply/divide any polynomial by any arbitrary constant, then the .zeroes of the polynomial never change

$\therefore p(x)=kx^2-3kx-10k$ where $p(x)=\frac{x^2}{k}-\frac{3}{k}x-\frac{10}{k}$, is any zero real number and k where .is any non zero real number k

Hence, the required number of polynomials are infinite i.e., more than 3.

Question 5: If one of the zeroes of the cubic polynomial ax^3+bx^2+cx+d is zero, then the product of the other two zeroes is

(A) $-\frac{c}{a}$ (B) $\frac{c}{a}$ (C) 0 (D) $-\frac{b}{a}$

Solution: (B) Let $p(x) = ax^3 + bx^2 + cx + d$.

Given that, one of the zeroes of the cubic polynomial $p(x)$ is zero.

Let α, β and γ are the zeroes of cubic polynomial $p(x)$, where $a \neq 0$.

We know that, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$\Rightarrow 0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{c}{a}$ $[\because \alpha = 0]$

$\Rightarrow 0 + \beta\gamma + 0 = \frac{c}{a}$

$\Rightarrow \beta\gamma = \frac{c}{a}$

Hence, the product of the other two zeroes is $\frac{c}{a}$.

Question 6: If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

(A) $b - a + 1$ (B) $b - a - 1$ (C) $a - b + 1$ (D) $a - b - 1$

Solution: (A) Let $p(x) = x^3 + ax^2 + bx + c$.

Given that, one of the zeroes of the cubic polynomial $p(x)$ is -1.

Let α, β and γ are the zeroes of cubic polynomial $p(x)$.

Given $\alpha = -1$.

one of the zeroes of the cubic polynomial is $-1 \Rightarrow p(-1) = 0 \Rightarrow (-1)^3 + a(-1)^2 + b(-1) + c = 0$

$\Rightarrow -1 + a - b + c = 0 \Rightarrow c = 1 - a + b$

We know that, product of all zeroes of cubic polynomial is $\alpha\beta\gamma = -c \Rightarrow (-1)\beta\gamma = -c$

$\Rightarrow \beta\gamma = c$

Hence, the product of the other two zeroes is c = b – a + 1

Question 7: The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(A) both positive (B) both negative (C) one positive and one negative (D) both equal

Solution: (B) Let given quadratic polynomial be $p(x) = x^2 + 99x + 127$.

On comparing $p(x)$ with $ax^2 + bx + c$, we get $a = 1, b = 99$ and $c = 127$.

We know that, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-99 \pm \sqrt{99^2 - 4(1)(127)}}{2(1)}$$

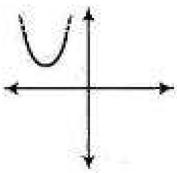
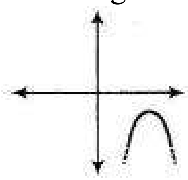
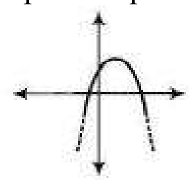
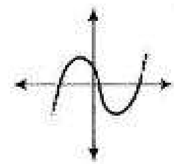
$$= \frac{-99 \pm \sqrt{9801 - 508}}{2}$$

$$= \frac{-99 \pm \sqrt{9293}}{2}$$

$$= \frac{-99 \pm 96.4}{2} = \frac{-99+96.4}{2}, \frac{-99-96.4}{2} = \frac{-2.6}{2}, \frac{-195.4}{2} = -1.3, -97.7$$

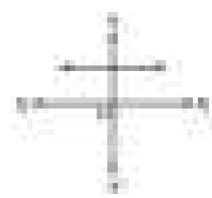
Hence, both zeroes of the given quadratic polynomial $p(x)$ are negative.

Question 8: Which of the following is not the graph of a quadratic polynomial?

(A)  (B)  (C)  (D) 

Solution: (D) For any quadratic polynomial ax^2+bx+c, $a \neq 0$, the graph of the Corresponding equation $y = ax^2+bx+c$, has one of the two shapes either open upwards like $\cup$ or open down wards like $\cap$ depending on whether $a > 0$ or $a < 0$. These curves are called parabolas. So, option (D) can not be possible.

Also, the curve of a quadratic polynomial crosses the X-axis on at most two points but in option (D) the curve crosses the X-axis on the three points, so it does not represent the quadratic polynomial.

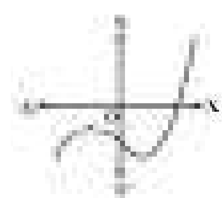


Question 9: The number of zeroes of the polynomial shown in the graph is

(A) 1 (B) 2 (C) 3 (D) 0

Solution: (D)

The graph does not intersect the $x-$ axis, so there are no zeroes in the polynomial.

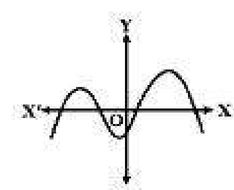


Question 10: The number of zeroes of the polynomial shown in the graph is

(A) 1 (B) 2 (C) 3 (D) 0

Solution: (A)

The graph intersects the $x-$ axis at one place, so there is one zero in the polynomial.



Question 11: The number of zeroes of the polynomial shown in the graph is

(A) 1 (B) 2 (C) 3 (D) 4

Solution: (D)

The graph intersects the $x-$ axis at four places, so there are four zeroes in the polynomial.

MULTIPLE CHOICE QUESTIONS

1. **Which of the following is not a polynomial**

   (A) $x^2+\sqrt{2}x+4$ (B) $x^2+2\sqrt{x}+4$ (C) $x^2+2x-\sqrt{2}$ (D) $\sqrt{2}\,x^2+2x+4$

2. **Which of the following is not a polynomial**

   (A) $2x^3+4x^2+5$ (B) $\frac{2}{x^3}+4x^2+4x+9$

   (C) $2x^3+4x^2+5\sqrt{x}+9$ (D) $2x^{-3}+4x^2+5$

3. **The degree of a polynomial** $4x^3-5x^2+x-1$

   (A) 1 (B) 2 (C) 3 (D) 4

4. **The degree of a quadratic polynomial is**
(A) 1 (B) 2 (C) 3 (D) 4

5. **The degree of a cubic polynomial is**
(A) 1 (B) 2 (C) 3 (D) 4

6. **The zero of a linear polynomial** $ax+b$ **is**
(A) $\frac{a}{b}$ (B) $-\frac{a}{b}$ (C) $\frac{b}{a}$ (D) $-\frac{b}{a}$

7. **If** $p(x)=x^2-5x-10$ **then the value of** $p(-2)$
(A) 1 (B) 2 (C) 3 (D) 4

8. **If** $p(x)=x^2-3x+1$ **then** $p(1)+p(-1)=$
(A) -1 (B) 0 (C) 5 (D) 4

9. **One zero of the polynomial** $p(x)=x^2+kx-8$ **is 4 then** $k=$
(A) 1 (B) 2 (C) -1 (D) -2

10. **The zeroes of a polynomial** x^2-9 **are**
(A) ± 3 (B) ± 9 (C) $0,9$ (D) ± 81

11. **The zeroes of a polynomial** x^2-2x-3 **are**
(A) $3,1$ (B) $-3,-1$ (C) $3,-1$ (D) $-3,1$

12. **The zeroes of a polynomial** $x^2-5\sqrt{2}x+12$ **are**
(A) $2\sqrt{2},3\sqrt{2}$ (B) $-2\sqrt{2},-3\sqrt{2}$ (C) $-2\sqrt{2},3\sqrt{2}$ (D) $2\sqrt{2},-3\sqrt{2}$

13. **The zero of a polynomial** $p(x)=x^2-10x+25$ **is**
(A) 5 (B) 6 (C) -5 (D) 4

14. **The zeroes of a polynomial** x^3-x^2 **are**
(A) $0,-3$ (B) $0,-1$ (C) $0,1$ (D) $1,-1$

15. **The zeroes of a polynomial** x^3-4x **are**
(A) $0,\pm\sqrt{2}$ (B) $0,\pm 1$ (C) $0,\pm 4$ (D) $0,\pm 2$

16. **The zeroes of a polynomial** $x^2+\frac{1}{6}x-2$ **are**
(A) $\frac{3}{2},\frac{4}{3}$ (B) $-\frac{3}{2},\frac{4}{3}$ (C) $\frac{3}{2},-\frac{4}{3}$ (D) $-\frac{3}{2},-\frac{4}{3}$

17. **The quadratic polynomial having zeroes** 2 **and** -3 **is**
(A) x^2-x-6 (B) x^2+x-6 (C) x^2+x+6 (D) x^2-x+6

18. **The quadratic polynomial having zeroes** $\frac{1}{4}$ **and** -1 **is**
(A) $4x^2+3x+1$ (B) $4x^2-3x+1$ (C) $4x^2-3x-1$ (D) $4x^2+3x-1$

19. **The quadratic polynomial having sum of zeroes** -3 **and product of zeroes** -10
(A) $x^2+3x+10$ (B) $x^2-3x+10$ (C) $x^2-3x-10$ (D) $x^2+3x-10$

20. **If sum of zeroes of a quadratic polynomial** ax^2+bx+c **is** 0 **then**
(A) $a=0$ (B) $b=0$ (C) $c=0$ (D) $a=c$

21. **If product of zeroes of a quadratic polynomial** ax^2+bx+c **is** 0 **then**
(A) $a=0$ (B) $b=0$ (C) $c=0$ (D) $a=c$

22. **The sum of zeroes of a quadratic polynomial** x^2-4x+3 **is**
(A) 3 (B) 4 (C) -3 (D) -4

23. **The sum of zeroes of a quadratic polynomial** x^2-4 **is**
(A) 2 (B) -2 (C) 4 (D) 0

24. **The quadratic polynomial having zeroes** 0 **and** $\sqrt{5}$ **is**
(A) $x^2+\sqrt{5}x$ (B) $x^2-\sqrt{5}x$ (C) x^2-5x (D) x^2+5x

25. **If** α,β **are zeroes of a quadratic polynomial** $\alpha+\beta=$ **then** x^2-x-6
(A) 1 (B) 2 (C) 6 (D) -1

26. **If** α,β **are zeroes of a quadratic polynomial** $\alpha^2\beta+\alpha\beta^2=$ **then** x^2+2x-8
(A) -8 (B) -2 (C) 16 (D) -16

27. **If** α,β **are zeroes of a quadratic polynomial** $\alpha^3+\beta^3=$ **then** x^2-3x+2
(A) 1 (B) 3 (C) 6 (D) 9

28. **If** α,β **are zeroes of a quadratic polynomial then** $3x^2+12x-12$
(A) $\alpha+\beta=\alpha\beta$ (B) $\alpha+\beta<\alpha\beta$ (C) $\alpha+\beta>\alpha\beta$ (D) $\alpha+\beta=-\alpha\beta$

29. **If** α,β **are zeroes of a quadratic polynomial** $\frac{1}{\alpha}+\frac{1}{\beta}=$ **then** $6x^2-5x+1$
(A) $\frac{5}{6}$ (B) $\frac{1}{6}$ (C) 5 (D) -5

30. **The sum of zeroes of a polynomial** $kx^2-(k+1)x-3$ **is** $\frac{7}{6}$ **then the value of is** k
(A) 7 (B) 6 (C) -7 (D) -6

31. **The zeroes of a polynomial** $x^2+(a+1)x+b$ **are** 3 **and** 4 **then the values of are** a,b
(A) $8,12$ (B) $8,-12$ (C) $-8,12$ (D) $-8,-12$

32. **One zero of a polynomial** $x^2-2kx+8$ **is 2 then** $k=$
(A) 3 (B) 2 (C) -4 (D) 4

33. **The sum of zeroes of a polynomial** $2x^3+kx^2-14x+8$ **is** $\frac{5}{2}$ **then** $k=$
(A) 7 (B) -2 (C) -7 (D) -5

34. **If** α,β,γ **are zeroes of a polynomial** $\alpha\beta+\beta\gamma+\gamma\alpha=$ **then** x^3+4x^2+5x-2
(A) 5 (B) -5 (C) 4 (D) -4

35. **If** α,β,γ **are zeroes of a polynomial** $\alpha+\beta+\gamma=$ **then** $2x^3+8x^2-6x-2$
(A) 5 (B) -5 (C) 4 (D) -4

36. **If** α,β,γ **are zeroes of a polynomialthen** $\alpha\beta+\beta\gamma+\gamma\alpha=0$ **and** x^3+5x^2+kx+4 **the value of is** k
(A) 2 (B) -2 (C) 0 (D) -1

37. **If** α,β,γ **are zeroes of a polynomial** $\alpha\beta\gamma=$ **then the value of** x^3+3x^2-x-2
(A) 2 (B) -2 (C) 3 (D) -1

38. **If** $p(x)=g(x)q(x)+r(x)$ **and** $\deg(p(x))=\deg(q(x))$ **then** $\deg(g(x))=$
(A) 0 (B) 1 (C) 2 (D) 3

39. **If** 0 **is the two zeroes of a polynomial** ax^3+bx^2+cx+d **then third zero is**
(A) $\frac{b}{a}$ (B) $-\frac{b}{a}$ (C) $\frac{c}{a}$ (D) $-\frac{c}{a}$

40. **If** $p(x)=x^2-5x+6, q(x)=x-2$ **and** $r(x)=0$ **then** $g(x)=$
(A) $x-3$ (B) $x-4$ (C) $x+2$ (D) $x+3$

41. **The degree of a polynomial** $p(x)=5x^3-x^2+6x-7$ **is**
(A) 1 (B) 2 (C) 3 (D) 4

42. **The coefficient of** x^5 **in** $p(x)=5x^7-6x^5+7x-6$ **is**
(A) 5 (B) 6 (C) -6 (D) 7

43. **The degree of a constant polynomial is**
(A) 1 (B) 2 (C) 3 (D) 0

44. **The degree of a linear polynomial is**
(A) 1 (B) 2 (C) 3 (D) 0

45. **The zero of a linear polynomial** $p(x)=2x-5$ **is**
(A) $\frac{2}{5}$ (B) $-\frac{2}{5}$ (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$

46. **The quadratic polynomial having zeroes** 1 **and** 3 **is**
(A) x^2+4x+3 (B) x^2-4x+3 (C) x^2-4x-3 (D) x^2+4x-3

47. **The sum of zeroes of a polynomial** $x^2+7x+10$ **is**
(A) 7 (B) 10 (C) -7 (D) -10

48. **One zero of a polynomial** $x^2-2x-15$ **is -3 then another zero is**
(A) 3 (B) 5 (C) -3 (D) -5

49. **If** $(-1,0)$ **is one point that cuts the then** x^2-3x-4 **by the curve axis** X^- **another point is**
(A) $(-4,0)$ (B) $(4,0)$ (C) $(-3,0)$ (D) $(3,0)$

50. **If the curveis** p **axis at only one point then the value of** X^- **touches the** x^2+6x+p
(A) 9 (B) -9 (C) 3 (D) -3

51. **The maximum number of terms in the polynomial** $p(x)$ **of degree** n **is**
(A) $2n$ (B) n (C) $n+1$ (D) $n-1$

52. **The quadratic polynomial having zeroes** $\sqrt{3}$ **and** $-\sqrt{3}$ **is**
(A) x^2+3 (B) x^2-3 (C) x^2+9 (D) x^2-9

53. **If the quadratic polynomial** $4x^2-4x+k$ **has only one zero then the value of k is**
(A) 3 (B) 2 (C) 1 (D) -1

54. **The minimum number of points that the cubic polynomial cuts the** X^- **axis in**
(A) 3 (B) 2 (C) 1 (D) 0

55. **The maximum number of points that the cubic polynomial cuts the** X^- **axis in**
(A) 3 (B) 2 (C) 1 (D) 0

56. **If the constant term is zero in a cubic polynomial then the product of zeroes is**
(A) 0 (B) 1 (C) 2 (D) not defined

57. **If** α,β,γ **are zeroes of a polynomial** $\alpha\beta+\beta\gamma+\gamma\alpha=$ **then** x^3+4x^2-5x-2
(A) 5 (B) -5 (C) 2 (D) -4

58. **If** α,β,γ **are zeroes of a polynomial** $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}=$ **then** x^3+4x^2-5x-2
(A) 2 (B) -2 (C) 4 (D) $-\frac{1}{2}$

59. **If two zeroes of a polynomial** x^3-5x^2+6x **are** $2,3$ **then third zero is**
(A) 0 (B) -2 (C) 1 (D) -3

60. **If two zeroes of a polynomial** $(x-1)(x^2-x-6)$ **are** $-2,3$ **then third zero is**
(A) 0 (B) 2 (C) 1 (D) −3

61. **If the coefficient of** x **is zero in a quadratic polynomial then the sum of zeroes is**
(A) 0 (B) 1 (C) 2 (D) not defined

62. **If** α,β,γ **are zeroes of a polynomial** $\alpha\beta+\beta\gamma+\gamma\alpha=$ **then** ax^3+bx^2+cx+d
(A) $\frac{b}{a}$ (B) $-\frac{b}{a}$ (C) $\frac{c}{a}$ (D) $\frac{d}{a}$

63. **If** α,β **are zeroes of a polynomial** $\frac{1}{\alpha}+\frac{1}{\beta}=$ **then** x^2-5x+4
(A) $-\frac{5}{4}$ (B) $\frac{4}{5}$ (C) $\frac{5}{4}$ (D) $-\frac{4}{5}$

64. **If the coefficient of** x^2 **is zero in a cubic polynomial then the sum of zeroes is**
(A) 0 (B) 1 (C) 2 (D) not defined

65. **If** $p(x)=g(x)q(x)+r(x)$ **and** $\deg(p(x))=5,$ **and** $\deg(q(x))=3$ **then** $\deg(g(x))=$
(A) 0 (B) 1 (C) 2 (D) 3

66. **If** $p(x)=g(x)q(x)+r(x)$ **and** $\deg(q(x))=3$ **and** $\deg(g(x))=1,$ **then** $\deg(p(x))=$
(A) 5 (B) 4 (C) 2 (D) 3

67. **If** $p(x)=g(x)q(x)+r(x)$ **and** $g(x)$ **is a factor of** $p(x)$ **then** $r(x)=$
(A) 0 (B) 1 (C) x (D) $g(x)$

68. 0 **is the reminder when** $p(x)=x^3-10x+k$ **is divided by** $(x-1)$ **then the value of is** k
(A) 43 (B) 7 (C) 9 (D) 5

69. **Ifcubic polynomial then represents a** ax^3+bx^2+cx+d **and** $a,b,c\in R$
(A) $a=0$ (B) $b=0$ (C) $a\neq 0$ (D) $d\neq 0$

70. **The product of zeroes of a cubic polynomial havingis** $3x^3-5x^2-11x-3$
(A) $\frac{5}{3}$ (B) $-\frac{5}{3}$ (C) $-\frac{11}{3}$ (D) 1

71. **The polynomial having zeroes** $0, 1$ **and** -1 **is**
(A) x^3-x^2+1 (B) x^3+x^2 (C) x^3-x^2 (D) x^3-x

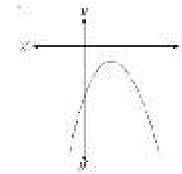
72. **The quadratic polynomial having zeroes** $\sqrt{2}+1$ **and** $\sqrt{2}-1$ **is**
(A) $x^2+2\sqrt{2}x+1$ (B) $x^2-2\sqrt{2}x-1$ (C) $x^2-2\sqrt{2}x+1$ (D) $x^2+2\sqrt{2}x-1$

73. **The quadratic polynomial having zeroes** 2 **and** -5 **is**
(A) $x^2-3x-10$ (B) $x^2+3x-10$ (C) x^2-2x-5 (D) x^2+2x+5

74. **The zeroes of a quadratic polynomial** $3x^2-10x+p$ **are reciprocals then** $p=$
(A) 10 (B) 3 (C) $\frac{1}{3}$ (D) −3

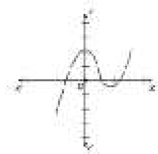
75. **If** α,β,γ **are zeroes of a polynomial** $\alpha^3+\beta^3+\gamma^3=$ **then** $p(x)=(x-1)(x-2)(x-3)$
(A) 9 (B) 27 (C) 35 (D) 36

76. The number of zeroes of the polynomial shown in the graph is



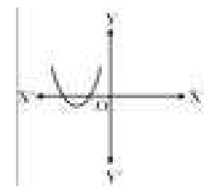
(A) 1 (B) 0 (C) 2 (D) 3

77. The number of zeroes of the polynomial shown in the graph is



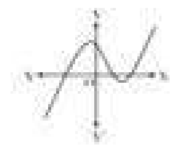
(A) 1 (B) 2 (C) 3 (D) 4

78. The number of zeroes of the polynomial shown in the graph is



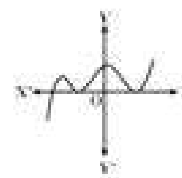
(A) 1 (B) 2
(C) 3 (D) 4

79. The number of zeroes of the polynomial shown in the graph is



(A) 1 (B) 2
(C) 3 (D) 4

80. The number of zeroes of the polynomial shown in the graph is



(A) 1 (B) 2
(C) 3 (D) 4

ANSWERS

1	B	2	C	3	C	4	B	5	C	6	D	7	D	8	D	9	D	10	A
11	C	12	A	13	A	14	C	15	D	16	B	17	B	18	D	19	D	20	B
21	C	22	B	23	D	24	B	25	A	26	C	27	D	28	A	29	C	30	B
31	C	32	A	33	D	34	A	35	D	36	C	37	A	38	A	39	B	40	A
41	C	42	C	43	D	44	A	45	C	46	B	47	C	48	B	49	B	50	A
51	C	52	B	53	C	54	C	55	A	56	A	57	B	58	B	59	A	60	C
61	A	62	C	63	C	64	A	65	C	66	B	67	A	68	C	69	C	70	D
71	C	72	C	73	B	74	B	75	D	76	B	77	C	78	B	79	C	80	C

* * *

3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

1. A pair of linear equations in two variables can be represented, and solved, by the:
(i) graphical method (ii) algebraic method

2. **Graphical Method:** The graph of a pair of linear equations in two variables is represented by two lines.
(i) If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.
(ii) If the lines coincide, then there are infinitely many solutions each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.
(iii) If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.

3. **Algebraic Methods:** We have discussed the following methods for finding the solution(s) of a pair of linear equations:
(i) Substitution Method
(ii) Elimination Method
(iii). Cross multiplication method

4. If a pair of linear equations is given by $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$; where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers, then the following situations can arise:
(i) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$: In this case, the pair of linear equations is consistent and it provides us the unique solution
(ii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$: In this case, the pair of linear equations is inconsistent and it provides no solution.
(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$: In this case, the pair of linear equations is dependent and consistent and it provides infinitely many solutions

5. There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.

SOLVED PROBLEMS

Question 1: Graphically, the pair of equations $6x-3y+10=0,\ 2x-y+9=0$ represents two lines which are
(A) intersecting at exactly one point (B) intersecting at exactly two points
(C) coincident (D) parallel

Solution: (D) The given equations are $6x-3y+10=0$

$\Rightarrow 2x - y + \frac{10}{3} = 0$ [dividingby3]…(i)

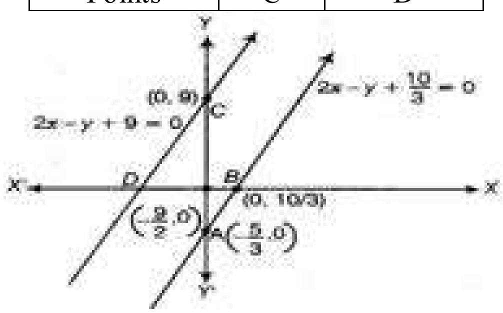
and $2x - y + 9 = 0$ …(ii)

Now, table for $2x - y + \frac{10}{3} = 0$

x	0	$-\frac{5}{3}$
$y = 2x + \frac{10}{3}$	$\frac{10}{3}$	0
Points	A	B

and table for $2x - y + 9 = 0$

x	0	$-\frac{9}{2}$
$y = 2x + 9$	9	0
Points	C	D



Hence, the pair of equations represents two parallel lines.

Question 2: The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has

(A) a unique solution (B) exactly two solutions (C) infinitely many solutions (D) no solution

Solution: (D) The given equations are $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$

Here, $a_1 = 1, b_1 = 2, c_1 = 5$ and $a_2 = -3, b_2 = -6, c_2 = 1$

Now, $\frac{a_1}{a_2} = \frac{-1}{3}, \frac{b_1}{b_2} = \frac{-2}{6} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, the pair of linear equations is inconsistent and has no solution.

Question 3: If a pair of linear equations is consistent, then the lines will be

(A) parallel (B) always coincident (C) intersecting or coincident (D) always intersecting

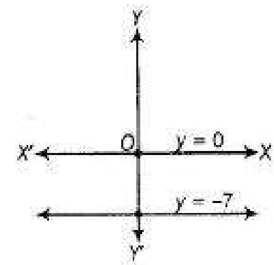
Solution: (C) Condition for a consistent pair of linear equations

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ [intersecting lines have unique solution] and $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ [coincident or dependant]

Question 4: The pair of equations $y=0$ and $y=-7$ has

(A) one solution (B) two solutions (C) infinitely many solutions

(D) no solution



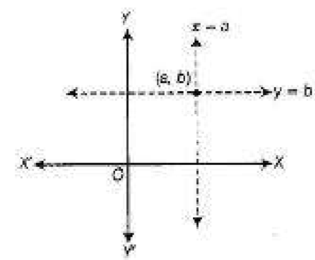
Solution: (D) The given pair of equations are $y=0$ and $y=-7$.

By graphically, both lines are parallel and having no solution

Question 5: The pair of equations $x=a$ and $y=b$ graphically represents lines which are

(A) parallel (B) intersecting at (b, a)

(C) coincident (D) intersecting at (a, b)



Solution: (D) By graphically in every condition, if

$a,b>>0; a,b<0; a>0, b<0; a<0, b>0$ but $a=b\neq 0$.

The pair of equations $x=a$ and $y=b$ graphically represents lines which are intersecting at (a,b).

If $a,b>0$;

Similarly, in all cases two lines intersect at (a,b).

Question 6: For what value of *k*, do the equations $3x-y+8=0$ and $6x-ky+16=0$ coincident.

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) -2

Solution: (C) The given lines are $3x-y+8=0$ …(i) and $6x-ky+16=0$ …(ii)

Here, $a_1=3, b_1=-1, c_1=8$ and $a_2=6, b_2=-k, c_2=16$

Condition for coincident lines is $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\Rightarrow\frac{3}{6}=\frac{-1}{-k}=\frac{8}{16}\Rightarrow\frac{1}{k}=\frac{1}{2}\Rightarrow k=2$

Question 7: If the lines given by $3x+2ky=2$ and $2x+5y=1$ are parallel, then the value of *k* is

(A) $-\frac{5}{4}$ (B) $\frac{2}{5}$ (C) $\frac{15}{4}$ (D) $\frac{3}{2}$

Solution: (C) The given lines are $3x+2ky=2$ …(i) and $2x+5y=1$ …(ii)

Here, $a_1=3, b_1=2k, c_1=-2$ and $a_2=2, b_2=5, c_2=-1$

Condition for parallel lines is $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}\Rightarrow\frac{3}{2}=\frac{2k}{5}\Rightarrow k=\frac{15}{4}$

Question 8: The value of *c* for which the pair of equations $cx-y=2$ and $6x-2y=3$ will have infinitely many solutions is

(A) 3 (B) –3 (C) -12 (D) no value

Solution: (D) The given lines are $cx-y=2$ …(i) and $6x-2y=3$ …(ii)

Here, $a_1=c, b_1=-1, c_1=-2$ and $a_2=6, b_2=-2, c_2=3$

Condition for infinitely many solutions is $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\Rightarrow\frac{c}{6}=\frac{-1}{-2}=\frac{-2}{-3}\Rightarrow\frac{c}{6}=\frac{1}{2}=\frac{2}{3}$

$\Rightarrow\frac{c}{6}=\frac{1}{2}\ \&\ \frac{c}{6}=\frac{2}{3}\Rightarrow c=3\ \&\ c=4$

Since, *c* has different values.

Hence, for no value of c the pair of equations will have infinitely many solutions.

Question 9: One equation of a pair of dependent linear equations is $-5x+7y-2=0$. The second equation can be

(A) $10x+14y+4=0$ (B) $-10x-14y+4=0$ (C) $-10x+14y+4=0$ (D) $10x-14y+4=0$

Solution: (D) The given line is $-5x+7y-2=0.$ …(i)

Here, $a_1=-5, b_1=7, c_1=-2$

Condition for dependent linear equations is $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}=\frac{1}{k} \Rightarrow \frac{-5}{a_2}=\frac{7}{b_2}=\frac{-2}{c_2}=\frac{1}{k}$

$\Rightarrow a_2=-5k, b_2=7k, c_2=-2k$ where k is any arbitrary constant.

Putting $k=2$ then $a_2=-10, b_2=14, c_2=-4$

Therefore, the required equation of line becomes $-10x+14y-4=0 \Rightarrow 10x-14y+4=0$

Question 10: A pair of linear equations which has a unique solution $x=2$ and $y=-3$ is

(A) $x+y=1$ and $2x-3y=-5$ (B) $2x+5y=-11$ and $4x+10y=-22$

(C) $2x-y=1$ and $3x+2y=0$ (D) $x-4y-14=0$ and $5x-y-13=0$

Solution: (B) If $x=2$ and $y=-3$ is a unique solution of any pair of equation, then these values must satisfy that pair of equations.

From option (B), LHS $=2(2)+5(-3)=4-15=-11=\text{RHS}$

and $\text{LHS}=4(2)+10(-3)=8-30=-22=\text{RHS}$

Question 11: If $x=a$ and $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then the values of a and b are, respectively

(A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) –1 and –3

Solution: (C) Since, $x=a$ and $y=b$ is the solution of the equations $x-y=2$ and $x+y=4$, then these values will satisfy that equations $a-b=2$,..(i) and $a+b=4$ …(ii)

On adding Eqs.(i) and (ii), we get $2a=6 \Rightarrow a=3$ and $b=4-a=4-3=1$

Question 12: Aruna has only 1 ` and 2 ` coins with her. If the total number of coins that she has is 50 and the amount of money with her is `75, then the number of 1 ` and 2 ` coins are, respectively

(A) 35 and 15 (B) 35 and 20 (C) 15 and 35 (D) 25 and 25

Solution: (D) Let number of 1 ` coins $=x$ and number of 2 ` coins $=y$

Now, by given conditions $x+y=50$ …(i)

Also, $x\times1+y\times2=75 \Rightarrow x+2y=75$ …(ii)

On subtracting Eq.(i) from Eq.(ii), we get $(x+2y)-(x+y)=75-50 \Rightarrow y=25$

When $y=25$,then $x=25$

Question13: The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are, respectively

(A) 4 and 24 (B) 5 and 30 (C) 6 and 36 (D) 3 and 24

Solution: (C) Let x yr be the present age of father and y yr be the present age of son.

Four years hence, it has relation by given condition, $x+4=4(y+4) \Rightarrow x-4y=12$ …(i)

and father's age is six times his son's age $\Rightarrow x=6y$ …(ii)

On putting the value of x from Eq.(ii) in Eq.(i), we get $6y-4y=12 \Rightarrow 2y=12 \Rightarrow y=6$

When $y=6$,then $x=36$

Hence, present age of father is 36 yr and age of son is 6 yr.

MULTIPLE CHOICE QUESTIONS

1. **5 pencils and 7 pens together cost Rs. 50 where as 7 pencils and 5pens together cost Rs.46 then the cost of 1 pencil is rupees.**

   (A) 1 (B) 2 (C) 3 (D) 4

2. **The area of a rectangle gets reduced by 80 sq. units if its length is reduced by 5 units and breadth is increased by 2 units. If we increase the length by 10 units and decrease the breadth by 5 units, the area will increases by 50Sq units, then the length of the rectangle =**

   (A) 20 units (B) 21 units (C) 19 units (D) None

3. **The cost of 2 kg apples and 1 kg of grapes on a day was found to be Rs.160. After a month, the cost of 4 Kg of apples and 2kg of grapes is the Rs.300. Which of the following equations represent this situations**

   (A) $x+2y=160, 4x+2y=300$ (B) $2x+y=160, 2x+4y=300$

   (C) $2x+y=160, 4x+2y=300$ (D) None

4. **The coach of a cricket team buys 3 bats and 6 balls for Rs.3900. Later he buys another bat and 3 more balls of the same kind for Rs.1300which of the following equations represent this situation.**

   (A) $3x+6y=3900, x+3y=1300$ (B) $6x+3y=3900, x+3y=1300$

   (C) $3x+6y=3900, 3x+y=1300$ (D) None

5. **10 students of class 10th took part in Mathematics quiz. If the number of girls is 4 more than the number of boys. Which of the following pairs represent the situation.**

   (A) $x+y=10, x-y=4$ (B) $x+y=4, x-y=10$

   (C) $4x+y=10, x-y=4$ (D) None

6. **Rubina went to a bank to withdraw Rs.2000. She asked the cashier to give the cash in Rs.50 and Rs.100 notes only. Rubina got 25 notes in all. Then number of Rs.50 those Rubina got**

   (A) 12 (B) 11 (C) 10 (D) 9

7. **The ratio of incomes of two persons is 9:7 and the ratio of their expenditures is 4:3. If each of them manages to save rs.2000 per month, their monthly incomes are**

   (A) 18000, 14000 (B) 36000, 28000 (C) 9000, 7000 (D) 27000, 21000

8. **Two angles are complementary. The larger angle is 3^0 less than twice the measure of the smaller angle. Then the greater angle is**

   (A) 54^0 (B) 36^0 (C) 41^0 (D) 59^0

9. **If** $\frac{1}{2x}-\frac{1}{y}=-1$ **and** $\frac{1}{x}+\frac{1}{2y}=8$ **where** $x\neq 0$ **and** $y\neq 0,$ **then the values of** x **and** y **are**

(A) $\frac{1}{2},\frac{1}{3}$ (B) $\frac{1}{6},\frac{1}{4}$ (C) $\frac{1}{4},\frac{1}{3}$ (D) None

10. **If** $\frac{10}{x+y}+\frac{2}{x-y}=4$ **and** $\frac{15}{x+y}-\frac{9}{x-y}=-2,$ **then** $x+y=$

(A) $\frac{15}{4}$ (B) $\frac{25}{4}$ (C) $\frac{5}{4}$ (D) None

11. **If** $\frac{5}{x-1}+\frac{1}{y-2}=2$ **and** $\frac{6}{x-1}-\frac{3}{y-2}=1,$ **then** $x=$

(A) 7 (B) 6 (C) 5 (D) 4

12. **If** $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$ **and** $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1,$ **then** $x^2=$

(A) 4 (B) 16 (C) 9 (D) none

13. **If** $\frac{xy}{x+y}=\frac{6}{5}$ **and** $\frac{xy}{y-x}=6$ **where** $x\neq 0$ **and** $y\neq 0,$ **then the value of** $y-x=$

(A) 1 (B) -1 (C) 0 (D) 2

14. **If the system of linear equations** $x-ky=2$ **and** $3x+2y=-5$ **has a unique solution, then** $k=$

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $-\frac{3}{2}$ (D) None

15. **If the system of linear equations** $3x-4y+7=0$ **and** $kx+3y-5=0$ **has no solution, then** $k=$

(A) $\frac{9}{4}$ (B) $-\frac{9}{4}$ (C) $\frac{4}{9}$ (D) $-\frac{4}{9}$

16. **If the system of linear equations** $5x+2y=k$ **and** $10x+4y=3$ **has infinitely many solutions then** $k=$

(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $-\frac{3}{2}$ (D) $-\frac{2}{3}$

17. **If the system of linear equations** $x+2y+7=0$ **and** $2x+ky+14=0$ **represent coincidental lines then** $k=$

(A) -4 (B) 4 (C) -6 (D) 6

18. **If the system of linear equations** $2x+3y=7$ **and** $2\alpha x+(\alpha+\beta)y=28$ **has infinitely many solutions then** $(\alpha,\beta)=$

(A) $(3,6)$ (B) $(2,4)$ (C) $(4,8)$ (D) None

19. **If the system of linear equations** $ax+by=c$ **and** $lx+my=n$ **have a unique solution, then which of the following is true**

(A) $am\neq bl$ (B) $am=bl$ (C) $al\neq bm$ (D) None

20. If the system of linear equations $\alpha x+3y=\alpha-3$ and $12x+\alpha y=\alpha$ has no solution, then the value of $\alpha =$

(A) 4 (B) –4 (C) 5 (D) −6

21. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Then the fraction is

(A) $\frac{5}{7}$ (B) $\frac{7}{5}$ (C) $-\frac{5}{7}$ (D) $-\frac{7}{5}$

22. On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains Rs.2000. But if he sells the T.V at 10% gain and a fridge at 5% loss, he gains Rs.1500 on the transaction. Then the original price of T.V. is Rs.

(A) 15000 (B) 25000 (C) 20000 (D) None

23. The sum of a two digit number and the number formed by interchanging the digits is 132. If 12 is added to the number, the new number becomes 5 times the sum of the digits. Then the number is

(A) 48 (B) 84 (C) 66 (D) both (A)&(B)

24. If twice the son's age in years is added to the father's age the sum is 70. But if twice the father's age is added to the son's age the sum is 95. Then the age of son is

(A) 10 years (B) 15 years (C) 5 years (D) None

25. Ten years later, A will be twice as old as B and five years ago, A was three times as old as B. then the present ages of A and B in years

(A) 50, 20 (B) 40, 30 (C) 60, 10 (D) None

26. Five years ago, Kamala was thrice old as Sonia. Ten years later, Kamala will twice as old as Sonia. Then the age of Sonia

(A) 35 (B) 30 (C) 25 (D) 20

27. The value of k for which the system of equations $x+2y=5$ and $3x+ky=-15$ has no solution

(A) 6 (B) −6 (C) $\frac{3}{2}$ (D) none

28. Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. The presnt ages of Nuri and Sonu are

(A) 50, 20 (B) 40, 30 (C) 60, 10 (D) None

29. The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits, then the number is

(A) 72 (B) 18 (C) 27 (D) 81

30. Meena went to a bank to withdraw 2000. She asked the cashier to give her 50 and 100 notes only. Meena got 25 notes in all. Number of 100 notes she received is

(A) 10 (B) 5 (C) 20 (D) 15

31. The denominator of a fraction is 4 more than twice the numerator. When both the numerator and denominator are decreased by 6 , then the denominator becomes 12 times the numerator then the fraction is

(A) $\frac{7}{18}$ (B) $\frac{5}{18}$ (C) $\frac{7}{15}$ (D) None

32. **A fraction becomes $\frac{4}{5}$, if 1 is added to both numerator and denominator. If, however, 5 is subtracted from both numerator and denominator the fraction becomes $\frac{1}{2}$, then that fraction is**

(A) $\frac{5}{9}$ (B) $\frac{7}{9}$ (C) $\frac{8}{9}$ (D) $\frac{4}{9}$

33. **3 bags and 4 pens together cost Rs. 257 where as 4 bags and 3 pens together cost Rs. 324. Then the cost of 1 book and 2 pens is**

(A) 156 (B) 157 (C) 155 (D) 154

34. **4 chairs and 3 tables cost Rs. 2100 and 5 chairs and 2 tables cost Rs. 1750. Then the cost of a chair**

(A) 150 (B) 200 (C) 250 (D) 300

35. **If $\frac{4}{x}+3y=14$ and $\frac{3}{x}-4y=23$ the value of $y=$**

(A) 2 (B) -2 (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$

36. **The larger of two supplementary angles exceeds the smaller by 18^0. Then the larger angle is**

(A) 99^0 (B) 81^0 (C) 72^0 (D) 90^0

37. **The system of linear equations $5x-4y+8=0$ and $7x+6y-9=0$**

(A) intersect at a point (B) parallel
(C) coincident (D)None

38. **The system of linear equations $9x+3y+12=0$ and $18x+6y+24=0$**

(A) intersect at a point (B) parallel
(C) coincident (D)None

39. **If a pair of linear equations in two variables is consistent, then the lines represented by two equations are**

(A) intersecting (B) parallel
(C) coincident (D) intersecting or coincident

40. **If the system of equations $2x+3y=7$ and $(a+b)x+(2a-b)y=21$ has infinitely many solutions, then $(a,b)=$**

(A) $(1,5)$ (B) $(5,1)$ (C) $(-1,5)$ (D) $(5,-1)$

41. **If the units and ten's digit of a two digit number are *y* and *x* respectively, then the number will be in the form of**

(A) $x+10y$ (B) $10x+y$ (C) xy (D) $10xy$

42. **If $x=2, y=3$ is a solution of a pair of lines $2x-3y+a=0$ and $2x+3y-b+2=0$, then the relationship between *a* and *b* is**

(A) $a=3b$ (B) $3a=b$ (C) $3a=2b$ (D) $2a=3b$

43. **The age of a son is one third the age of his mother. If the present age of mother is *x* years, then the age of the son after 12 years is**

(A) $3x+12$ (B) $12x+3$ (C) $\frac{x}{3}+12$ (D) $12x-3$

44. **If $ad \neq bc$, then the pair of linear equations $ax+by=p$ and $cx+dy=p$ has solutions?**

(A) finite (B) no (C) unique (D) infinite

45. **The pair of linear equations $3x+5y=3$, $6x+ky=8$ do not have solutions if *k* =**

(A) 12 (B) 3 (C) 8 (D) 10

46. The sum of the two digits of a two digit number is 12. The number obtained by interchanging the two digits exceeds the given number by 18, the number is
(A) 57 (B) 48 (C) 75 (D) 84

47. If the system of linear equations $(k-3)x+3y=k,\ kx+ky=12$ has infinite number of solutions then the value of *k* is
(A) 6 (B) 4 (C) 7 (D) 8

48. The condition if the pair of linear equations, $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0;$ has a unique solution is
(A) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ (B) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ (C) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (D) None

49. The condition if the pair of linear equations, $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0;$ has no solution is
(A) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ (B) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ (C) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (D) None

50. The condition if the pair of linear equations, $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0;$ has infinite solutions is
(A) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ (B) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ (C) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ (D) None

ANSWERS

1	C	2	A	3	C	4	A	5	A	6	C	7	A	8	D	9	B	10	A
11	D	12	B	13	A	14	C	15	B	16	A	17	B	18	C	19	A	20	A
21	A	22	C	23	A	24	B	25	A	26	D	27	A	28	A	29	B	30	D
31	A	32	B	33	C	34	A	35	B	36	A	37	A	38	B	39	D	40	B
41	B	42	B	43	C	44	C	45	D	46	A	47	A	48	A	49	B	50	C

* * *

4 QUADRATIC EQUATIONS

SYNOPSIS

1. A **quadratic equation** in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.
2. A real number α a is said to be a **zero or root** of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.
3. If we can factorise $ax^2 + bx + c = 0$, $a \neq 0$ into a product of two linear factors, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating each factor to zero.
4. **Quadratic formula:** The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ provided $b^2 - 4ac \geq 0$
5. Nature of the roots of quadratic equation $ax^2 + bx + c = 0, a \neq 0$, depends upon the value of $b^2 - 4ac$, which is known as the discriminant of the quadratic equation.
6. A quadratic equation $ax^2 + bx + c = 0$ has
   (i) two distinct real roots, if $b^2 - 4ac > 0$,
   (ii) two equal roots (i.e., coincident roots), if $b^2 - 4ac = 0$, and
   (iii) no real roots, if $b^2 - 4ac < 0$.
7. A quadratic equation have at most 2 zeros.
8. If α and β are the roots of a quadratic equation $ax^2 + bx + c = 0$ then the sum of roots is $\alpha + \beta = -\dfrac{b}{a}$ and product of the roots is $\alpha\beta = \dfrac{c}{a}$.
9. The quadratic equation whose roots are α and β is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

SOLVED PROBLEMS

Question 1: Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (B) $-2x^2 = (5 - x)\left(2x - \dfrac{2}{5}\right)$

(C) $(k + 1)x^2 + \dfrac{3}{2}x = 7$, where $k = -1$ (D) $x^3 - x^2 = (x - 1)^3$

Solution:(D)

(A) The given equation is $x^2 + 2x + 1 = (4 - x)^2 + 3$
$\Rightarrow x^2 + 2x + 1 = 16 + x^2 - 8x + 3$
$\Rightarrow 10x - 18 = 0$
Which is not of the form $ax^2 + bx + c = 0,\ a \neq 0$.
Thus, the equation is not a quadratic equation

(B) The given equation is $-2x^2 = (5 - x)\left(2x - \dfrac{2}{5}\right)$
$\Rightarrow -2x^2 = 10x - 2x^2 + \dfrac{2x}{5} - 2$
$\Rightarrow 52x - 10 = 0$
Which is not of the form $ax^2 + bx + c = 0,\ a \neq 0$.
Thus, the equation is not a quadratic equation.

(C) The given equation is $(k+1)x^2+\frac{3}{2}x=7$, where $k=-1$

$\Rightarrow (-1+1)x^2+\frac{3}{2}x=7$

$\Rightarrow 3x-14=0$

Which is not of the form $ax^2+bx+c=0,\ a\neq 0$.

Thus, the equation is not a quadratic equation

(D) The given equation is $x^3-x^2=(x-1)^3$

$\Rightarrow x^3-x^2=x^3-3x^2+3x-1$

$\Rightarrow 2x^2-3x+1=0$

Which is of the form $ax^2+bx+c=0,\ a\neq 0$.

Thus, the equation is a quadratic equation

Question 2: Which of the following is a quadratic equation?

(A) $2(x-1)^2=4x^2-2x+1$ (B) $2x-x^2=x^2+5$

(C) $(\sqrt{2}x+\sqrt{3})^2=3x^2-5x$ (D) $(x^2+2x)^2=x^4+3+4x^2$

Solution: (D)

(A) The given equation is $2(x-1)^2=4x^2-2x+1$

$\Rightarrow 2(x^2-2x+1)=4x^2-2x+1$

$\Rightarrow 2x^2-4x+2=4x^2-2x+1$

$\Rightarrow 2x^2+2x-1=0$

Which is of the form $ax^2+bx+c=0,\ a\neq 0$.

Thus, the equation is a quadratic equation

(B) The given equation is $2x-x^2=x^2+5$

$\Rightarrow 2x^2-2x+5=0$

Which is of the form $ax^2+bx+c=0,\ a\neq 0$.

Thus, the equation is a quadratic equation.

(C) The given equation is $(\sqrt{2}x+\sqrt{3})^2=3x^2-5x$

$\Rightarrow 2x^2+3+2\sqrt{6}x=3x^2-5x$

$\Rightarrow x^2-(5+2\sqrt{6})x-3=0$

Which is of the form $ax^2+bx+c=0,\ a\neq 0$.

Thus, the equation is a quadratic equation

(D) The given equation is $(x^2+2x)^2=x^4+3+4x^2$

$\Rightarrow x^4+4x^2+4x^3=x^4+4x^2+3$

$\Rightarrow 4x^3-3=0$

Which is not of the form $ax^2+bx+c=0,\ a\neq 0$.

Thus, the equation is not a quadratic equation

Question 3: Which of the following equations has 2 as a root?

(A) $x^2-4x+5=0$ (B) $x^2-3x+12=0$

(C) $2x^2-7x+6=0$ (D) $3x^2-6x-2=0$

Solution: (C)

(A) Substitute $x=2$ in the equation $x^2-4x+5=0$, we get $2^2-4(2)+5=4-8+5=1\neq 0$

So, $x=2$ is not a root of the equation $x^2-4x+5=0$

(B) Substitute $x=2$ in the equation $x^2-3x+12=0$, we get

$2^2-3(2)+12=4-6+12=10\neq 0$

So, $x=2$ is not a root of the equation $x^2-3x+12=0$

(C) Substitute $x=2$ in the equation $2x^2-7x+6=0$, we get $2(2)^2-7(2)+6=8-14+6=0$

So, $x=2$ is a root of the equation $2x^2-7x+6=0$

(D) Substitute $x = 2$ in the equation $3x^2 - 6x - 2 = 0$, we get

$3(2)^2 - 6(2) - 2 = 12 - 12 - 2 = -2 \neq 0$

So, $x = 2$ is not a root of the equation $3x^2 - 6x - 2 = 0$

Question 4: If $\frac{1}{2}$ is a root of the equation $x^2 - kx + \frac{5}{4} = 0$, then the value of k is

(A) 2 (B) -2 (C) 3 (D) -3

Solution: (C) Since, $\frac{1}{2}$ is a root of the quadratic equation $x^2 - kx + \frac{5}{4} = 0$.

Then $\left(\frac{1}{2}\right)^2 - k\left(\frac{1}{2}\right) + \frac{5}{4} = 0 \Rightarrow \frac{1}{4} - \frac{k}{2} + \frac{5}{4} = 0 \Rightarrow \frac{1+5}{4} = \frac{k}{2} \Rightarrow \frac{k}{2} = \frac{6}{4} \Rightarrow \frac{k}{2} = \frac{3}{2} \Rightarrow k = 3$

Question 5: Which of the following equations has the sum of its roots as 3?

(A) $2x^2 - 3x + 6 = 0$ (B) $-x^2 + 3x - 3 = 0$ (C) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (D) $3x^2 - 3x + 3 = 0$

Solution: (B)

(A) Given that, $2x^2 - 3x + 6 = 0$

On comparing with $ax^2 + bx + c = 0$, we get, $a = 2, b = -3$ and $c = 6$.

Therefore, sum of roots is $= -\frac{b}{a} = \frac{-(-3)}{2} = \frac{3}{2}$

So, sum of roots of the quadratic equation $2x^2 - 3x + 6 = 0$ is not 3, so it is not the answer.

(B) Given that, $-x^2 + 3x - 3 = 0$

On comparing with $ax^2 + bx + c = 0$, we get, $a = -1, b = 3$ and $c = -3$.

Therefore, sum of roots is $= -\frac{b}{a} = \frac{-3}{-1} = 3$

So, sum of roots of the quadratic equation $-x^2 + 3x - 3 = 0$ is 3, so it is the answer.

(C) Given that, $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$

On comparing with $ax^2 + bx + c = 0$, we get, $a = \sqrt{2}, b = -\frac{3}{\sqrt{2}}$ and $c = 1$.

Therefore, sum of roots is $= -\frac{b}{a} = \frac{-(-3/\sqrt{2})}{\sqrt{2}} = \frac{3}{2}$

So, sum of roots of the quadratic equation $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ is not 3, so it is not the answer.

(D) Given that, $3x^2 - 3x + 3 = 0$

On comparing with $ax^2 + bx + c = 0$, we get, $a = 3, b = -3$ and $c = 3$.

Therefore, sum of roots is $= -\frac{b}{a} = \frac{-(-3)}{3} = \frac{3}{3} = 1$

So, sum of roots of the quadratic equation $2x^2 - 3x + 6 = 0$ is not 3, so it is not the answer.

Question 6: Value (s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is/are

(A) 0 (B) 4 (C) 8 (D) 0, 8

Solution: (D)

Given equation is $2x^2 - kx + k = 0$

On comparing with $ax^2 + bx + c = 0$, we get, $a = 2, b = -k$ and $c = k$.

For equal roots, the discriminant must be zero *i.e.*, $b^2 - 4ac = 0$

$\Rightarrow (-k)^2 - 4(2)(k) = 0$

$\Rightarrow k^2 - 8k = 0$
$\Rightarrow k(k-8) = 0$
$\Rightarrow k = 0, k = 8$

Hence, the required values of k are 0 and 8.

Question 7: The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(A) two distinct real roots (B) two equal real roots
(C) no real roots (D) more than 2 real roots

Solution: (C)

Given equation is $2x^2 - \sqrt{5}x + 1 = 0$

On comparing with $ax^2 + bx + c = 0,$ we get, $a = 2, b = -\sqrt{5}$ and $c = 1.$

The discriminant is $D = b^2 - 4ac$

$D = (-\sqrt{5})^2 - 4(2)(1) = 5 - 8 = -3 < 0$

Since, the discriminant is negative, therefore the quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has no real roots.

Question 8: Which of the following equations has two distinct real roots?

(A) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (B) $x^2 + x - 5 = 0$
(C) $x^2 + 3x + 2\sqrt{2} = 0$ (D) $5x^2 - 3x + 1 = 0$

Solution:(B)

(A) Given equation is $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

On comparing with $ax^2 + bx + c = 0,$ we get, $a = 2, b = -3\sqrt{2}$ and $c = \frac{9}{4}.$

The discriminant is $D = b^2 - 4ac$

$D = (-3\sqrt{2})^2 - 4(2)\left(\frac{9}{4}\right) = 18 - 18 = 0$

Since, the discriminant is zero, therefore the quadratic equation $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ has real and equal roots.

(B) Given equation is $x^2 + x - 5 = 0$

On comparing with $ax^2 + bx + c = 0,$ we get, $a = 1, b = 1$ and $c = -5.$

The discriminant is $D = b^2 - 4ac$

$D = (1)^2 - 4(1)(-5) = 1 + 20 = 21 > 0$

Since, the discriminant is positive, therefore the quadratic equation $x^2 + x - 5 = 0$ has real and distinct roots.

(C) Given equation is $x^2 + 3x + 2\sqrt{2} = 0$

On comparing with $ax^2 + bx + c = 0,$ we get, $a = 1, b = 3$ and $c = 2\sqrt{2}.$

The discriminant is $D = b^2 - 4ac$

$D = (3)^2 - 4(1)(2\sqrt{2}) = 9 - 8\sqrt{2} < 0$

Since, the discriminant is negative, therefore the quadratic equation $x^2 + 3x + 2\sqrt{2} = 0$ has no real roots.

(D) Given equation is $5x^2 - 3x + 1 = 0$

On comparing with $ax^2 + bx + c = 0,$ we get, $a = 5, b = -3$ and $c = 1.$

The discriminant is $D = b^2 - 4ac$

$D = (3)^2 - 4(5)(1) = 9 - 20 = -11 < 0$

Since, the discriminant is negative, therefore the quadratic equation $5x^2 - 3x + 1 = 0$ has no real roots.

Question 9: Which of the following equations has no real roots?
(A) $x^2-4x+3\sqrt{2}=0$ (B) $x^2+4x-3\sqrt{2}=0$ (C) $x^2-4x-3\sqrt{2}=0$ (D) $3x^2+4\sqrt{3}x+4=0$
Solution:(A)

(A) Given equation is $x^2-4x+3\sqrt{2}=0$
On comparing with $ax^2+bx+c=0,$ we get, $a=1, b=-4$ and $c=3\sqrt{2}.$
The discriminant is $D=b^2-4ac$
$D=(-4)^2-4(1)(3\sqrt{2})=16-12\sqrt{2}<0$
Since, the discriminant is negative, therefore the quadratic equation $x^2-4x+3\sqrt{2}=0$ has no real roots.

(B) Given equation is $x^2+4x-3\sqrt{2}=0$
On comparing with $ax^2+bx+c=0,$ we get, $a=1, b=4$ and $c=-3\sqrt{2}.$
The discriminant is $D=b^2-4ac$
$D=(4)^2-4(1)(-3\sqrt{2})=16+12\sqrt{2}>0$
Since, the discriminant is positive, therefore the quadratic equation $x^2+4x-3\sqrt{2}=0$ has real and distinct roots.

(C) Given equation is $x^2-4x-3\sqrt{2}=0$
On comparing with $ax^2+bx+c=0,$ we get, $a=1, b=-4$ and $c=-3\sqrt{2}.$
The discriminant is $D=b^2-4ac$
$D=(-4)^2-4(1)(-3\sqrt{2})=16+12\sqrt{2}>0$
Since, the discriminant is positive, therefore the quadratic equation $x^2-4x-3\sqrt{2}=0$ has real and distinct roots.

(D) Given equation is $3x^2+4\sqrt{3}x+4=0$
On comparing with $ax^2+bx+c=0,$ we get, $a=3, b=4\sqrt{3}$ and $c=4.$
The discriminant is $D=b^2-4ac$
$D=(4\sqrt{3})^2-4(3)(4)=48-48=0$
Since, the discriminant is zero, therefore the quadratic equation $3x^2+4\sqrt{3}x+4=0$ has real and equal roots.

Question 10: If $(m+1)x^3+6x^2+5x=16$ represents the quadratic equation then the value of m is
(A) 1 (B) -1 (C) 2 (D) 0
Solution: (B)
The given equation contains 3 as exponent of the highest degree term, hence degree of the equation is $(m+1)x^3+6x^2+5x=16$ is 3
Since $(m+1)x^3+6x^2+5x=16$ represents the quadratic equation, it must have degree 2 only when $m+1=0$
$\Rightarrow m=-1.$

Question 11: The roots of the equation $3x^2+2x-1=0$ are
(A) $-1, \frac{1}{3}$ (B) $-1, -\frac{1}{3}$ (C) $3, -1$ (D) $-3, -1$
Solution: (A)
If α and β are the roots of the equation $\alpha\beta=\frac{c}{a}$. and $\alpha+\beta=-\frac{b}{a}$ then $3x^2+2x-1=0,$
On comparing with $ax^2+bx+c=0,$ we get, $a=3, b=2$ and $c=-1.$
By inspection it is observed that $\alpha=-1$ and $\beta=\frac{1}{3}$

Question 12: The sum of roots of the equation $3x^2+2x-1=0$ is

(A) $-\frac{2}{3}$ (B) $-\frac{4}{3}$ (C) 2 (D) -4

Solution: (A)

If α and β are the roots of the equation $\alpha\beta=\frac{c}{a}$. and $\alpha+\beta=-\frac{b}{a}$ then $3x^2+2x-1=0$,

On comparing with $ax^2+bx+c=0$, we get, $a=3, b=2$ and $c=-1$.

By inspection it is observed that $\alpha+\beta=-\frac{b}{a}=\frac{-2}{3}$

MULTIPLE CHOICE QUESTIONS

1. **Which of the following is a Q.E**

   (A) $5+\frac{3}{x}=x^2$ (B) $x^2+\frac{1}{x^2}=\frac{17}{4}$

   (C) $x(x+3)=6x+3$ (D) $x(2x+3)=2x^2-7$

2. **Which of the following is a Q.E**

   (A) $x^2-6x-4=0$ (B) $2x^2=7x$ (C) $(2x+1)(3x+2)=0$ (D) All the above

3. **Which of the following is not a Q.E**

   (A) $x(x-3)=x^2+7$ (B) $x(x-5)=2x^2+4$

   (C) $(2x+1)(3x+2)=0$ (D) All the above

4. **For what value of is not a Q.E** $ax^2+bx+c=0$ a,

   (A) 1 (B) 2 (C) 3 (D) 0

5. **Which of the following is a Q.E**

   (A) $(x+3)^3=x+4$ (B) $(x-2)^2+1=2x-3$

   (C) $x(x+1)+8=(x+2)(x-2)$ (D) $(x+2)^2-(x-2)^2=0$

6. **If** $(m+1)x^3+6x^2+5x=16$ **represents the quadratic equation then the value of** m **is**

   (A) 1 (B) -1 (C) 2 (D) 0

7. **If** $ax(x^2-4)+dx=2x^3+bx^2+10, b\neq 0$ **represents the Q.E then the value of** a **is**

   (A) 1 (B) -1 (C) 2 (D) 0

8. **The product of two consecutive positive numbers is** 132 **then the Q.E to find the numbers is**

   (A) $x^2+x-132=0$ (B) $x^2-x+132=0$ (C) $x^2-x-132=0$ (D) $x^2+x+132=0$

9. **The product of two consecutive odd numbers is** 399 **then the Q.E to find the numbers is**

   (A) $x^2+2x-399=0$ (B) $x^2+2x+399=0$ (C) $x^2+x-399=0$ (D) $x^2-2x+399=0$

10. **The product of two consecutive even numbers is** 120 **then the Q.E to find the numbers is**

    (A) $x^2+4x-120=0$ (B) $x^2+2x-120=0$ (C) $x^2-4x+120=0$ (D) $x^2-2x+120=0$

11. **The sum of a number and its reciprocal is** $\frac{5}{2}$ **represents by the Q.E is**

    (A) $2x^2-5x+1=0$ (B) $2x^2-5x+2=0$ (C) $2x^2+5x+2=0$ (D) $5x^2-2x+5=0$

12. **The sum of a number and its reciprocal is 2 represents by the Q.E is**
(A) $x^2-2x+1=0$ (B) $x^2+2x+1=0$ (C) $x^2+2x-1=0$ (D) $x^2+2x+2=0$

13. **The sum of squares of two consecutive odd numbers is 290 then the Q.E to find the numbers is**
(A) $x^2+(x+2)^2=290$ (B) $x^2+(x+2)^2=290^2$
(C) $x^2-(x+2)^2=290$ (D) $x^2-(x+2)^2=290^2$

14. **The sum of a number and its square is 56 then the Q.E is**
(A) $x^2+2x=56$ (B) $2x^2+x=56$ (C) $x^2+x=56$ (D) $x^2-x=56$

15. **The present age of a father is twice his daughter. After four years the product of their ages is 306 then the Q.E for this data is**
(A) $3x^2-14x-162=0$ (B) $3x^2-14x+145=0$
(C) $3x^2+28x-306=0$ (D) $x^2+6x-145=0$

16. **The difference of two numbers is 5, the sum of their squares is 325 then the Q.E to find the big number is**
(A) $x^2+(x+5)^2=325$ (B) $x^2+(x-5)^2=325$
(C) $x^2-(x+5)^2=325$ (D) $x^2-(x+5)^2=325$

17. **The roots of the Q.E $(x-4)(x+2)=0$ are**
(A) $-4,2$ (B) $-4,-2$ (C) $4,2$ (D) $4,-2$

18. **The roots of the Q.E $x^2-5x+6=0$ are**
(A) $-3,2$ (B) $-3,-2$ (C) $3,2$ (D) $3,-2$

19. **The roots of the Q.E $2x^2-6x=0$ are**
(A) $-3,0$ (B) $3,0$ (C) $6,2$ (D) $0,2$

20. **If α,β are the roots of the Q.E $x^2+6x+5=0$ then $\alpha+\beta=$**
(A) 5 (B) -6 (C) 6 (D) 1

21. **If α,β are the roots of the Q.E $x^2-5x+6=0$ then $\alpha-\beta=$**
(A) 5 (B) 3 (C) 1 (D) -2

22. **If α,β are the roots of the Q.E $x^2-3x-10=0$ then $\alpha^2+\beta^2=$**
(A) 25 (B) 10 (C) 21 (D) 29

23. **If α,β are the roots of the Q.E $x^2-3x-4=0$ then $\alpha^3+\beta^3=$**
(A) 63 (B) 64 (C) -1 (D) 17

24. **If α,β are the roots of the Q.E $x^2+4x+4=0$ then**
(A) $\alpha=\beta$ (B) $\alpha=-2,\beta=-2$ (C) $\alpha+\beta=-4$ (D) All the above

25. **If α,β are the roots of the Q.E $x^2-6x+8=0$ then $\alpha\beta=$**
(A) 6 (B) -6 (C) -8 (D) 8

26. **One root of the Q.E $2x^2-5x+3=0$ is**
(A) -1 (B) 1 (C) 0 (D) 2

27. **If one root of the Q.E 4 is $x^2+2kx+16=0$ then the value of k is**
(A) 4 (B) -4 (C) 16 (D) 32

28. **If one root of the Q.E $\sqrt{2}$ is $x^2+2\sqrt{2}\,x-k=0$ then the value of k is**
(A) 6 (B) -6 (C) $2\sqrt{2}$ (D) $-2\sqrt{2}$

29. **The Q.Es $ax^2+ax+8=0$ and $x^2+x+c=0$ have a common root 1 then the value of $a.c$ is**
(A) 8 (B) 4 (C) -8 (D) -4

30. **For any value of a which of the following is one root of the Q.E** $(a+2)x^2-ax-2=0$

(A) 0 (B) 2 (C) -1 (D) 1

31. **If the roots of the Q.E $c=$ are equal then** $ax^2+bx+c=0$

(A) $-\frac{b}{2a}$ (B) $\frac{b}{2a}$ (C) $-\frac{b^2}{4a}$ (D) $\frac{b^2}{4a}$

32. **The discriminate of the Q.E** $ax^2+bx+c=0$ **is**

(A) $b-4ac$ (B) b^2-4c (C) b^2-4ac (D) b^2+4ac

33. **If the roots of the Q.Eare equal then** $ax^2+bx+c=0$

(A) $b^2-4ac\le 0$ (B) $b^2-4ac<0$ (C) $b^2-4ac>0$ (D) $b^2-4ac=0$

34. **If the roots of the Q.E are equal then** $ax^2+bx+c=0$ **one root is**

(A) $-\frac{b}{2a}$ (B) $\frac{b}{2a}$ (C) $-\frac{b^2}{4a}$ (D) $\frac{b^2}{4a}$

35. **The product of the digits in a two digit number is 6, if we add 9 to the number then the digits may interchanged then the number is**

(A) 16 (B) 23 (C) 32 (D) 61

36. **In a right angled triangle one side is 3 cm more than the other side and the hypotenuse is 15 cm then which of the following Q.E is used to find the small side**

(A) $3x^2+6x-108=0$ (B) $x^2+6x-108=0$

(C) $x^2+3x-108=0$ (D) $2x^2+3x+108=0$

37. **The Q.E used to find the two numbers if their sum is 27 and product is 182**

(A) $x(x-27)=182$ (B) $x(x+27)=182$

(C) $x(27-x)=182$ (D) $x(27-x)=182(x+27)$

38. **The condition that the Q.E $3x^2+6x+k=0$ has real and distinct roots is**

(A) $k<3$ (B) $k>3$ (C) $k=3$ (D) $k>4$

39. **The condition that the Q.E $x^2+kx-25=0$ has real roots is**

(A) $k^2-100=0$ (B) $k^2+100<0$ (C) $k^2+100>0$ (D) $k^2+100\ge 0$

40. **The maximum value of p to find the real roots of the Q.E is** $2x^2-8x+p=0$

(A) 8 (B) -8 (C) 64 (D) -64

41. **In a triangle base is 4 cm more than the height and the area is 48 sq.cm then which of the following Q.E is used to find the height**

(A) $x^2+4x=96$ (B) $x^2+4x-96=0$ (C) $\frac{1}{2}x(x+4)=48$ (D) All the above

42. **If the roots of the Q.E $kx^2-6x+9=0$ are not real then**

(A) $k=0$ (B) $k<1$ (C) $k>1$ (D) $k^2-1=0$

43. **If the roots of the Q.E $3x^2+6x+k=0$ are not real**

(A) $k<0$ (B) $k<3$ (C) $k>3$ (D) $k=3$

44. **If the Q.E $2x^2+kx+3=0$ has two real and equal roots then the value ofis k**

(A) 24 (B) $\pm 6\sqrt{2}$ (C) $\pm 2\sqrt{3}$ (D) $\pm 2\sqrt{6}$

45. **If the Q.E $kx(x-2)+6=0$ has two real and equal roots then the value ofis k**

(A) 2 (B) 6 (C) 4 (D) -6

46. If one root of the Q.E -3 is $x^2-k^2=0$ then the other root is

(A) 9 (B) 3 (C) $\sqrt{3}$ (D) $-\sqrt{3}$

47. Which of the following Q.E has two equal roots

(A) $x^2+4x+4=0$ (B) $x^2-4x-4=0$ (C) $x^2+3x+9=0$ (D) $x^2+4x+8=0$

48. Which of the following Q.E has two real and distinct roots

(A) $2x^2-4x+6=0$ (B) $2x^2+4x+6=0$ (C) $2x^2-6x+3=0$ (D) $2x^2+6x+8=0$

49. The nature of the roots of the Q.E $2x^2-3x+5=0$ is

(A) real and distinct (B) real and equal (C) not real (D) None

50. The nature of the roots of the Q.E $3x^2-4\sqrt{3}x+4=0$ is

(A) real and distinct (B) real and equal (C) not real (D) None

51. The nature of the roots of the Q.E $2x^2+6x+3=0$ is

(A) real and distinct (B) real and equal (C) not real (D) None

52. The roots of the Q.E $ax^2+bx+c=0$ are

(A) $\frac{-b\pm\sqrt{b^2-4ac}}{2}$ (B) $\frac{-b\pm\sqrt{b^2+4ac}}{2a}$ (C) $\frac{b\pm\sqrt{b^2-4ac}}{2a}$ (D) $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

53. If the roots of the Q.E $ax^2+bx+c=0$ are real and equal then

(A) $b^2>4ac$ (B) $b^2<4ac$ (C) $b^2=4ac$ (D) $a^2=b^2+c^2$

54. If the discriminate of the Q.E $ax^2+bx+c=0$ is $b^2-4ac>0$ then the roots are

(A) real and distinct (B) real and equal (C) not real (D) None

55. If the discriminate of the Q.E $ax^2+bx+c=0$ is $b^2-4ac<0$ then the roots are

(A) real and distinct (B) real and equal (C) not real (D) None

56. If the discriminate of the Q.E $ax^2+bx+c=0$ is $b^2-4ac=0$ then the roots are

(A) real and distinct (B) real and equal (C) not real (D) None

57. The discriminate of the Q.E $2x^2-4x+3=0$ is

(A) -4 (B) -8 (C) 16 (D) 40

58. The discriminate of the Q.E $\sqrt{3}x^2-6x+12\sqrt{3}=0$ is

(A) $12\sqrt{3}$ (B) 72 (C) 36 (D) -108

59. If the discriminate of the Q.E $x^2-3x-k=0$ is 25 then $k=$

(A) -4 (B) 4 (C) 9 (D) -9

60. If one root of the Q.E 2 is $3x^2-6x=0$ then then the other root is

(A) 0 (B) 3 (C) 6 (D) -2

61. If one root of the Q.E 1 is $3x^2-5x+2=0$ then then the other root is

(A) $-\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) -1

62. Which of the following is true for the Q.E $\sqrt{3}x^2+11x+6\sqrt{3}=0$

(A) $3\sqrt{3}$ is a root (B) real and equal (C) not a Q.E (D) $-3\sqrt{3}$ is a root

63. Which of the following is not true for the Q.E $x^2-x-20=0$

(A) -4 and are the roots 5 (B) real and distinct
(C) real and equal (D) (A) and (B)

64. **If the roots of the Q.E $ax^2+2x+a=0$ are equal then**

(A) $a=\pm 1$ (B) $a=0$ (C) $a=0,-1$ (D) $a=1,0$

65. **If the roots of the Q.E $x^2+2x+(k^2+1)=0$ are equal then the value of k is**

(A) 0 (B) 1 (C) 2 (D) 3

66. **If the roots of the Q.E $x^2+4x+k=0$ are real and distinct then**

(A) $k>4$ (B) $k<4$ (C) $k\geq 4$ (D) $k\leq 4$

67. **If the Q.E $x^2+6x+\lambda=0$ is a perfect square then the value of λ is**

(A) 3 (B) 6 (C) 9 (D) 36

68. **If the Q.E $4x^2+4\lambda x+25=0$ is a perfect square then the value of λ is**

(A) 2 (B) 16 (C) 4 (D) ± 5

69. **If the Q.E $3x^2-4\lambda x+4=0$ is a perfect square then the value of λ is**

(A) $\sqrt{2}$ (B) 3 (C) 4 (D) $\sqrt{3}$

70. **The discriminate of the Q.E $(2x+3)^2=0$ is**

(A) 0 (B) -3 (C) 1 (D) 2

71. **The discriminate of the Q.E $3x^2+2\sqrt{5}x-5=0$ is**

(A) 20 (B) -40 (C) 40 (D) 80

72. **The roots of the Q.E $(3x-2)^2=-2(3x-2)^2$ are**

(A) $\pm\frac{2}{3}$ (B) $\frac{2}{3},\frac{2}{3}$ (C) $\frac{3}{2},\frac{3}{2}$ (D) $-\frac{2}{3},-\frac{2}{3}$

73. **The roots of the Q.E $3(x-4)^2=(x-4)^2+8$ are**

(A) ± 2 (B) ± 4 (C) $-2,-6$ (D) $2,6$

74. **The roots of the Q.E $(x+2)^2-9=0$ are**

(A) 1 (B) -5 (C) 5 (D) (A) and (B)

75. **The roots of the Q.E $x^2-4x+2=0$ are**

(A) $2\pm 2\sqrt{2}$ (B) $2\pm\sqrt{2}$ (C) $4\pm\sqrt{2}$ (D) $2\pm\sqrt{3}$

76. **The roots of the Q.E $x^2+4x-4=0$ are**

(A) $-2\pm 2\sqrt{2}$ (B) $2\pm 2\sqrt{2}$ (C) ± 2 (D) $-2,-2$

77. **The roots of the Q.E $3x^2-6x+2=0$ are**

(A) $3\pm\sqrt{3}$ (B) $\frac{3\pm\sqrt{3}}{2}$ (C) $\frac{3\pm\sqrt{3}}{3}$ (D) $\frac{3\pm\sqrt{3}}{6}$

78. **If one root of the Q.E $\frac{3+\sqrt{5}}{2}$ is $x^2-3x+1=0$ then the other root is**

(A) $\frac{-3-\sqrt{5}}{2}$ (B) $3+\sqrt{5}$ (C) $3-\sqrt{5}$ (D) $\frac{3-\sqrt{5}}{2}$

79. **The number of diagonals of a polygon having $\frac{n(n-3)}{2}$ sides is n then the number of sides of a polygon having 5 diagonals is**

(A) 4 (B) 5 (C) 10 (D) 15

80. **If the roots of the Q.E** $2x^2 - 2\sqrt{2}x + k = 0$ **are equal then the roots are**

(A) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (B) $\sqrt{2}, \sqrt{2}$ (C) $\frac{1}{2}, \frac{1}{2}$ (D) $1,1$

81. $(2x+1)^3 = px^3 + 5$ **is a Q.E then the value of** p **is**

(A) 0 (B) 2 (C) 4 (D) 8

82. **Maximum number of roots of the Q.E is**

(A) 0 (B) 1 (C) 2 (D) 3

83. **The roots of the Q.E** $(2x-1)(2x+1) = 0$ **are**

(A) $\frac{1}{2}, -\frac{1}{2}$ (B) $2, -2$ (C) $\frac{1}{2}, \frac{1}{2}$ (D) $1,1$

84. **The discriminate of the Q.E** $px + qx^2 + r = 0$ **is**

(A) $p^2 + 4qr$ (B) $q^2 - 4pr$ (C) $p^2 - 4qr$ (D) $r^2 - 4pq$

85. **The discriminate of the Q.E** $2x^2 - 4x - 3 = 0$ **is**

(A) -8 (B) 16 (C) -24 (D) 40

86. **The discriminate of the Q.E** $3x^2 - 2x + k = 0$ **is** k **then the value of** 0 **is**

(A) -3 (B) $\frac{1}{3}$ (C) 3 (D) $-\frac{1}{3}$

87. **If the roots of the Q.E** $4x^2 + 4\sqrt{3}x + k = 0$ **are equal then the roots are**

(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

88. **If the roots of the Q.E** $4x^2 + 20x + k^2 = 0$ **are equal then the roots are**

(A) 5 (B) -5 (C) $\frac{5}{2}$ (D) $-\frac{5}{2}$

89. **If one root of the Q.E** 5 **is** $x^2 + kx + 50 = 0$ **then the value of** k **is**

(A) 5 (B) -5 (C) 15 (D) -15

90. **If** α, β **are the roots of the Q.E** $x^2 - 7x + 12 = 0$ **then** $\alpha\beta =$

(A) 7 (B) -7 (C) -12 (D) 12

91. **The area of a rectangular plot is 528** m^2**. The length of the plot (in metres) is one more than twice its breadth, the Q.E for this situation is**

(A) $2x^2 + x - 528 = 0$ (B) $2x^2 + x + 528 = 0$

(C) $2x^2 + x + 258 = 0$ (D) $2x^2 - x - 528 = 0$

92. **If one root of the Q.E** k **then the value of** 3 **is** $kx(x-2) + 6 = 0$ **is**

(A) -1 (B) 1 (C) 2 (D) -2

93. **John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124, the Q.E for this situation is**

(A) $x^2 - 45x + 324 = 0$ (B) $x^2 - 45x + 200 = 0$

(C) $x^2 - 4x + 324 = 0$ (D) $x^2 - 40x + 324 = 0$

94. **The Q.E used to find the two numbers if their sum is** 27 **and product is** 182 **is** $x^2 - kx + 182 = 0$ **then the value of** k **is**

(A) 27 (B) 182 (C) -27 (D) -182

95. **The discriminate of the Q.E** $\sqrt{x^2 + x + 1} = 2$ **is**

(A) 13 (B) -3 (C) 11 (D) None

96. If the roots of the Q.E are $\frac{p}{q}, \frac{q}{p}$ then the equation is

(A) $qx^2 - (p^2 + q^2)x + p = 0$ (B) $px^2 - (p^2 + q^2)x + q = 0$

(C) $pqx^2 - (p^2 + q^2)x + pq = 0$ (D) $p^2q^2x^2 - (p^2 + q^2)x + p^2q^2 = 0$

97. If one root of the Q.E is reciprocal to other $3x^2 + 2x + k = 0$ then k is

(A) 3 (B) -3 (C) 2 (D) 6

98. The roots of the Q.E $x - \frac{3}{x} = 2$ are

(A) $1, 3$ (B) $3, -1$ (C) $2, 2$ (D) $1, 2$

99. If one root of the Q.E is 3 times the other $px^2 + qx + r = 0$ then $3q^2$

(A) $12pr$ (B) $14pr$ (C) $16pr$ (D) $18pr$

100. If α, β are the roots of the Q.E $x^2 - 3x - 1 = 0$ then $\frac{1}{\alpha} + \frac{1}{\beta} =$

(A) 3 (B) -3 (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

ANSWERS

1	C	2	D	3	A	4	D	5	B	6	B	7	C	8	A	9	A	10	B
11	B	12	A	13	B	14	C	15	D	16	B	17	D	18	C	19	B	20	B
21	C	22	D	23	A	24	D	25	D	26	B	27	B	28	A	29	A	30	D
31	D	32	C	33	D	34	A	35	B	36	C	37	C	38	A	39	D	40	A
41	D	42	C	43	C	44	D	45	B	46	B	47	A	48	C	49	D	50	B
51	A	52	D	53	C	54	A	55	D	56	B	57	B	58	D	59	B	60	A
61	C	62	D	63	C	64	A	65	A	66	B	67	C	68	D	69	D	70	A
71	D	72	B	73	D	74	D	75	B	76	A	77	C	78	D	79	B	80	A
81	D	82	C	83	A	84	B	85	D	86	B	87	A	88	D	89	D	90	D
91	A	92	D	93	A	94	A	95	A	96	C	97	A	98	B	99	A	100	B

* * *

5. ARITHMETIC PROGRESSIONS

SYNOPSIS

1. An **arithmetic progression** (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term.
The general form of an AP is $a, a+d, a+2d, ...$
Here a is the **first term,** fixed number d is called the **common difference**.

2. A given list of numbers $a_1, a_2, a_3, ...$ is an AP, if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3, ...$ give the same value, i.e., if $a_{k+1} - a_k$ is the same for different values of k.

3. In an AP with first term a and common difference d, the n^{th} term (or the general term) is given by $a_n = a + (n-1)d$.

4. The sum of the first n terms of an AP is given by: $S_n = \frac{n}{2}[2a + (n-1)d]$.

5. If l is the last term of the finite AP, say the n^{th} term, then the sum of all terms of the AP is given by : $S_n = \frac{n}{2}[a + l]$.

6. If a, b, c are in AP, then $b = \frac{a+c}{2}$ and b is called the arithmetic mean of a and c.

7. The n^{th} term of an AP from the end is $a_n = l - (n-1)d$ where l is the last term and d is the common difference of the finite AP

SOLVED PROBLEMS

Question 1: In an AP, if $d = -4, n = 7$ and $a_n = 4$, then a is equal to

(A) 6 (B) 7 (C) 20 (D) 28

Solution: (D)

Given if $d = -4, n = 7$ and $a_n = 4$.

In an AP, $a_n = a + (n-1)d$

$\Rightarrow 4 = a + (7-1)(-4)$

$\Rightarrow 4 = a - 24$

$\Rightarrow 4 + 24 = a$

$\Rightarrow a = 28$

Question 2: In an AP, if $a = 3.5, d = 0$ and $n = 101$ then a_n is equal to

(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

Solution: (B)

Given if $a = 3.5, d = 0$ and $n = 101$

In an AP, $a_n = a + (n-1)d$

$\Rightarrow a_n = 3.5 + (101-1)(0)$

$\Rightarrow a_n = 3.5 + (100)(0)$

$\Rightarrow a_n = 3.5 + 0$

$\Rightarrow a_n = 3.5$

Question 3: The list of numbers –10, –6, –2, 2, … is

(A) an AP with $d = -16$ (B) an AP with $d = 4$

(C) an AP with $d = -4$ (D) not an AP

Solution: (B) The given numbers are –10, –6, –2, 2, …

Here, $a_1 = -10, a_2 = -6, a_3 = -2$ and $a_4 = 2,...$

Since, $a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$a_3 - a_2 = a_2 - a_1 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

… … … … … … … … … … … …

Each successive term of given list has same difference *i.e.*, 4

So, the given list forms an AP with common difference, $d = 4$

Question 4: The 11th term of an AP $-5, -\frac{5}{2}, 0, \frac{5}{2}, ...$

(A) −20 (B) 20 (C) −30 (D) 30

Solution: Given AP, $-5, -\frac{5}{2}, 0, \frac{5}{2}, ...$

Here, $a = -5, d = -\frac{5}{2} + 5 = \frac{5}{2}$

Therefore, $a_{11} = -5 + (11-1) \times \frac{5}{2}$ $[\because a_n = a + (n-1)d]$

$= -5 + 10 \times \frac{5}{2} = -5 + 25 = 20$

Question 5: The first four terms of an AP whose first term is –2 and the common difference is −2 are

(A) −2, 0, 2, 4 (B) −2, 4, −8, 16

(C) −2, −4, −6, −8 (D) −2, −4, −8, −16

Solution: (C) Let the first four terms of an AP are $a, a+d, a+2d$ and $a+3d$.

Given, that first term, $a = -2$ and common difference, $d = -2$, then we have an AP as follows:

$-2, -2-2, -2+2(-2), -2+3(-2) = -2, -4, -6, -8$

Question 6: The 21st term of an AP whose first two terms are –3 and 4, is

(A) 17 (B) 137 (C) 143 (D) −143

Solution: (B) Given, first two terms of an AP are $a = -3$ and $a + d = 4$.

$\Rightarrow -3 + d = 4 \Rightarrow d = 4 + 3 \Rightarrow d = 7$

Therefore, $a_{21} = -3 + (21-1) \times 7$ $[\because a_n = a + (n-1)d]$

$\Rightarrow a_{21} = -3 + (20) \times 7 \Rightarrow a_{21} = -3 + 140 \Rightarrow a_{21} = 137$

Question 7: If the 2nd term of an AP is 13 and 5th term is 25, what is its 7th term?

(A) 30 (B) 33 (C) 37 (D) 38

Solution: (B) Given, $a_2 = 13$ and $a_5 = 25$

i.e., $a + (2-1)d = 13$ and $a + (5-1)d = 25$ $[\because a_n = a + (n-1)d]$

$\Rightarrow a + d = 13$ … (i) and $a + 4d = 25$ … (ii)

On subtracting eq (i) from eq (ii), we get $3d = 25 - 13 = 12 \Rightarrow d = 4$

From eq (i), $a = 13 - 4 = 9$

Therefore, $a_7 = 9 + (7-1) \times 4$ $[\because a_n = a + (n-1)d]$

$\Rightarrow a_7 = 9 + 6 \times 4 = 9 + 24 = 33$

Question 8: Which term of an AP: 21, 42, 63, 84,…is 210?

(A) 9th (B) 10th (C) 11th (D) 12th

Solution: (B) Let n^{th} term of the given AP be 210 *i.e.*, $a_n = 210$

Here, first term, $a = 21$ and common difference, $d = 42 - 21 = 21$

We have, $a_n = a + (n-1)d$

$\Rightarrow 210 = 21 + (n-1)21 \Rightarrow 210 - 21 = (n-1)21 \Rightarrow 189 = (n-1)21$

$\Rightarrow n - 1 = \frac{189}{21} = 9 \Rightarrow n = 9 + 1 = 10$

Hence, the 10th term of an AP is 210.

Question 9: If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(A) 5 (B) 20 (C) 25 (D) 30

Solution: (C) Given, the common difference of AP *i.e.*, $d = 5$

Now, $a_{18} - a_{13} = [a + (18-1)d] - [a + (13-1)d]$ $[\because a_n = a + (n-1)d]$

$= [a + 17d] - [a + 12d] = 5d = 5 \times 5 = 25$

Question 10: What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(A) 8 (B) -8 (C) -4 (D) 4

Solution: (A) Given, $a_{18} - a_{14} = 32$

$\Rightarrow [a + (18-1)d] - [a + (14-1)d] = 32$ $[\because a_n = a + (n-1)d]$

$\Rightarrow [a + 17d] - [a + 13d] = 32$

$\Rightarrow 4d = 32$

$\Rightarrow d = \frac{32}{4} = 8$ which is the required common difference of an AP.

Question 11: Two APs have the same common difference. The first term of one of these is –1 and that of the other is –8. The difference between their 4th terms is

(A) -1 (B) -8 (C) 7 (D) -9

Solution: (C) Let the common difference of two APs are d_1 and d_2, respectively.

By condition, $d_1 = d_2 = d$ … (i)

Let the first term of first AP $(a_1) = -1$ and the first term of second AP $(a_2) = -8$

We know that, the n^{th} term of an AP, $a_n = a + (n-1)d$

$\therefore$ 4th term of first AP, $a_4 = a_1 + (4-1)d$ and 4th term of second AP, $a_4' = a_2 + (4-1)d$

Now, the difference between their 4th terms is *i.e.*, $|a_4 - a_4^1| = |[a_1 + 3d] - [a_2 + 3d]| = |a_1 - a_2|$

$= |-1 - (-8)| = |-1 + 8| = 7$

Hence, the required difference is 7.

Question 12: If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be

(A) 7 (B) 11 (C) 18 (D) 0

Solution: (D) According to the question, $7a_7 = 11a_{11}$

$\Rightarrow 7[a + (7-1)d] = 11[a + (11-1)d]$ $[\because a_n = a + (n-1)d]$

$\Rightarrow 7[a + 6d] = 11[a + 10d]$

$\Rightarrow 7a + 42d = 11a + 110d$

$\Rightarrow 4a + 68d = 0$

$\Rightarrow 4(a + 17d) = 0$

$\Rightarrow a + 17d = 0$

$\Rightarrow a_{18} = 0$

Question 13: The 4th term from the end of an AP –11, -8, $-5, \ldots$, 49 is

(A) 37 (B) 40 (C) 43 (D) 58

Solution: (B) We know that, the n^{th} term of an AP from the end is $a_n = l - (n-1)d$ … (i)

Here, last term, $l = 49$ and common difference, $d = -8 - (-11) = -8 + 11 = 3$

From Eq, (i), $a_4 = l - (4-1)d = 49 - 3(3) = 49 - 9 = 40$

Question 14: The famous mathematician associated with finding the sum of the first 100 natural numbers is

(A)'Pythagoras (B) Newton (C) Gauss (D) Euclid

Solution: (C) Gauss is the famous mathematician associated with finding the sum of the first100 Natural numbers *i.e.*, 1, 2, 3, …, 100.

Question 15: If the first term of an AP is –5 and the common difference is 2, then the sum of the first 6 terms is

(A) 0 (B) 5 (C) 6 (D) 15

Solution: (A)

Given, first term, $a = -5$ and common difference, $d = 2$

Now, $S_6 = \frac{6}{2}[2(-5)+(6-1)2] \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$= 3[-10+10] = 3(0) = 0$

Question 16: The sum of first 16 terms of the AP 10, 6, 2, … is

(A) -320 (B) 320 (C) -352 (D) -400

Solution: (A) Given, AP is 10, 6, 2,…

Given, first term, $a = 10$ and common difference, $d = 6-10 = -4$

Now, $S_{16} = \frac{16}{2}[2(10)+(16-1)(-4)] \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$= 8[20-60] = 8(-40) = -320$

Question 17: In an AP, if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is equal to

(A) 19 (B) 21 (C) 38 (D) 42

Solution: (C) Given, $a = 1$, $a_n = 20$ and $S_n = 399$

Since, $a_n = 20 \Rightarrow a+(n-1)d = 20$

$\Rightarrow 1+(n-1)d = 20$

$\Rightarrow (n-1)d = 20-1$

$\Rightarrow (n-1)d = 19$

Now, $S_n = \frac{n}{2}[2(1)+(n-1)(d)] \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$\Rightarrow 399 = \frac{n}{2}[2+19]$

$\Rightarrow \frac{399\times 2}{21} = n$

$\Rightarrow n = 38$

Question 18: The sum of first five multiples of 3 is

(A) 45 (B) 55 (C) 65 (D) 75

Solution: (A) The first five multiples of 3 are 3, 6, 9, 12 and 15.

Here, first term, $a = 3$, common difference, $d = 6-3 = 3$ and number of terms, $n = 5$

Now, $S_5 = \frac{5}{2}[2(3)+(5-1)(3)] \left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$= \frac{5}{2}[6+12] = \frac{5}{2}(18) = 5\times 9 = 45$

Question 19: The sum of first 100 positiveintegers is

(A) 4050 (B) 5050 (C) 5000 (D) 4950

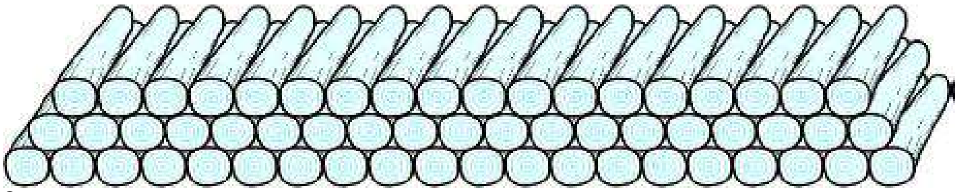
Solution: (B) The first 100 positiveintegers are 1, 2, 3, … and 100.

The sum of first n positive integers is $1+2+3+\ldots+n = \frac{n(n+1)}{2}$

The first 100 positiveintegers 1, 2, 3, … and 100 is $1+2+3+\ldots+100 = \frac{100(100+1)}{2}$

$$= \frac{100(101)}{2} = 5050$$

Question 20. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on as shown in the Fig. in how many rows are the 200 logs placed?



(A) 12 (B) 16 (C) 20 (D) 25

Solution: (B) We have, $S_n = \frac{n}{2}[2a + (n-1)d]$

Take $S_n = 200, a = 20, d = -1$

$\Rightarrow 200 = \frac{n}{2}[2(20) + (n-1)(-1)]$

$\Rightarrow 200 = \frac{n}{2}[40 - (n-1)]$

$\Rightarrow 400 = n(41 - n)$

$\Rightarrow n^2 - 41n + 400 = 0$

$\Rightarrow (n-16)(n-25) = 0$

$\Rightarrow n = 16 \ \ or \ \ n = 25$

For , $n = 25$ we have $a + (n-1)d = 20 + (25-1)(-1) = 20 - 24 = -4,$ which is not possible. Therefore, $n = 16$.

MULTIPLE CHOICE QUESTIONS

1. **Which of the following is an A.P**
   (A) $1,3,6,10,15,...$ (B) $100,80,60,40,...$ (C) $2,4,8,16,...$ (D) $3,3,4,4,5,5,...$
2. **Which of the following is an A.P**
   (A) $1,2,3,4,...$ (B) $3,3,3,3,...$ (C) $6,3,0,-3,...$ (D) $6,4,1,-3,...$
3. **Which of the following is an A.P**
   (A) $4,7,10,13,...$ (B) $11,6,1,-4,...$ (C) $13,19,25,...$ (D) All the above
4. **Which of the following is an A.P**
   (A) $1,\frac{1}{2},\frac{1}{4},\frac{1}{8},...$ (B) $1,\frac{1}{2},0,-\frac{1}{2},...$ (C) $4,8,16,32,...$ (D) $1,\frac{1}{5},\frac{1}{25},\frac{1}{125},...$
5. **The common difference of an A.P** $1,-1,-3,-5,...$ **is**
   (A) 1 (B) -1 (C) 2 (D) -2
6. **The** k^{th} **term of an A.P** $5,2,-1,-4,...$ **is**
   (A) $2-3k$ (B) $8-3k$ (C) $3k-2$ (D) $2+3k$
7. **Which of the following is an infinite A.P**
   (A) $3,7,11,15,...$ (B) $6,9,12,15,...39$ (C) $100,95,90,...-10$ (D) $1,2,3,4,...65$
8. **The** 10^{th} **term of an A.P** $4,10,16,22,...$ **is**
   (A) 70 (B) 64 (C) 58 (D) 52
9. **The common difference of an A.P** $2x,4x,6x,8x,...$ **is**
   (A) x (B) $2x$ (C) $-x$ (D) $-2x$

10. The common difference of an A.P $\frac{1}{4},-\frac{1}{4},-\frac{3}{4},-\frac{5}{4},...$ **is**

(A) $\frac{1}{4}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{4}$

11. The common difference of an A.P $0.6, 1.7, 2.8, 3.9,...$ **is**

(A) 0.6 (B) 1.7 (C) 1.1 (D) 0.1

12. The common difference of an A.P $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2},...$ **is**

(A) 3 (B) $3+\sqrt{2}$ (C) $\sqrt{2}$ (D) $3\sqrt{2}$

13. The next term of $\sqrt{2},\sqrt{8},\sqrt{18},\sqrt{32},...$ **is**

(A) $\sqrt{64}$ (B) $\sqrt{72}$ (C) $\sqrt{50}$ (D) $\sqrt{84}$

14. The 10^{th} **term of an A.P** $5,1,-3,-7,...$ **is**

(A) -35 (B) -31 (C) -27 (D) 41

15. The next term of an A.P $2,\frac{5}{2},3,\frac{7}{2},4,...$ **is**

(A) 2 (B) 5 (C) $\frac{9}{2}$ (D) $\frac{1}{2}$

16. Which term of an A.P $21,18,15,...$ **is** -81

(A) 33 (B) 34 (C) 35 (D) 36

17. Which term of an A.P $21,18,15,...$ **is** 0

(A) 7 (B) 8 (C) 9 (D) 10

18. The sum of first n **natural numbers is**

(A) n^2 (B) $\frac{n(n-1)}{2}$ (C) $\frac{n(n+1)}{3}$ (D) $\frac{n(n+1)}{2}$

19. The sum of first 10 **natural numbers is**

(A) 10 (B) 55 (C) 45 (D) 50

20. The sum of first 100 **natural numbers is**

(A) 5005 (B) 55 (C) 500500 (D) 5050

21. The first term of an A.P is 3.5, **common difference is** 0 **then** 108^{th} **term is**

(A) 105 (B) 3.5 (C) 0 (D) 111.5

22. The first term of an A.P is 4, **common difference is** -3 **then** 4^{th} **term is**

(A) -5 (B) -8 (C) 16 (D) -2

23. In an A.P $a_1 = 2$ **and** $a_3 = 18$ **then** $a_2 =$

(A) 20 (B) 10 (C) 16 (D) 36

24. The number of terms of an A.P: $3,8,13,18,...,78$ **is**

(A) 16 (B) 15 (C) 17 (D) 34

25. The number of terms of an A.P: $7,13,19,...,205$

(A) 31 (B) 35 (C) 32 (D) 34

26. The first three terms of an A.P are $x+2,\ 2x,\ 2x+2$ **then** $x=$

(A) 4 (B) 5 (C) 6 (D) 8

27. The first three terms of an A.P are $x+1,\ 3x,\ 4x+2$ **then** $x=$

(A) 0 (B) 1 (C) 2 (D) 3

28. Which term of an A.P: 25, 20, 15,... **is first negative number**

(A) 5 (B) 6 (C) 7 (D) 8

29. The n^{th} **term of an A.P is** $a_n = 2n+3$ **then the** 12^{th} **term is**

(A) 23 (B) 165 (C) 27 (D) 38

30. The n^{th} term of an A.P is $a_n = 7-2n$ then common difference is
(A) -2 (B) 2 (C) -7 (D) 7

31. The n^{th} term of an A.P is $a_n = 3+2n$ then sum of three terms is s
(A) 12 (B) 9 (C) 21 (D) 25

32. The three terms of an A.P are x, y, z then
(A) $y = \frac{x+z}{2}$ (B) $2y = x+z$ (C) $y-x = z-y$ (D) All the above

33. Which of the following is true
(A) $a_n = S_n + S_{n-1}$ (B) $a_n = a+(n-1)d$ (C) $S_n = n[2a+(n-1)d]$ (D) All the above

34. The n^{th} term of an A.P is $a_n = 3+2n$ then sum of 24 terms is
(A) 652 (B) 762 (C) 51 (D) 672

35. The sum of first ten terms of an A.P $2, 7, 12, \ldots$ is
(A) 245 (B) 490 (C) 47 (D) 295

36. In an A.P: $a = -1.25$ and $d = -0.25$ then $a_4 =$
(A) -2 (B) -1.75 (C) -2.25 (D) -0.25

37. In an A.P: $a_1 = 2$ and $a_3 = 18$ then $a_2 =$
(A) 20 (B) 10 (C) 16 (D) 36

38. In an A.P: $a_n = 0$ and $a_7 = -4$ $a_2 = 6,$ then the value of n is
(A) 4 (B) 5 (C) 6 (D) 8

39. In an A.P, the 21 term is 17^{th} more than 10^{th} then the common difference is
(A) 2 (B) 3 (C) -2 (D) -3

40. The number of multiples of 4 lie in between $1$and 250 is
(A) 59 (B) 60 (C) 61 (D) 62

41. The $-11, -8, -5, \ldots 49$ term from last of an A.P: 4^{th} is
(A) 40 (B) 43 (C) 46 (D) 58

42. The sum of first twelve terms of an A.P $-37, -33, -29, \ldots$ is
(A) 180 (B) -180 (C) 7 (D) -7

43. The sum of first eighteen terms of an A.P $3, 7, 11, \ldots$ is
(A) 766 (B) 666 (C) 718 (D) 659

44. In an A.P, $a_1 = 7$ and $a_{13} = 35$ then $S_{13} =$
(A) 546 (B) 464 (C) 273 (D) 672

45. In an A.P, $a_{12} = 37$ and $d = 3$ then $S_{12} =$
(A) 41 (B) 256 (C) 276 (D) 246

46. In an A.P, 4^{th} term is $a_n = 9-5n$ then the sum of first fifteen terms is
(A) 465 (B) -465 (C) -66 (D) 66

47. The sum of first 40 positive integers which are divisible by 6 is
(A) 4920 (B) 5920 (C) 5290 (D) 4290

48. The sum of n term of an A.P is then the common difference is $2n^2+3n$
(A) 3 (B) 4 (C) 5 (D) 9

49. The sum of n term of an A.P is then the second term is $3n^2+5n$
(A) 8 (B) 14 (C) 20 (D) 22

50. In an A.P $a_7 = 4, d = 2$ and $S_8 = -8$ then $S_9 =$
(A) -6 (B) -12 (C) -14 (D) 0

51. The sum of odd numbers lie in between 100 andis 200
(A) 750 (B) 7500 (C) 5500 (D) 8050

52. The first and last terms of an A. P are 17 and 350 respectively, common difference is $S_n =$ then 9
(A) 5238 (B) 6973 (C) 6138 (D) 6813

53. The number of two digited numbers those are divisible by 3
(A) 20 (B) 30 (C) 25 (D) 35

54. The number of three digited numbers those are divisible by 7
(A) 12 (B) 13 (C) 14 (D) None

55. The number of multiples of 4 those lie between 10 and 250 is
(A) 60 (B) 70 (C) 40 (D) 80

56. If 7th and 13th terms of an A.P be 34 and 64 respectively, then its 18th term is
(A) 87 (B) 88 (C) 89 (D) 90

57. If the first term of an A.P is 2 and common difference is 4, then the sum of its 40 terms is
(A) 3200 (B) 1600 (C) 200 (D) 2800

58. The number of terms of the A. P. 3, 7, 11, 15, … to be taken so that the sum is 406 is
(A) 5 (B) 10 (C) 12 (D) 14

59. Sum of n terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\ldots$ is
(A) $\frac{n(n+1)}{2}$ (B) $\sqrt{2}n(n+1)$ (C) $\frac{n(n+1)}{\sqrt{2}}$ (D) 1

60. The 9th term of an A.P is 449 and 449th term is 9. The term in A.P which is equal to zero is
(A) 501th (B) 502th (C) 508th (D) None

61. If the first term of an A.P is *a* and n th term is *b*. Then its common difference is
(A) $\frac{b-a}{n+1}$ (B) $\frac{b-a}{n-1}$ (C) $\frac{b-a}{n}$ (D) $\frac{b+a}{n+1}$

62. The sum of first n odd natural number is
(A) $2n-1$ (B) $2n+1$ (C) n^2 (D) n^2-1

63. If $18, a, b, -3$ are in A.P then $a+b=$
(A) 19 (B) 7 (C) 11 (D) 15

64. If $\frac{5+9+13+\ldots+n\,terms}{7+9+11+\ldots+(n+1)\,terms}=\frac{17}{16}$ then $n=$
(A) 8 (B) 7 (C) 10 (D) 11

65. If in an A.P 11th term is 38 and the 16th term is 73, then the 31st term in that A.P is
(A) 178 (B) 176 (C) 175 (D) 174

66. The n th terms of two A.Ps 63, 65, 67,… and 3, 10, 17, … are equal then $n=$
(A) 10 (B) 11 (C) 12 (D) 13

67. In an A.P 4th term is $\frac{1}{5}$ and 5th term is $\frac{1}{4}$, then sum of 20 terms in the progression is
(A) $\frac{1}{20}$ (B) $\frac{1}{21}$ (C) $\frac{21}{2}$ (D) $\frac{19}{2}$

68. If the sum of first 7 terms of an A.P is 49 and that of 17 terms is 289, then the sum of n terms
(A) $2n+1$ (B) n^2 (C) $2n-1$ (D) $(n-1)^2$

69. If 14 and 18 are the 2nd and 3rd terms of an A.P, then the sum of 51 terms of that A.P is
(A) 5610 (B) 5611 (C) 5612 (D) None

70. If p,q,r,s,t,u,v are in A.P, then $q+r+s+t+u=$
(A) $\frac{5(p+q)}{2}$ (B) $\frac{2(r-p)}{5}$ (C) $\frac{5p}{2}$ (D) None

71. Sum of natural numbers between 1 and 101 those are divisible by 2 or 5 is
(A) 1050 (B) 2050 (C) 3050 (D) 4050

72. $7, 2, a, b, 3$ **are in A.P then** $(a, b) =$
(A) $(5, 4)$ (B) $(5.2, 4.4)$ (C) $(5.8, 4.4)$ (D)None

73. Reena applied for a job and got selected. She has been offered the job with a starting monthly salary of Rs 8000, with an annual increment of Rs 500. What would be her monthly salary for the fifth year?
(A) 10000 (B) 10500 (C) 9500 (D) 11000

74. Which of the following AP has 3rd term is 5 and the 7th term is 9.
(A) $1, 3, 5, 6, 7, 8, 9, 11, 15, ...$ (B) $3, 4, 5, 6, 7, ...$
(C) $1, 3, 5, 6, 7, 8, 9, ...$ (D) None

75. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. Number of rows in the flower bed is
(A) 7 (B) 8 (C) 9 (D) 10

76. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?
(A) 13 (B) 14 (C) 15 (D) 16

77. The 31st term of an AP whose 11th term is 38 and the 16th term is 73 is
(A) 175 (B) 177 (C) 178 (D) 179

78. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106, then the 29th term is
(A) 58 (B) 60 (C) 62 (D) 64

79. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?
(A) 10^{th} (B) 11^{th} (C) 12^{th} (D) 9^{th}

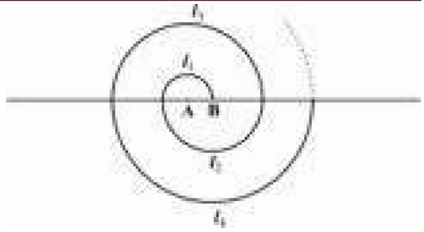
80. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?
(A) 130 (B) 120 (C) 100 (D) 110

81. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?
(A) 27650 (B) 27570 (C) 27750 (D) 27670

82. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, then the least value of of the prize is
(A) 120 (B) 60 (C) 20 (D) 40

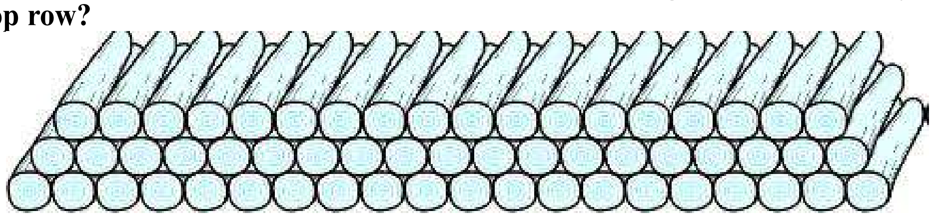
83. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?
(A) 234 (B) 224 (C) 214 (D) 244

84. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, . . . as shown in the following Fig. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = 22/7$)



(A) $134 cm$ (B) $143 cm$ (C) $164 cm$ (D) $154 cm$

85. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on as shown in the Fig. Then how many logs are in the top row?



(A) 2 (B) 3 (C) 7 (D) 5

86. How many terms of the AP: 9, 17, 25, . . . must be taken to give a sum of 636?
(A) 30 (B) 14 (C) 10 (D) 12

87. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, then the number of terms in the AP is
(A) 39 (B) 37 (C) 38 (D) 40

88. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, then the sum of terms in the AP is
(A) 6397 (B) 6379 (C) 6977 (D) 6973

89. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, then the total production in first 7 years is
(A) 4355 (B) 4375 (C) 4775 (D) 4575

90. The 11th term from the last term (towards the first term) of the AP: 10, 7, 4, . . ., – 62 is
(A) – 22 (B) – 52 (C) – 32 (D) – 42

ANSWERS

1	B	2	D	3	D	4	B	5	D	6	B	7	A	8	C	9	B	10	B
11	C	12	C	13	C	14	B	15	C	16	C	17	B	18	D	19	B	20	D
21	B	22	A	23	B	24	A	25	D	26	A	27	D	28	C	29	C	30	A
31	C	32	D	33	B	34	D	35	A	36	A	37	B	38	B	39	B	40	D
41	D	42	B	43	B	44	C	45	D	46	B	47	A	48	D	49	D	50	D
51	A	52	B	53	B	54	D	55	A	56	C	57	B	58	D	59	C	60	D
61	B	62	C	63	D	64	B	65	A	66	D	67	C	68	B	69	D	70	A
71	C	72	D	73	A	74	B	75	D	76	D	77	C	78	D	79	B	80	C
81	C	82	D	83	A	84	B	85	D	86	D	87	C	88	D	89	B	90	C

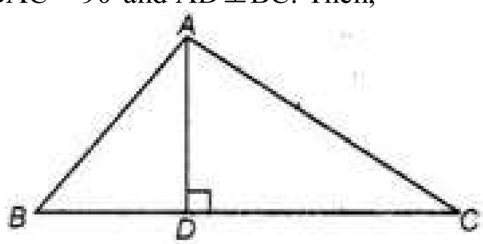
6. TRIANGLES

SYNOPSIS

1. Two figures having the same shape but not necessarily the same size are called similar figures.
2. All the congruent figures are similar but the converse is not true.
3. Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e., proportion).
4. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
5. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
6. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio and hence the two triangles are similar (AAA similarity criterion).
7. If in two triangles, two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are similar (AA similarity criterion).
8. If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS similarity criterion).
9. If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are similar (SAS similarity criterion).

SOLVED PROBLEMS

Question 1: In figure, if $\angle BAC = 90°$ and $AD \perp BC$. Then,



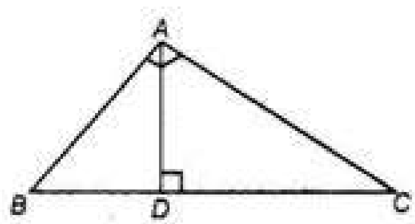
(A) $BD.CD=BC^2$ (B) $AB.AC=BC^2$
(C) $BD.CD = AD^2$ (D) $AB.AC=AD^2$

Solution: (C) In ΔADB and ΔADC,

$\angle D = 90°$; $\angle DBA = \angle DAC$; ΔADB = ΔADC

$\therefore \frac{BD}{AD} = \frac{AD}{CD}$

$\Rightarrow BD.CD = AD^2$



Question 2: If Δ ABC ~ Δ EDF and ΔABC is not similar to Δ DEF, then which of the following is not true?

(A) $BC{\cdot}EF = AC{\cdot}FD$ (B) $AB{\cdot}EF = AC{\cdot}DE$
(C) $BC{\cdot}DE = AB{\cdot}EF$ (D) $BC{\cdot}DE = AB{\cdot}FD$

Solution: (C)

Given, ΔABC $\sim$ ΔEDF,

$\therefore \frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$

If $\dfrac{AB}{ED} = \dfrac{BC}{DF} \Rightarrow$ AB.DF = ED.BC or BC.DE = AB.DF

So, option (D) is true.

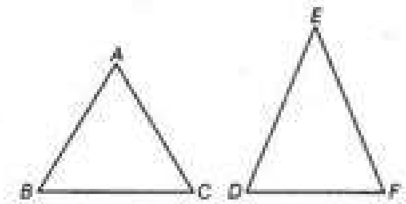
If $\dfrac{BC}{DF} = \dfrac{AC}{EF} \Rightarrow$ BC.EF = AC.DF

So, option (A) is true.

If $\dfrac{AB}{ED} = \dfrac{AC}{EF} \Rightarrow$ AB.EF = ED.AC

So, option (B) is true.

Hence, option (C) is not true.



Question 3: If in two ΔABC and ΔPQR, $\dfrac{AB}{QR} = \dfrac{BC}{PR} = \dfrac{CA}{PQ}$ then

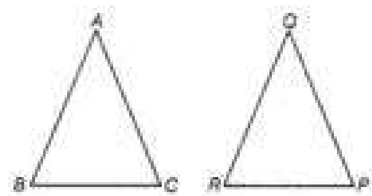
(A) ΔPQR ~ ΔCAB (B) ΔPQR ~ ΔABC

(C) ΔCBA ~ ΔPQR (D) ΔBCA ~ ΔPQR

Solution: (A) Given, in two Δ ABC and ΔPQR, $\dfrac{AB}{QR} = \dfrac{BC}{PR} = \dfrac{CA}{PQ}$

Which shows that sides of one triangle are proportional to the side of the other triangle, then their corresponding angles are also equal.

So by SSS similarity, triangles are similar, i.e., ΔCAB ~ ΔPQR



Question 4: Two line segments AC and BD intersect each other at the point P such that PA = 6 *cm*, PB = 3 *cm*, PC = 2.5 *cm*, PD = 5 *cm*, ∠ APB = 50° and ∠ CDP=30°.Then, ∠ PBA is equal to

(A) 50° (B) 30° (C) 60° (D) 100°

Solution: (D) In Δ APB and Δ CPD, ∠ APB = ∠ CPD = 50°

$\dfrac{AP}{PD} = \dfrac{6}{5}$ and $\dfrac{BP}{CP} = \dfrac{3}{2.5} = \dfrac{6}{5}$

$\therefore \dfrac{AP}{PB} = \dfrac{BP}{CP}$

$\therefore \Delta\,APB \sim \Delta\,DPC$

$\therefore \angle A = \angle D = 30^0$

In Δ APB, $\angle A + \angle B + \angle APB = 180^0$

$\Rightarrow 30^0 + \angle B + 50^0 = 180^0$

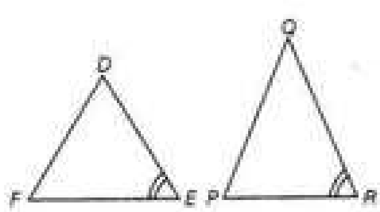
$\therefore \angle B = 180^0 - 30^0 - 50^0 = 100^0$

i.e., $\angle PBA = 100^0$

Question 5: If in two Δ DEF and Δ PQR, ∠ D = ∠ Q and ∠ R = ∠ E, then which of the following is not true?

(A) $\dfrac{EF}{PR} = \dfrac{DF}{PQ}$ (B) $\dfrac{DE}{PQ} = \dfrac{EF}{RP}$ (C) $\dfrac{DE}{QR} = \dfrac{DF}{PQ}$ (D) $\dfrac{EF}{RP} = \dfrac{DE}{QR}$

Solution: (A) Given, in ΔDEF and Δ PQR, ∠ D = ∠ Q and ∠ R = ∠ E



$\therefore \Delta\,DEF \,\square\, \Delta\,QRP$

$\Rightarrow \angle F = \angle P$

$\frac{DF}{PQ} = \frac{DE}{RQ} = \frac{EF}{RP}$

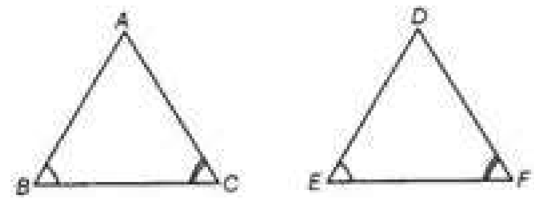
Question 6: In Δ ABC and Δ DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, the two triangles are

(A) congruent but not similar (B) similar but not congruent

(C) neither congruent nor similar (D) congruent as well as similar

Solution: (B) In Δ ABC and Δ DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE.

We know that, if in two triangles corresponding two angles are same, then they are similar by AAA similarity criterion. Also, Δ ABC and Δ DEF do not satisfy any rule of congruency, (SAS, ASA, SSS), so both are not congruent.



Question 7: If Δ ABC ~ Δ PQR with $\frac{BC}{QR} = \frac{1}{3}$, then $\frac{ar(\Delta PRQ)}{ar(\Delta BCA)}$ is equal to

(A) 9 (B) 3 (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

Solution: (A) Given, Δ ABC ~ Δ PQR and $\frac{BC}{QR} = \frac{1}{3}$.

We know that, the ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

$\therefore \frac{ar(\Delta PRQ)}{ar(\Delta BCA)} = \frac{(QR)^2}{(BC)^2} = \left(\frac{QR}{BC}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1} = 9$

Question 8:

If Δ ABC ~ Δ DFE, $\angle A = 30°$, $\angle C = 50°$, AB = 5 *cm*, AC = 8 *cm* and DF = 7.5 *cm*. Then, which of the following is true?

(A) DE = 12 *cm*, $\angle F = 50°$ (B) DE = 12 *cm*, $\angle F = 100°$

(C) EF = 12 *cm*, $\angle D = 100°$ (D) EF = 12 *cm*, $\angle D = 30°$

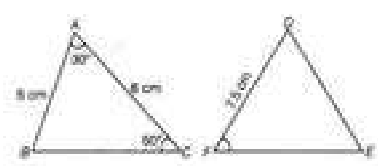
Solution: (B) Given, Δ ABC ~ Δ DFE, then $\angle A = \angle D = 30°$, $\angle C = \angle E = 50°$

$\therefore \angle B = \angle F = 180° - (30° + 50°) = 100°$

Also, $\frac{AB}{DF} = \frac{AC}{DE}$

$\Rightarrow \frac{5}{7.5} = \frac{8}{DE} \Rightarrow DE = \frac{8 \times 7.5}{5} = 12\,cm$

Hence, $DE = 12\,cm$ and $\angle F = 100°$

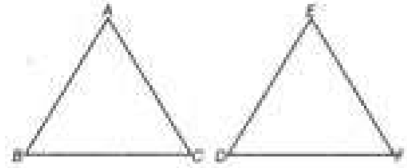


Question 9: If in Δ ABC and Δ DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(A) $\angle B = \angle E$ (B) $\angle A = \angle D$ (C) $\angle B = \angle D$ (D) $\angle A = \angle F$

Solution: (C) Given, in Δ ABC and Δ EDF, $\frac{AB}{DE} = \frac{BC}{FD}$

If Δ ABC ~ Δ EDF, then $\angle B = \angle D$, $\angle A = \angle E$ and $\angle C = \angle F$



Question 10: If Δ ABC ~ Δ QRP, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{4}$, AB = 18 *cm* and BC = 15 *cm*, then PR is equal to

(A) 10 *cm* (B) 12 *cm* (C) $\frac{20}{3}$ *cm* (D) 8 *cm*

Solution: (A) Given, Δ ABC ~ Δ QRP, AB = 18 *cm* and BC = 15 *cm*,

We know that, the ratio of area of two similar triangles is equal to the ratio of square of their corresponding sides.

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{BC}{RP}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \left(\frac{15}{RP}\right)^2 = \frac{9}{4} \Rightarrow (RP)^2 = \frac{225 \times 4}{9} = 100 = (10)^2$$

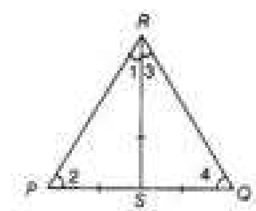
$\therefore RP = 10\ cm$

Question 11: If S is a point on side PS of a Δ PQR such that PS = QS = RS, then

(A) $PR.QR = RS^2$ (B) $QS^2 + RS^2 = QR^2$

(C) $PR^2 + QR^2 = PQ^2$ (D) $PS^2 + RS^2 = PR^2$

Solution: (C) Given, in ΔPQR, $PS = QS = RS$



in ΔPSR, $PS = RS \Rightarrow \angle 1 = \angle 2$

similarly in $\Delta RSQ \Rightarrow \angle 3 = \angle 4$

Now, in ΔPQR, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^0$

$\Rightarrow \angle 1 + \angle 1 + \angle 3 + \angle 3 = 180^0$

$\Rightarrow 2(\angle 1 + \angle 3) = 180^0$

$\Rightarrow \angle 1 + \angle 3 = 90^0$

$\therefore \angle R = 90^0$

Therefore, by Pythagoras theorem, $PR^2 + QR^2 = PQ^2$

MULTIPLE CHOICE QUESTIONS

1. **Which of the following are not similar figures**

   (A) circles (B) squares (C) equilateral triangles (D) isosceles triangles

2. **Two polygons of the same number of sides are similar, if**

   (A) their corresponding angles are equal (B) their corresponding sides are proportional

   (C) both (A) and (B) (D) None

3. **Two triangles are similar, if**

   (A) their corresponding angles are equal and
   (B) their corresponding sides are in the same ratio
   (C) both (A) and (B) (D) None

4. **Which of the following are similar triangles**

   (A) scalen (B) right angled (C) equilateral (D) isosceles

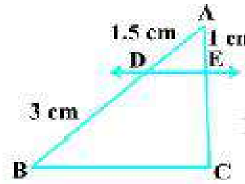
5. **All circles are**

   (A) congruent (B) not similar (C) similar (D) none of these

6. **All squares are**

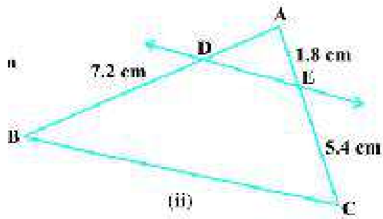
   (A) congruent (B) not similar (C) similar (D) none of these

7. In the given Fig. if DE || BC, then EC =



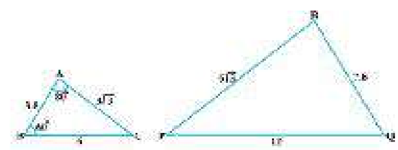
(A) 2 *cm* (B) 2.5 *cm* (C) 3 *cm* (D) 4 *cm*

8. In the given Fig. if DE || BC, then AD =



(A) 2.8 *cm* (B) 2.4 *cm* (C) 3.6 *cm* (D) 4.8 *cm*

9. In the given Fig. ∠ P =



(A) 60^0 (B) 80^0

(C) 40^0 (D) 100^0

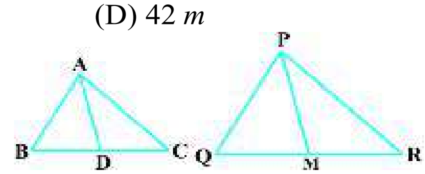
10. If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ ΔPQR, then

(A) $\frac{AB}{PM} = \frac{AD}{PQ}$ (B) $\frac{AB}{PQ} = \frac{AD}{PM}$ (C) both (A) and (B) (D) None

11. A vertical pole of length 6 *m* casts a shadow 4 *m* long on the ground and at the same time a tower casts a shadow 28 *m* long, then the height of the tower is

(A) 28 *m* (B) 24 *m* (C) 36 *m* (D) 42 *m*

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR as in Fig, then



(A) ΔABC ~ ΔPQR (B) ΔABD ~ ΔPQM

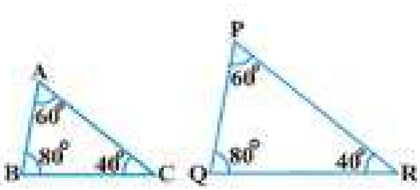
(C) ΔADC ~ ΔPMR (D) ΔADB ~ ΔPMQ

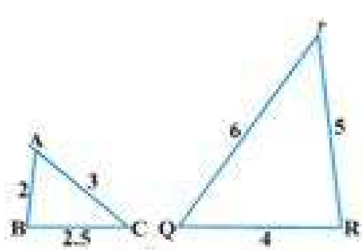
13. In Fig, altitudes AD and CE of ΔABC intersect each other at the point P, then which of the following is true

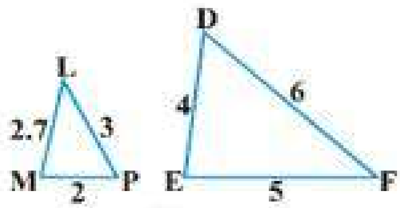
(A) Δ AEP ~ Δ CDP (B) Δ ABD ~ Δ CBE

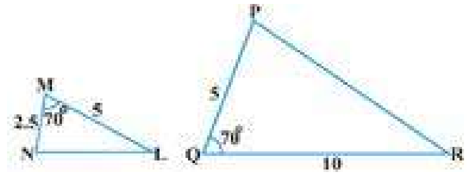
(C) Δ AEP ~ Δ ADB (D) None

14. Which of the following are not similar figures

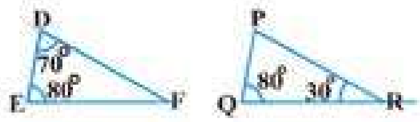
(A) 

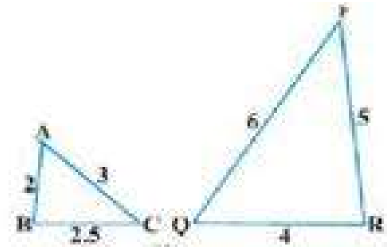
(B) 

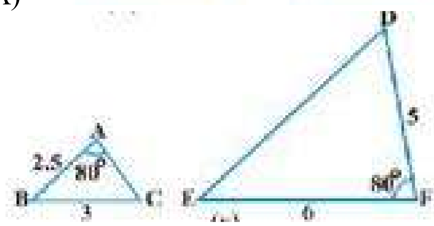
(C) 

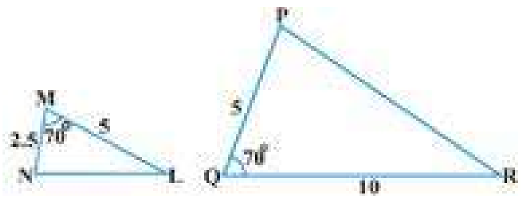
(D) 

15. Which of the following are not similar figures

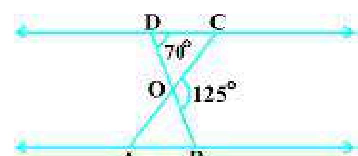
(A) 

(B) 

(C) 

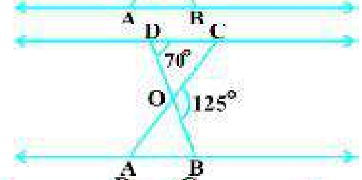
(D) 

16. In Fig, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$, then $\angle DOC$ =



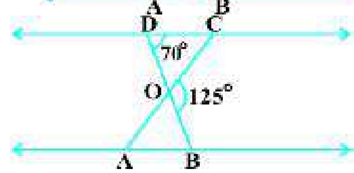
(A) 55^0 (B) 70^0 (C) 65^0 (D) 40^0

17. In Fig, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$, then $\angle DCO$ =



(A) 55^0 (B) 70^0 (C) 65^0 (D) 40^0

18. In Fig, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\angle CDO = 70°$, then $\angle OAB$ =



(A) 55^0 (B) 70^0 (C) 65^0 (D) 40^0

19. A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 *m/s*. If the lamp is 3.6 m above the ground, then the length of her shadow after 4 seconds is

(A) 2.8 *m* (B) 2.4 *m* (C) 1.6 *m* (D) 4.2 *m*

20. In the Fig, OA .OB = OC.OD, then

(A) $\angle A = \angle C$ (B) $\angle B = \angle D$ (C) both (A) and (B) (D) None

ANSWERS

1	D	2	C	3	C	4	C	5	C	6	C	7	A	8	B	9	C	10	B
11	D	12	A	13	D	14	C	15	C	16	A	17	A	18	A	19	C	20	C

* * *

7. CO-ORDINATE GEOMETRY

SYNOPSIS

1. The abscissa and ordinate of a given point are the distances of the point from x - axis and y - axis respectively.
2. Any point on x - axis is of the form $(x, 0)$.
3. Any point on y -axis is of the form $(0, y)$.
4. The distance between points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
5. Distance of a point $P(x, y)$ from the origin $O(0, 0)$ is given by $OP = \sqrt{x^2 + y^2}$.
6. The point which divides the join of points $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio $m:n$ is $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.
7. The point which divides the join of points $P(x_1, y_1)$ and $Q(x_2, y_2)$ externally in the ratio $m:n$ is $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$.
8. The midpoint of line segment joining the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

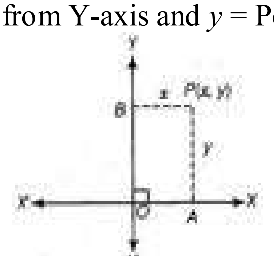
SOLVED PROBLEMS

Question1: The distance of the point P (2, 3) from the X-axis is

(A) 2 (B) 3 (C) 1 (D) 5

Solution: (B) We know that, if (x, y) is any point on the Cartesian plane in first quadrant.

Then, x = Perpendicular distance from Y-axis and y = Perpendicular distance from X-axis



Distance of the point P(2, 3) from the X-axis = Ordinate of a point P(2, 3) = 3.

Question 2: The distance between the points A (0, 6) and B (0, −2) is

(A) 6 (B) 8 (C) 4 (D) 2

Solution: (B) The distance between points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Here, $x_1 = 0, y_1 = 6$ and $x_2 = 0, y_2 = -2$

Therefore, distance between points A (0, 6) and B (0, −2) is $\text{AB} = \sqrt{(0-0)^2 + (-2-6)^2}$

$= \sqrt{0^2 + (-8)^2} = \sqrt{0 + 64} = \sqrt{64} = 8$ units.

Question 3: The distance of the point P (−6,8) from the origin is

(A) 8 (B) $2\sqrt{7}$ (C) 10 (D) 6

Solution: (C) The distance between points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Here, $x_1 = -6, y_1 = 8$ and $x_2 = 0, y_2 = 0$

Therefore, distance between points P (– 6,8) and O (0, 0) is $OP = \sqrt{(0-(-6))^2 + (0-8)^2}$

$= \sqrt{6^2 + (-8)^2} = \sqrt{36+64} = \sqrt{100} = 10$ units.

Question 4: The distance between the points (0, 5) and (– 5, 0) is

(A) 5 (B) $5\sqrt{2}$ (C) $2\sqrt{5}$ (D) 10

Solution: (B) The distance between points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Here, $x_1 = 0, y_1 = 5$ and $x_2 = -5, y_2 = 0$

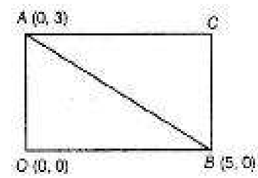
Therefore, distance between points A (0, 5) and B (– 5, 0) is $AB = \sqrt{(-5-0)^2 + (0-5)^2}$

$= \sqrt{(-5)^2 + (-5)^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$ units.

Question 5: If AOBC is a rectangle whose three vertices are A (0, 3), O (0, 0) and B (5, 0), then the length of its diagonal is

(A) 5 (B) 3 (C) $\sqrt{34}$ (D) 4

Solution: (C)



Now, length of the diagonal AB = Distance between the points A (0, 3) and B (5, 0).

The distance between points (x_1, y_1) and (x_2, y_2) is

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Here, $x_1 = 0, y_1 = 3$ and $x_2 = 5, y_2 = 0$

Therefore, distance between points A (0, 3) and B (5, 0) is $AB = \sqrt{(5-0)^2 + (0-3)^2}$

$= \sqrt{(5)^2 + (-3)^2} = \sqrt{25+9} = \sqrt{34}$ units.

Hence, the required length of its diagonal is $\sqrt{34}$.

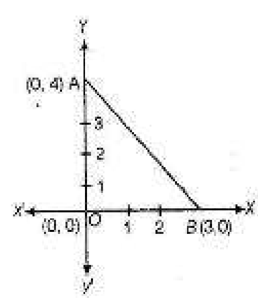
Question 6: The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(A) 5 (B) 12 (C) 11 (D) $7+\sqrt{5}$

Solution: (B) We have, distance between points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

We Further, adding all the distance of a triangle to get the perimeter of a triangle. We plot the vertices of a triangle i.e., A (0, 4), O (0, 0) and B (3, 0) as shown below.



Now, perimeter of ΔAOB = Sum of the length of all its sides = d(AO) + d(OB) + d(AB)

$= \sqrt{(0-0)^2 + (0-4)^2} + \sqrt{(3-0)^2 + (0-0)^2} + \sqrt{(3-0)^2 + (0-4)^2}$

$= \sqrt{0^2 + (-4)^2} + \sqrt{(3)^2 + (0)^2} + \sqrt{(3)^2 + (-4)^2} = \sqrt{0+16} + \sqrt{9+0} + \sqrt{9+16}$

$= \sqrt{16} + \sqrt{9} + \sqrt{9+16} = 4 + 3 + 5 = 12$ units.

Hence, the required perimeter of triangle is 12 units..

Question 7: The points (– 4, 0), (4, 0) and (0, 3) are the vertices of a

(A) right angled triangle (B) isosceles triangle
(C) equilateral triangle (D) scalene triangle

Solution: (B) Let A (– 4, 0), B (4, 0), C (0, 3) are the given vertices.

We have, distance between points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Now, distance between A (– 4, 0) and B (4, 0) is $AB = \sqrt{(4-(-4))^2 + (0-0)^2}$

$= \sqrt{(8)^2 + (0)^2} = \sqrt{64+0} = \sqrt{64} = 8$ units.

Also, distance between B (4, 0) and C (0, 3) is $BC = \sqrt{(0-4)^2 + (3-0)^2}$

$= \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units.

Now, distance between A (– 4, 0) and C (0, 3) is $AC = \sqrt{(0-(-4))^2 + (3-0)^2}$

$= \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ units.

$\therefore$ BC = AC

Hence, ΔABC is an isosceles triangle because an isosceles triangle has two sides equal.

Question 8: The point which divides the line segment joining the points (7, –6) and (3, 4) in ratio1:2 internally lies in the

(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

Solution: (D) If P (*x*, *y*) divides the line segment joining A(x_1, y_1) and B(x_2, y_2) internally in the ratio $m : n$ internally are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$.

Here, $x_1 = 7, y_1 = -6;\ x_2 = 3, y_2 = 4$ and $m = 1, n = 2$

$\therefore P = \left(\frac{1(3)+2(7)}{1+2}, \frac{1(4)+2(-6)}{1+2}\right) = \left(\frac{3+14}{3}, \frac{4-12}{3}\right) = \left(\frac{17}{3}, -\frac{8}{3}\right)$ which lies in IV quadrant

Question 9: The point which lies on the bisector of the line segment joining the points A (– 2, – 5) and B (2, 5) is

(A) (0, 0) (B) (0, 2) (C) (2, 0) (D) (– 2, 0)

Solution: (A) We know that, the perpendicular bisector of the any line segment divides the segment into two equal parts i.e., the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

We have, the midpoint of line segment joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Mid-point of the line segment joining the points A (– 2, – 5) and B (2, 5) is

$\left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

Question10: The fourth vertex D of a parallelogram ABCD whose three vertices are A (– 2, 3), B (6, 7) and C (8, 3) is

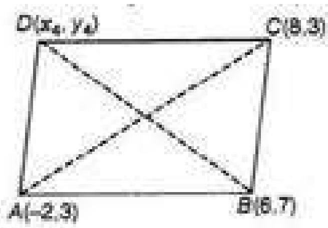
(A) (0, 1) (B) (0, – 1) (C) (– 1, 0) (D) (1,0)

Solution: (B) Let the fourth vertex of parallelogram, D ≡ (x_4, y_4) and L, M be the middle points of AC and BD, respectively.

We have, midpoint of line segment joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

$$L = \left(\frac{-2+8}{2}, \frac{3+3}{2}\right) = \left(\frac{6}{2}, \frac{6}{2}\right) = (3,3) \text{ and } M = \left(\frac{6+x_4}{2}, \frac{7+y_4}{2}\right)$$

Since, ABCD is a parallelogram, therefore diagonals AC and BD will bisect each other. Hence, L and M are the same points.



$$\therefore (3,3) = \left(\frac{6+x_4}{2}, \frac{7+y_4}{2}\right)$$

$$\Rightarrow 3 = \frac{6+x_4}{2} \text{ and } 3 = \frac{7+y_4}{2}$$

$$\Rightarrow 6 = 6 + x_4 \text{ and } 6 = 7 + y_4$$

$$\Rightarrow x_4 = 0 \text{ and } y_4 = -1$$

Hence, the fourth vertex of parallelogram is D (x_4, y_4) is (0, − 1).

Question11: If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then

(A) AP = $\frac{1}{3}$AB (B) AP = PB (C) PB = $\frac{1}{3}$AB (D) AP = $\frac{1}{2}$AB

Solution: (D) Given that, the point P (2, 1) lies on the line segment joining the points A (4, 2) and B (8, 4).

We have, distance between points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Now, distance between A (4, 2) and P (2, 1) is

$$AP = \sqrt{(2-4)^2 + (1-2)^2} = \sqrt{(-2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5} \text{ units.}$$

Distance between A (4, 2) and B (8, 4) is

$$AB = \sqrt{(8-4)^2 + (4-2)^2} = \sqrt{(4)^2 + (2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} \text{ units.}$$

Distance between B (8, 4) and P (2, 1) is

$$BP = \sqrt{(2-8)^2 + (1-4)^2} = \sqrt{(-6)^2 + (-3)^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5} \text{ units.}$$

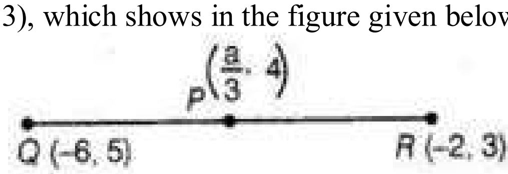
$$AB = 2AP$$

Hence, required condition is $AP = \frac{1}{2}AB$

Question 12: If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q (− 6, 5) and R (− 2, 3), then the value of a is

(A) − 4 (B) − 12 (C) 12 (D) − 6

Solution: (B) Given that, $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q (− 6, 5) and R (− 2, 3), which shows in the figure given below



We have, midpoint of line segment joining (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Mid-point of the line segment joining the points Q (– 6, 5) and R (– 2, 3)

is $\left(\frac{-6-2}{2}, \frac{5+3}{2}\right) = \left(\frac{-8}{2}, \frac{8}{2}\right) = (-4, 4)$

But, mid-point $P\left(\frac{a}{3}, 4\right)$ is given.

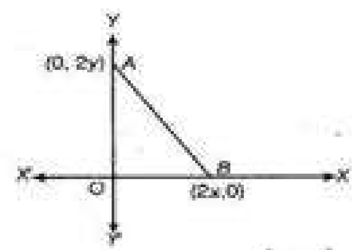
$\therefore \left(\frac{a}{3}, 4\right) = (-4, 4)$

$\Rightarrow \frac{a}{3} = -4$

$\Rightarrow a = -12$

Hence, the required value of a is – 12.

Question 13: The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is



(A) (x, y) (B) (y, x)

(C) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (D) $\left(\frac{y}{2}, \frac{x}{2}\right)$

Solution: (A) Let the coordinate of the point which is equidistant from the three vertices 0 (0, 0), A (0, 2y) and B (2x, 0) is P (h, k).

Then, PO = PA = PB

$\Rightarrow$ $(PO)^2 = (PA)^2 = (PB)^2$ …(i)

By distance formula, $\sqrt{(h-0)^2+(k-0)^2} = \sqrt{(h-0)^2+(k-2y)^2} = \sqrt{(h-2x)^2+(k-0)^2}$ …(ii)

$\Rightarrow \sqrt{h^2+k^2} = \sqrt{h^2+(k-2y)^2} = \sqrt{(h-2x)^2+k^2}$

$\Rightarrow h^2+k^2 = h^2+(k-2y)^2 = (h-2x)^2+k^2$

Taking first two equations, $h^2+k^2 = h^2+(k-2y)^2 \Rightarrow k^2 = k^2+4y^2-4yk$

$\Rightarrow 4y^2-4yk = 0$

$\Rightarrow 4y(y-k) = 0$

$\Rightarrow y(y-k) = 0$

$\Rightarrow y = 0, y = k$

$\Rightarrow y = k\ [\because y \neq 0]$

Taking first and third equations, $h^2+k^2 = (h-2x)^2+k^2 \Rightarrow h^2 = h^2+4x^2-4xh$

$\Rightarrow 4x^2-4xh = 0$

$\Rightarrow 4x(x-h) = 0$

$\Rightarrow x(x-h) = 0$

$\Rightarrow x = 0, x = h$

$\Rightarrow x = h\ [\because x \neq 0]$

Hence, the required point is $(h, k) = (x, y)$.

MULTIPLE CHOICE QUESTIONS

1. **The point which lies on x^{-} axis is**
   (A) $(0,3)$ (B) $(-4,0)$ (C) $(3,5)$ (D) $(0,-4)$
2. **The point which lies on y^{-} axis is**
   (A) $(0,3)$ (B) $(-4,0)$ (C) $(0,0)$ (D) All the above
3. **The point $(3,-5)$ lies on the quadrant**
   (A) I (B) II (C) III (D) IV
4. **The point lies on the quadrant III is**
   (A) $(1,3)$ (B) $(-2,3)$ (C) $(-3,-5)$ (D) $(3,-4)$
5. **The distance between the points $(-4,0)$ andis $(4,0)$**
   (A) 0 (B) 8 (C) 2 (D) 16
6. **The distance between the points $(0,-3)$ andis $(0,-8)$**
   (A) 3 (B) 5 (C) 8 (D) 11
7. **If the distance between the points $(3,8)$ and k then the value of $6,$is $(k,8)$ is**
   (A) 5 (B) 6 (C) 8 (D) 9
8. **The distance from $(0,0)$ tois $(3,4)$**
   (A) 3 (B) 4 (C) 5 (D) 7
9. **The distance between the points (a,b) andis $(-a,-b)$**
   (A) $\sqrt{a^2+b^2}$ (B) $2\sqrt{a+b}$ (C) $4\sqrt{a^2+b^2}$ (D) $2\sqrt{a^2+b^2}$
10. **The point which lies 3 units distance from $(5,7)$ is**
    (A) $(8,4)$ (B) $(0,5)$ (C) $(3,0)$ (D) $(8,7)$
11. **The point which lies on x^{-} axis and having 5 units distance from $(2,3)$ is**
    (A) $(6,0)$ (B) $(5,0)$ (C) $(-2,0)$ (D) $(-2,0)$
12. **The points $(0,0),(5,0)$ are $(0,7)$ vertices of a triangle**
    (A) Right angled (B) Right angled Isosceles (C) Isosceles (D) Equilateral
13. **If $\mathrm{A}(4,2)$ and $\mathrm{B}(7,5)$ then the length of $\overline{\mathrm{AB}}$ is**
    (A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $5\sqrt{2}$ (D) 18
14. **If $\mathrm{A}(x_1,y_1)$ and $\overline{\mathrm{AB}}$ then the length of $\mathrm{B}(x_2,y_2)$ is**
    (A) $\sqrt{(x_2+x_1)^2+(y_2+y_1)^2}$ (B) $\sqrt{(x_2-x_1)^2+(y_2+y_1)^2}$
    (C) $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ (D) $\sqrt{(x_2+x_1)+(y_2-y_1)^2}$
15. **If $\mathrm{A}(x_1,y_1)$ and $\mathrm{B}(x_2,y_2)$ then the point which divides $\overline{\mathrm{AB}}$ in the ratiointernally $m:n$ is**
    (A) $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2+ny_1}{m+n}\right)$ (B) $\left(\frac{mx_2-nx_1}{m-n},\frac{my_2+ny_1}{m+n}\right)$
    (C) $\left(\frac{mx_2+nx_1}{m+n},\frac{my_2-ny_1}{m-n}\right)$ (D) $\left(\frac{mx_2-nx_1}{m-n},\frac{my_2-ny_1}{m-n}\right)$

16. **If** $A(x_1, y_1)$ **and** $B(x_2, y_2)$ **then the point which divides** $\overline{AB}$ **in the ratio** $m:n$ **isexternally**

(A) $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ (B) $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2+ny_1}{m+n}\right)$

(C) $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2-ny_1}{m-n}\right)$ (D) $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

17. **The line joining points** $A(x_1, y_1)$ **and** x^- **divided by** $B(x_2, y_2)$ **axis in the ratio**

(A) $-x_1 : y_1$ (B) $-x_1 : x_2$ (C) $-y_1 : y_2$ (D) $-x_2 : y_2$

18. **The line joining points** $A(x_1, y_1)$ **and** y^- **divided by** $B(x_2, y_2)$ **axis in the ratio**

(A) $-x_1 : y_1$ (B) $-x_1 : x_2$ (C) $-y_1 : y_2$ (D) $-x_2 : y_2$

19. **If** $A(3,5)$ **and** $\overline{AB}$ **then the point which divides** $B(8,10)$ **in the ratiointernally** $2:3$ **are**

(A) $(7,5)$ (B) $(5,8)$ (C) $(8,6)$ (D) $(5,7)$

20. **If** $A(x_1, y_1)$ **and** $B(x_2, y_2)$, **then the midpoint of** $\overline{AB}$ **is**

(A) $\left(\frac{x_2-x_1}{2}, \frac{y_2-y_1}{2}\right)$ (B) $\left(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2}\right)$

(C) $\left(\frac{x_1+x_2}{3}, \frac{y_1+y_2}{3}\right)$ (D) $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

21. **If** $A(3,0)$ **and** $\overline{AB}$ **then the midpoint of** $B(-5,8)$ **is**

(A) $(-1,4)$ (B) $(-2,8)$ (C) $(4,-1)$ (D) $(8,-2)$

22. **The origin divides the join of** $A(6,9)$ **andin the ratio** $B(-6,-9)$

(A) $2:3$ (B) $3:2$ (C) $1:1$ (D) $1:2$

23. **The** x^- **axis divides the join of** $(7,3)$ **andin the ratio** $(6,-5)$

(A) $6:7$ (B) $7:6$ (C) $5:3$ (D) $3:5$

24. **The** y^- **axis divides the join of** $(5,-6)$ **and** $(-1,-4)$ **in the ratio**

(A) $1:5$ (B) $5:1$ (C) $2:3$ (D) $3:2$

25. **The distance between the points** $(a\cos\theta, 0)$ **andis** $(0, a\sin\theta)$

(A) a (B) $\frac{a}{2}$ (C) a^2 (D) $\sqrt{a}$

26. **The point** $(-4,6)$ **divides the join of** $(-6,10)$ **andin the ratio** $(3,-8)$

(A) $2:1$ (B) $2:7$ (C) $2:8$ (D) $7:2$

27. **If** $A(-2,3), B(6,7)$ **and** $C(8,3)$ **are three vertices of a parallelogram** $ABCD$, **then the fourth vertex** $D =$

(A) $(0,1)$ (B) $(0,-1)$ (C) $(1,0)$ (D) $(-1,0)$

28. **If the points** $(1,2), (-1,m)$ **and** $(-3,-4)$ **are collinear, then the value of** m **is**

(A) -2 (B) 2 (C) 1 (D) -1

29. **If the points** $(7,-2), (5,1)$ **and** $(3,k)$ **are collinear, then the value of** k **is**

(A) 3 (B) 6 (C) 4 (D) -2

30. **If the points** $(t, 2t), (-2,6)$ **and** $(3,1)$ **are collinear, then the value of** t **is**

(A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$

31. **The perimeter of a triangle with vertices** $(0,0),(2,0),(0,2)$ **is**

(A) 4 units (B) $4+2\sqrt{2}$ units (C) $2\sqrt{2}$ units (D) $4+2\sqrt{2}$ units

32. $(0,3),(3,3),(3,p)$ **are three vertices of an equilateral triangle, then the value of** p **is**

(A) 2 (B) 3 (C) 6 (D) $-\sqrt{3}$

33. **The area of the rectangle formed by the vertices** $(0,-1),(-2,3),(6,7),(8,3)$ **is**

(A) 20 sq. units (B) 40 sq. units (C) 80 sq. units (D) 1600 sq. units

34. **The area of the square formed by the vertices** $(3,2),(0,5),(-3,2),(0,-1)$ **is**

(A) 9 sq. units (B) 18 sq. units (C) $\sqrt{46}$ sq. units (D) $\sqrt{18}$ sq. units

35. **The points** $(4,8),(7,5),(1,-1),(-2,k)$ **are vertices of a rectangle, then the value of** k **is**

(A) 1 (B) 2 (C) 3 (D) 4

36. **The length of the diagonal of a rectangle having vertices** $A(0,3), B(0,0), C(5,0)$ **is**

(A) 3 units (B) 5 units (C) 8 units (D) $\sqrt{34}$ units

37. **If** $(-2,-1),(a,0),(4,b),(1,2)$ **are the vertices of a parallelogram, then** $(a,b)=$

(A) $(3,1)$ (B) $(1,3)$ (C) $(-1,-3)$ (D) $(-3,1)$

38. **If** A, B **and** ΔABC **are collinear, then the area of** C **is**

(A) 1 (B) 2 (C) 3 (D) 0

39. **If** $A(x_1,y_1)$ **and** $B(x_2,y_2)$, **then the bisector point of** $\overline{AB}$ **is**

(A) $\left(\frac{x_2-x_1}{2},\frac{y_2-y_1}{2}\right)$ (B) $\left(\frac{x_1-x_2}{2},\frac{y_1-y_2}{2}\right)$

(C) $\left(\frac{x_1+x_2}{3},\frac{y_1+y_2}{3}\right)$ (D) $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

40. **The ratio in which segment joining** $(-2,6)$ **and** $(3,1)$ **divided by** $y-$ **axis is**

(A) $2:3$ (B) $1:6$ (C) $6:1$ (D) $-2:3$

ANSWERS

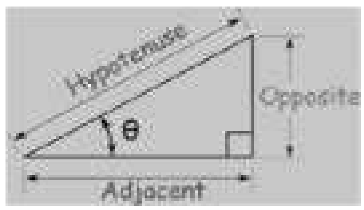
1	B	2	A	3	D	4	C	5	B	6	B	7	D	8	C	9	D	10	D
11	A	12	A	13	B	14	C	15	A	16	D	17	C	18	B	19	D	20	D
21	A	22	C	23	D	24	B	25	A	26	B	27	B	28	D	29	C	30	C
31	D	32	C	33	B	34	B	35	B	36	D	37	B	38	D	39	D	40	A

* * *

8. INTRODUCTION TO TRIGONOMETRY

SYNOPSIS

1. An angle is considered as the figure obtained by rotating a given ray about its end point. The revolving ray is called the generating line of the angle. The initial position OA is called the terminal side of the angle.
2. The measure of an angle is the amount of rotation from the initial side to the terminal side.
3. If Δ ABC is a right angled triangle right angle at B and $\angle BAC = \theta$, then with reference to angle we have .



$\cos\theta = \dfrac{Adjacent\ side}{Hypotenuse}$ $\sin\theta = \dfrac{Opposite\ side}{Hypotenuse}$

$\tan\theta = \dfrac{Opposite\ side}{Adjacent\ side}$

$\sec\theta = \dfrac{Hypotenuse}{Adjacent\ side}$ $\cot\theta = \dfrac{Adjacent\ side}{Opposite\ side}$

$\text{cosec}\,\theta = \dfrac{Hypotenuse}{Opposite\ side}$

4. We have

(i) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ or $\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$ (ii) $\sec\theta = \dfrac{1}{\cos\theta}$ or $\cos\theta = \dfrac{1}{\sec\theta}$

(iii) $\tan\theta = \dfrac{1}{\cot\theta}$ or $\cot\theta = \dfrac{1}{\tan\theta}$ (iv) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ or $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

5. The trigonometric ratios for angles are given in the following table

Ratio	0^0	30^0	45^0	60^0	90^0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
$\text{cosec}\,\theta$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

6. The values of $\sin\theta$ and $\cos\theta$ never exceed 1, where as the values of $\sec\theta$ and $\text{cosec}\,\theta$ are always greater than or equal to 1.
7. Also,

   (i) $\sin(90^0-\theta)=\sin\theta$ (ii) $\cos(90^0-\theta)=\cos\theta$

   (iii) $\tan(90^0-\theta)=\tan\theta$ (iv) $\cot(90^0-\theta)=\cot\theta$

   (v) $\sec(90^0-\theta)=\sec\theta$ (vi) $\text{cosec}(90^0-\theta)=\text{cosec}\,\theta$
8. An equation is called an identity if its true for all values of variables involved.
9. An equation involving trigonometric ratios of an angle θ is called trigonometric identity. It is true for all the values of the angle.
10. Some important Trigonometric Identities

    (i) $\sin^2\theta+\cos^2\theta=1$ or $1-\cos^2\theta=\sin^2\theta$ or $1-\sin^2\theta=\cos^2\theta$

    (ii) $\sec^2\theta-\tan^2\theta=1$ r o $1+\tan^2\theta=\sec^2\theta$ or $\sec^2\theta-1=\tan^2\theta$

    (iii) $\text{cosec}^2\theta-\cot^2\theta=1$ r o $1+\cot^2\theta=\text{cosec}^2\theta$ or $\text{cosec}^2\theta-1=\cot^2\theta$

SOLVED PROBLEMS

Question 1: If $\cos A=\dfrac{4}{5}$, then the value of tanA is

(A) $\dfrac{3}{5}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{4}{3}$ (D) $\dfrac{5}{3}$

Solution: (B) Given $\cos A=\dfrac{4}{5}$

We have $\sin A=\sqrt{1-\cos^2 A}=\sqrt{1-\left(\dfrac{4}{5}\right)^2}=\sqrt{1-\dfrac{16}{25}}=\sqrt{\dfrac{25-16}{25}}=\sqrt{\dfrac{9}{25}}=\dfrac{3}{5}$

Now, $\tan A=\dfrac{\sin A}{\cos A}=\dfrac{3/5}{4/5}=\dfrac{3}{4}$

Question 2: If $\sin A=\dfrac{1}{2}$, then the value of cot A is

(A) $\sqrt{3}$ (B) $\dfrac{1}{\sqrt{3}}$ (C) $\dfrac{\sqrt{3}}{2}$ (D) 1

Solution: (A) Given, $\sin A=\dfrac{1}{2}$

We have $\cos A=\sqrt{1-\sin^2 A}=\sqrt{1-\left(\dfrac{1}{2}\right)^2}=\sqrt{1-\dfrac{1}{4}}=\sqrt{\dfrac{4-1}{4}}=\sqrt{\dfrac{3}{4}}=\dfrac{\sqrt{3}}{2}$

Now, $\cot A=\dfrac{\cos A}{\sin A}=\dfrac{\sqrt{3}/2}{1/2}=\sqrt{3}$

Question 3: The value of the expression

$\text{cosec}(75^0+\theta)-\sec(15^0-\theta)-\tan(55^0+\theta)+\cot(35^0-\theta)$ is

(A) 1 (B) 0 (C) 1 (D) 3/2

Solution: (B) Given expression:

$\text{cosec}(75^0+\theta)-\sec(15^0-\theta)-\tan(55^0+\theta)+\cot(35^0-\theta)$

$=\text{cosec}[90^0-(15^0-\theta)]-\sec(15^0-\theta)-\tan[90^0-(35^0-\theta)]+\cot(35^0-\theta)$

$=\sec(15^0-\theta)-\sec(15^0-\theta)-\cot(35^0-\theta)+\cot(35^0-\theta)=0$

$\left[\because \operatorname{cosec}(90^0-\theta)=\sec\theta;\tan(90^0-\theta)=\cot\theta\right]$

Hence, the required value of the given expression is 0.

Question 4: If $\sin\ \theta=\dfrac{a}{b}$, then the value of $\cos\ \theta$ is

(A) $\dfrac{b}{\sqrt{b^2-a^2}}$ (B) $\dfrac{a}{b}$ (C) $\dfrac{\sqrt{b^2-a^2}}{b}$ (D) $\dfrac{a}{\sqrt{b^2-a^2}}$

Solution: (A) Given, $\sin\ \theta=\dfrac{a}{b}$

We have $\cos\theta=\sqrt{1-\sin^2\theta}=\sqrt{1-\left(\dfrac{a}{b}\right)^2}=\sqrt{1-\dfrac{a^2}{b^2}}=\sqrt{\dfrac{b^2-a^2}{b^2}}=\dfrac{\sqrt{b^2-a^2}}{b}$

Question 5: If $\cos(\alpha+\beta)=0$, then $\sin(\alpha-\beta)$ can be reduced to

(A) $\cos\beta$ (B) $\cos 2\beta$ (C) $\sin\alpha$ (D) $\sin 2\alpha$

Solution: (B) Given, $\cos(\alpha+\beta)=0$

$\Rightarrow \cos(\alpha+\beta)=0=\cos 90^0$

$\Rightarrow \alpha+\beta=90^0$

$\Rightarrow \alpha=90^0-\beta$

Now, $\sin(\alpha-\beta)=\sin(90^0-\beta-\beta)=\sin(90^0-2\beta)=\cos 2\beta$

Hence, $\sin(\alpha-\beta)$ can be reduced to $\cos 2\beta$.

Question 6: The value of (tan l° tan 2° tan 3°…tan 89°) is

(A) 0 (B) 1 (C) 2 (D) $\dfrac{1}{2}$

Solution: (B) tan l° tan 2° tan 3°…tan 89°

= tan l° tan 2° tan 3°… tan 44° tan45° tan 46°…tan 88° tan 89°

= tan l° tan 2° tan 3°… tan 44° (1) tan (90°−44°)…tan (90°−2°) tan (90°−1°) $\left(\because \tan 45^0=1\right)$

= tan l° tan 2° tan 3°… tan 44° cot44°…cot 2° cot 1° $\left(\because \tan(90^0-\theta)=\cot\theta\right)$

= tan l° tan 2° tan 3°… tan 44° $\dfrac{1}{\tan 44°}\ \ldots\dfrac{1}{\tan 2°}\ \dfrac{1}{\tan 1°}$ $\left(\because \cot\theta=\dfrac{1}{\tan\theta}\right)$

$=1$

Question 7: If $\cos 9\alpha=\sin\alpha$ and $9\alpha<90^\circ$, then the value of $\tan 5\alpha$ is

(A) $\dfrac{1}{\sqrt{3}}$ (B) $\sqrt{3}$ (C) 0 (D) 1

Solution: (D) Given, $\cos 9\alpha=\sin\alpha$ and $9\alpha<90^\circ$

$\Rightarrow \sin(90^\circ-9\alpha)=\sin\alpha\left(\because \cos\theta=\sin(90^\circ-\theta)\right)$

$\Rightarrow 90^\circ-9\alpha=\alpha$

$\Rightarrow 90^\circ=9\alpha+\alpha$

$\Rightarrow 10\alpha=90^\circ$

$\Rightarrow \alpha=\dfrac{90^\circ}{10}=9^\circ$

$\therefore \tan 5\alpha=\tan(5\times 9^\circ)=\tan 45^\circ=1$

Question 8: If Δ ABC is right angled at C, then the value of cos (A+B) is

(A) 0 (B) 1 (C) $\dfrac{1}{2}$ (D) $\dfrac{\sqrt{3}}{2}$

Solution: (A) We know that in Δ ABC, sum of three angles $=180^\circ$

i.e., $\angle A + \angle B + \angle C = 180^\circ$

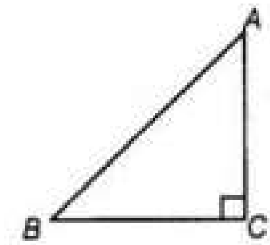
But is right angled at C *i.e.*, $\angle C = 90^\circ$

$\angle A + \angle B + 90^\circ = 180^\circ$

$\Rightarrow \angle A + \angle B = 180^\circ - 90^\circ$

$\Rightarrow \angle A + \angle B = 90^\circ$

Now, $\cos(A+B) = \cos 90^\circ = 0$.



Question 9: If sin A + $\sin^2$A=1, then the value of ($\cos^2$ A+$\cos^4$ A) is

(A) 1 (B) 3 (C) 2 (D) 0

Solution: (A) Given, $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A$

$\Rightarrow \sin A = \cos^2 A$

Now, $\cos^2 A + \cos^4 A = \sin A + \sin^2 A = 1$

Question 10: If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$ then the value of ($\alpha + \beta$) is

(A) 0^0 (B)30^0 (C) 60^0 (D) 90^0

Solution: (D) Given $\sin\alpha = \frac{1}{2} = \sin 30^0 \Rightarrow \alpha = 30^0$

and $\cos\beta = \frac{1}{2} = \cos 60^0 \Rightarrow \beta = 60^0$

$\therefore \alpha + \beta = 30^0 + 60^0 = 90^0$

Question 11: The value of the expression $\left(\frac{\sin^2 22^0 + \sin^2 68^0}{\cos^2 22^0 + \cos^2 68^0} + \sin^2 63^0 + \cos 63^0 \sin 27^0\right)$ is

(A) 1 (B) 2 (C) 3 (D) 0

Solution: (B) Given expression: $\frac{\sin^2 22^0 + \sin^2 68^0}{\cos^2 22^0 + \cos^2 68^0} + \sin^2 63^0 + \cos 63^0 \sin 27^0$

$= \frac{\sin^2 22^0 + \sin^2(90^0 - 22^0)}{\cos^2 22^0 + \cos^2(90^0 - 22^0)} + \sin^2 63^0 + \cos 63^0 \sin(90^0 - 63^0)$

$= \frac{\sin^2 22^0 + \cos^2 22^0}{\cos^2 22^0 + \sin^2 22^0} + \sin^2 63^0 + \cos 63^0 \cos 63^0 \left[\because \sin(90^0 - \theta) = \cos\theta; \cos(90^0 - \theta) = \sin\theta\right]$

$= 1 + \sin^2 63^0 + \cos^2 63^0 = 1 + 1 \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= 2$

Question 12: If $4\tan\theta = 3$, then $\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}$ is equal to

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Solution: (C) Given, $4\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{4}$

Now, $\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \frac{4\left(\frac{\sin\theta}{\cos\theta}\right) - 1}{4\left(\frac{\sin\theta}{\cos\theta}\right) + 1} = \frac{4\tan\theta - 1}{4\tan\theta + 1} = \frac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \frac{3-1}{3+1} = \frac{2}{4} = \frac{1}{2}$

Question 13: If $\sin\theta - \cos\theta = 0$, then the value of ($\sin^4\theta + \cos^4\theta$) is

(A) 1 (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Solution: (C) Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \theta = 45^0$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^0 + \cos^4 45^0 = \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Question 14: $\sin(45^0 + \theta) - \cos(45^0 - \theta)$ is equal to

(A) $2\cos\theta$ (B) 0 (C) $2\sin\theta$ (D) 1

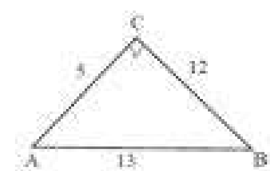
Solution: (B) $\sin(45^0 + \theta) - \cos(45^0 - \theta) = \sin[90^0 - (45^0 - \theta)] - \cos(45^0 - \theta)$

$= \cos(45^0 - \theta) - \cos(45^0 - \theta)$ $\left[\because \sin(90^0 - \theta) = \cos\theta\right]$

$= 0$

MULTIPLE CHOICE QUESTIONS

1. **In the following figure, the value of cot A is**



(A) $\frac{12}{5}$ (B) $\frac{13}{12}$

(C) $\frac{5}{12}$ (D) $\frac{12}{13}$

2. **If in ΔABC, $\angle B = 90^0$, AB = 12 cm and BC= 5cm, then the value of cos C is**

(A) $\frac{5}{12}$ (B) $\frac{13}{12}$ (C) $\frac{12}{5}$ (D) $\frac{12}{13}$

3. **If** $\cot\theta = \frac{b}{a}$, **then** $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$ **is equal to**

(A) $\frac{b-a}{b+a}$ (B) $\frac{a+b}{a-b}$ (C) $\frac{a-b}{a+b}$ (D) $\frac{b+a}{b-a}$

4. **The maximum value of sin θ is**

(A) 1 (B) 2 (C) 0 (D) −1

5. **If A is an acute angle of a ΔABC, right angled at B, then the value of sin A + cos A is**

(A) <1 (B) $=1$ (C) ≤ 1 (D) >1

6. **The value of** $\frac{2\tan 30^0}{1+\tan^2 30^0}$ **is equal to**

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1

7. **If sin** $\theta = \frac{1}{2}$**, then the value of** $(\tan\theta + \cot\theta)^2$ **is**

(A) $\frac{3}{16}$ (B) $\frac{16}{3}$ (C) 4 (D) $\frac{36}{5}$

8. **The value of** $\frac{2\tan 30^0}{1-\tan^2 30^0}$ **is equal to**

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$ (C) $\sqrt{3}$ (D) 1

9. **If $\theta = 45^0$ then the value of** $\frac{1-\cos 2\theta}{\sin 2\theta}$ **is equal to**

(A) −1 (B) 1 (C) 0 (D) $\frac{1}{\sqrt{2}}$

10. If tan θ = cot θ, then the value of Sec θ is

(A) $\frac{2}{\sqrt{3}}$ (B) 2 (C) $\sqrt{2}$ (D) 0

11. If A+B = 90^0, cot B = 3/4, then tan A is equal to

(A) $\frac{4}{3}$ (B) $\frac{3}{7}$ (C) $\frac{3}{4}$ (D) $\frac{4}{7}$

12. If sin $(x-20^0)$ = cos $(3x-10)^0$. Then x is

(A) 30^0 (B) 5^0 (C) 10^0 (D) 45^0

13. The value of 1+tan5^0 cot85^0 is equal to

(A) 2 (B) $\frac{1}{2}$ (C) 4 (D) $\frac{1}{\sqrt{3}}$

14. In any triangle ABC, the value of sin $\left(\frac{B+C}{2}\right)$ is

(A) $\cos\frac{B}{2}$ (B) $\cos\left(\frac{B+C}{2}\right)$ (C) 0 (D) $\cos\frac{A}{2}$

15. If cos $\theta = \frac{a}{b}$, then cosec θ is equal to

(A) $\frac{b}{\sqrt{b^2-a^2}}$ (B) $\frac{a}{b}$ (C) $\frac{a}{\sqrt{b^2-a^2}}$ (D) $\frac{\sqrt{b^2-a^2}}{b}$

16. The value of cos20^{0}cos70^0− sin20^{0}sin70^0 is equal to

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) −1

17. The value of tan5^0 tan 25^0 tan45^0 tan65^0 tan 85^0 is

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) −1

18. If tan θ + cot θ = 5 then the value of tan$^2\theta$ + cot$^2\theta$ is

(A) 2 (B) 29 (C) 23 (D) 27

19. If cosec θ = 2 and cot $\theta = \sqrt{3}p$ where θ is an acute angle, then the value of p is

(A) $\frac{2}{\sqrt{3}}$ (B) 1 (C) $\sqrt{2}$ (D) 0

20. $\sqrt{\frac{1+\sin A}{1-\sin A}}$ is equal to

(A) $\sec A-\tan A$ (B) 0 (C) $\sec A+\tan A$ (D) 1

21. If cosec θ − cot θ = 1/4 then the value of cosecθ+ cot θ is

(A) $\frac{17}{4}$ (B) $\frac{4}{17}$ (C) 4 (D) 2

22. sin 45^0+ cos 45^0 =

(A) $\frac{1}{\sqrt{2}}$ (B) 1 (C) 0 (D) $\sqrt{2}$

23. 2 tan^2 45^0+ cos^2 30^0 − sin^2 60^0 =

(A) $\frac{1}{\sqrt{2}}$ (B) 2 (C) $\frac{\sqrt{3}}{2}$ (D) 0

24. sin (90^0−A) =

(A) sin A (B) cos A (C) cosec A (D) sec A

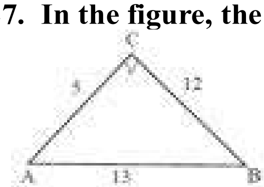
25. If sin A = cos B then, the value of A + B =

(A) 0^0 (B) 30^0 (C) 60^0 (D) 90^0

26. If $\sec\theta = \frac{m+n}{2\sqrt{mn}}$**, then sin** θ **=**

(A) $\frac{m+n}{m-n}$ (B) $\frac{m-n}{m+n}$ (C) $\frac{1}{m+n}$ (D) $\frac{1}{\sqrt{m+n}}$

27. In the figure, the value of sec A is



(A) $\frac{12}{5}$ (B) $\frac{13}{12}$ (C) $\frac{5}{12}$ (D) $\frac{13}{5}$

28. If sin 2A = 1/2 tan^{2}45^0, where A is an acute angle then the value of A is

(A) 5^0 (B) 30^0 (C) 45^0 (D) 15^0

29. The maximum value of 1/secθ**,** $0^0 < \theta < 90^0$ **is**

(A) 0 (B) 2 (C) 1 (D) $\sqrt{2}$

30. $\frac{1-\cos^2\theta}{\sin^2\theta}$ **is equal to**

(A) 0 (B) 1 (C) 2 (D) $\frac{1}{\sqrt{2}}$

31. If cot θ =1 then $\frac{1+\sin\theta}{\cos\theta}$ **is equal to**

(A) $\sqrt{2}-1$ (B) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (C) $\frac{\sqrt{2}-1}{\sqrt{2}}$ (D) $\sqrt{2}+1$

32. $\mathbf{Sec^2\theta - 1 =}$

(A) $\cot^2\theta$ (B) $\tan^2\theta$ (C) $\cos^2\theta$ (D) $\sin^2\theta$

33. If sec θ **+ tan** θ **=** p**, then the value of sec** θ **– tan** θ **=**

(A) $\frac{p}{\sqrt{p^2-1}}$ (B) 0 (C) $\frac{1}{p}$ (D) $\frac{1}{\sqrt{p}}$

34. The value of sin A or cos A never exceeds

(A) 1 (B) –1 (C) 0 (D) $\frac{1}{\sqrt{2}}$

35. sec (90^0– A) =

(A) sin A (B) cos A (C) cosec A (D) sec A

36. $9\sec^2\theta - 9\tan^2\theta =$

(A) 0 (B) 9 (C) $\frac{1}{9}$ (D) 1

37. $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta) =$

(A) 0 (B) 1 (C) 2 (D) –1

38. $(\sec A + \tan A)(1-\sin A) =$

(A) $\sec A$ (B) $\cos A$ (C) $\sin A$ (D) $\operatorname{cosec} A$

39. $\frac{1+\tan^2\theta}{1+\cot^2\theta} =$

(A) $\tan^2\theta$ (B) $\cot^2\theta$ (C) 0 (D) 1

40. $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} =$

(A) $2\sec A$ (B) $2\cos A$ (C) $2\sin A$ (D) $2\operatorname{cosec} A$

41. $\frac{\cos A - \sin A}{\cos A + \sin A} =$
(A) $\sec A - \tan A$ (B) $\sec A - \tan A$ (C) $2\sin A$ (D) $2\operatorname{cosec} A$

42. $\sqrt{\frac{1-\sin A}{1+\sin A}} =$
(A) $\sec A + \tan A$ (B) 0 (C) $\sec A - \tan A$ (D) 1

43. $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} =$
(A) $\sec A + \tan A$ (B) 0 (C) $\sec A - \tan A$ (D) 1

44. $\sin 60^0 \cos 30^0 + \sin 30^0 \cos 60^0 =$
(A) 2 (B) 0 (C) 1 (D) -1

45. If sin (A – B) = $\frac{1}{2}$, cos (A + B) = $\frac{1}{2}$,0 ° < A + B < 90° and A > B, the values of A and B are respectively
(A) $60^0 \& 30^0$ (B) $45^0 \& 30^0$ (C) $60^0 \& 45^0$ (D) $45^0 \& 15^0$

46. If tan (A – B) = $\frac{1}{\sqrt{3}}$, tan (A + B) = $\sqrt{3}$,0 ° < A + B < 90° and A > B, the values of A and B are respectively
(A) $60^0 \& 30^0$ (B) $45^0 \& 30^0$ (C) $60^0 \& 45^0$ (D) $45^0 \& 15^0$

47. sin 2A = 2 sin A is true and A is an acute angle then the value of A is
(A) 0^0 (B) 30^0 (C) 45^0 (D) 60^0

48. $\frac{\sin 30^0 + \tan 45^0}{\cos 60^0 + \cot 45^0} =$
(A) 2 (B) 0 (C) 1 (D) -1

49. If 15 cot A = 8, then the value of sin A is
(A) $\frac{15}{8}$ (B) $\frac{15}{17}$ (C) $\frac{17}{15}$ (D) $\frac{8}{17}$

50. If $\cot\theta = \frac{7}{8}$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is
(A) $\frac{64}{49}$ (B) 15 (C) $\frac{49}{64}$ (D) $\frac{1}{15}$

ANSWERS

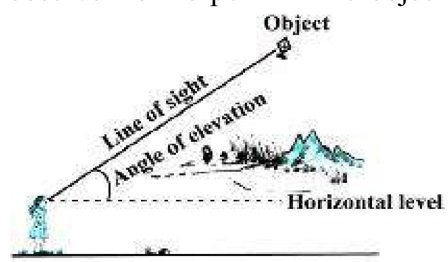
1	C	2	A	3	D	4	A	5	D	6	C	7	B	8	C	9	B	10	C
11	C	12	A	13	A	14	D	15	D	16	B	17	A	18	C	19	B	20	C
21	C	22	D	23	B	24	B	25	D	26	B	27	D	28	D	29	C	30	B
31	D	32	B	33	C	34	A	35	C	36	B	37	C	38	B	39	A	40	A
41	B	42	B	43	C	44	C	45	D	46	D	47	C	48	C	49	B	50	C

* * *

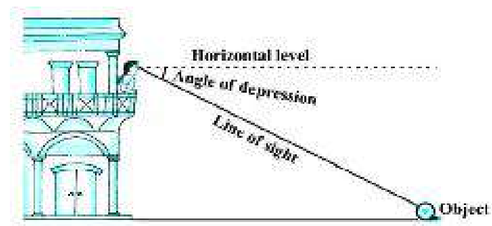
9. SOME APPLICATIONS OF TRIGONOMETRY

SYNOPSIS

1. The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.
2. The **angle of elevation** of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level, i.e., the case when we raise our head to look at the object.



3. The **angle of depression** of an object viewed, is the angle formed by the line of sight with the horizontal when it is below the horizontal level, i.e., the case when we lower our head to look at the object.



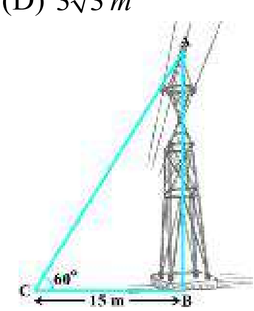
4. The height or length of an object or the distance between two distant objects can be determined with the help of trigonometric ratios.

SOLVED PROBLEMS

Question 1: A tower stands vertically on the ground. From a point on the ground, which is 15 *m* away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°, then the height of the tower is

(A) $5\sqrt{3}\,m$ (B) $15\sqrt{3}\,m$ (C) $10\sqrt{3}\,m$ (D) $3\sqrt{3}\,m$

Solution: (B) Here AB represents the tower, CB is the distance of the point from the tower a n d Δ ACB is the angle of elevation. We need to determine the height of the tower, i.e., AB. Also, ACB is a triangle, right-angled at B.



From the given data $\tan 60^0 = \frac{AB}{AC}$

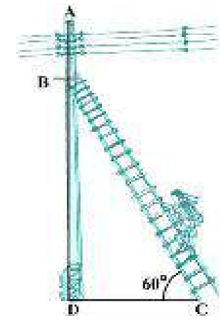
$\frac{AB}{15} \Rightarrow \sqrt{3} =$

$= 15\sqrt{3} \Rightarrow AB$

Hence, the height of the tower is $15\sqrt{3}\,m$

Question 2: An electrician has to repair an electric fault on a pole of height 5 m. She needs to reach a point 1.3m below the top of the pole to undertake the repair work. What should be the length of the ladder that she should use which, when inclined at an angle of 60° to the horizontal? (Take $\sqrt{3}$ = 1.73)

(A) $4.28\,m$ (B) $2.14\,m$ (C) $3.14\,m$ (D) $5\,m$



Solution: (A) The electrician is required to reach the point B on the pole AD.

So, BD = AD – AB = (5 – 1.3) m = 3.7 m.

Here, BC represents the ladder. We need to find its length, i.e., the hypotenuse of the right triangle BDC.

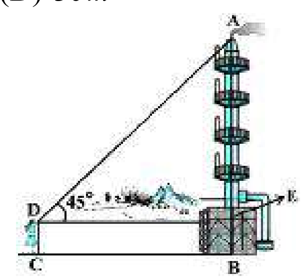
We have $\sin 60^0 = \frac{BD}{BC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3.7}{BC}$

Therefore, $BC = \frac{3.7 \times 2}{\sqrt{3}} = \frac{3.7 \times 2}{1.73} = 4.28\ m$ (approx.)

i.e., the length of the ladder should be 4.28 m.

Question 3: An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°, then the height of the Chimney is

(A) 27 m (B) 25 m (C) 14 m (D) 30 m



Solution: (D) Here, AB is the chimney, CD the observer and Δ ADE the angle of elevation. In this case, ADE is a triangle, right-angled at E and we are required to find the height of the chimney.

We have AB = AE + BE = AE + 1.5 and DE = CB = 28.5 m

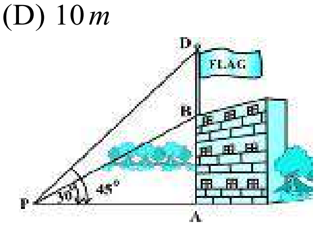
Now, $\tan 45^0 = \frac{AE}{DE} \Rightarrow 1 = \frac{AE}{28.5}$

Therefore, AE = 28.5

So the height of the chimney (AB) = (28.5 + 1.5) m = 30 m.

Question 4: From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. The distance of the building from the point P is (Take $\sqrt{3}$ = 1.732)

(A) 17.64 m (B) 17.32 m (C) 7.32 m (D) 10 m



Solution: (B) Here, AB denotes the height of the building, BD the flagstaff and P the given point. Note that there are two right triangles PAB and PAD. We are required to find the distance of the building from the point P, i.e., PA.

We have $\tan 30^0 = \frac{AB}{AP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{AP}$

$\Rightarrow AP = 10(1.732) = 17.32 = 10\sqrt{3}$

Therefore, the distance of the building from the point P, i.e., PA $= 17.32$

Question 5: The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°, then the height of the tower is

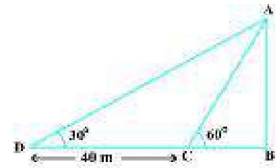
(A) $20\sqrt{3}\,m$ (B) $15\sqrt{3}\,m$ (C) $10\sqrt{3}\,m$ (D) $5\sqrt{3}\,m$

Solution: (A) Here, AB is the tower and BC is the length of the shadow when the Sun's altitude is 60°, i.e., the angle of elevation of the top of the tower from the tip of the shadow is 60° and DB is the length of the shadow, when the angle of elevation is 30°.

Now, let AB be h m and BC be x m. According to the question, DB is 40 m longer than BC.

So, DB = $(40 + x)$ m

Now, we have two right triangles ABC and ABD.



In ΔABC, $\tan 60^0 = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x}$ (1)

In ΔABD, $\tan 30^0 = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+40}$ (2)

From (1), we have $h = x\sqrt{3}$

Putting this value in (2), we get $(x\sqrt{3})\sqrt{3} = x + 40$, i.e., $3x = x + 40$

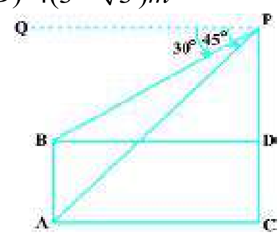
i.e., $x = 20$

So, $h = 20\sqrt{3}$ [From (1)]

Therefore, the height of the tower is $20\sqrt{3}m$.

Question 6: The angles of depression of the top and the bottom of an 8 *m* tall building from the top of a multi-storeyed building are 30° and 45°, respectively, then the distance between the two buildings is

(A) $4(\sqrt{3}+1)m$ (B) $4(\sqrt{3}-1)m$ (C) $4(\sqrt{3}+3)m$ (D) $4(3-\sqrt{3})m$



Solution: (C) Here, PC denotes the multi-storyed building and AB denotes the 8 *m* tall building. We are interested to determine the height of the multi-storeyed building, i.e., PC and the distance between the two buildings, i.e., AC. Observe that PB is a transversal to the parallel lines PQ and BD.

Therefore, $\angle$QPB and $\angle$PBD are alternate angles, and so are equal.

So $\angle$ PBD = 30°. Similarly, $\angle$ PAC = 45°.

In right Δ PBD, we have $\tan 30^0 = \frac{PD}{BD}$ or $\frac{1}{\sqrt{3}} = \frac{PD}{BD}$

$\Rightarrow BD = PD\sqrt{3}$

In right Δ PAC, we have $\tan 45^0 = \frac{PC}{AC}$ orPC $1 = \frac{PC}{AC}$

i.e., PC = AC

Also, PC = PD + DC, therefore, PD + DC = AC.

Since, AC = BD and DC = AB = 8 *m*, we get PD + 8 = $BD = PD\sqrt{3}$

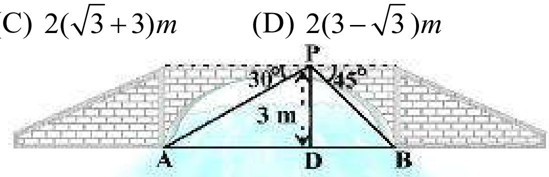
This gives PD $= \frac{8}{\sqrt{3}-1} = \frac{8(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = 4(\sqrt{3}+1)$

So, the height of the multi-storeyed building is $= 4(\sqrt{3}+1)+8 = 4(\sqrt{3}+3)$ *m* and the distance between the two buildings is also $4(\sqrt{3}+3)$)*m*.

Question 7: From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 *m* from the banks, then the width of the river is

(A) $3(\sqrt{3}+1)m$ (B) $3(\sqrt{3}-1)m$ (C) $2(\sqrt{3}+3)m$ (D) $2(3-\sqrt{3})m$

Solution: (A) Let A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, i.e., DP = 3 *m*.



We are interested to determine the width of the river, which is the length of the side AB of the Δ APB.

Now, AB = AD + DB

In right ΔAPD, $\angle$A = 30°.

So, $\tan 30^0 = \frac{PD}{AD}$ or $\frac{1}{\sqrt{3}} = \frac{3}{AD}$

$\Rightarrow AD = 3\sqrt{3}$

In right Δ PBD, $\angle$B = 45°

we have $\tan 45^0 = \frac{PD}{BD}$ or $BD = PD = 3m$

Now, BD + AD = AB.= $3 + 3\sqrt{3} = 3(\sqrt{3}+1)\,m$

Therefore, the width of the river is $3(\sqrt{3}+1)\,m$.

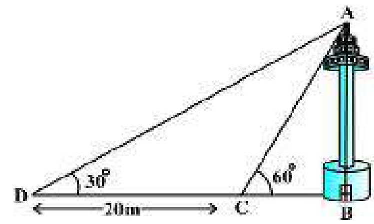
MULTIPLE CHOICE QUESTIONS

1. **A circus artist is climbing a 20 *m* long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. If the angle made by the rope with the ground level is 30°, then the height of the pole is**
   (A) $5m$ (B) $40m$ (C) $10m$ (D) $20m$
2. **A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 *m*. The height of the tree is**
   (A) $8\sqrt{3}\,m$ (B) $\frac{\sqrt{3}}{8}m$ (C) $\frac{8}{\sqrt{3}}m$ (D) $5\sqrt{3}\,m$
3. **The angle of elevation of the top of a tower from a point on the ground, which is 30 *m* away from the foot of the tower, is 30°, then the height of the tower is**
   (A) $5\sqrt{3}\,m$ (B) $\frac{\sqrt{3}}{10}m$ (C) $\frac{10}{\sqrt{3}}m$ (D) $10\sqrt{3}\,m$
4. **A kite is flying at a height of 60 *m* above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Assuming that there is no slack in the string, then the length of the string is**
   (A) $40\sqrt{3}\,m$ (B) $5\sqrt{3}\,m$ (C) $20\sqrt{3}\,m$ (D) $10\sqrt{3}\,m$
5. **A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building, then the distance he walked towards the building is**
   (A) $5\sqrt{3}\,m$ (B) $10\sqrt{3}\,m$ (C) $20\sqrt{3}\,m$ (D) $19\sqrt{3}\,m$
6. **From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 *m* high building are 45° and 60° respectively, then the height of the tower is**
   (A) $20(\sqrt{3}+1)m$ (B) $20(\sqrt{3}-1)m$ (C) $10(\sqrt{3}+3)m$ (D) $10(3-\sqrt{3})m$
7. **A statue, 1.6 *m* tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°, then the height of the pedestal is**
   (A) $0.8(\sqrt{3}+1)m$ (B) $0.8(\sqrt{3}-1)m$ (C) $1.6(\sqrt{3}+3)m$ (D) $1.6(3-\sqrt{3})m$
8. **The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 *m* high, then the height of the building is**
   (A) $8.33m$ (B) $16.33\,m$ (C) $16.66m$ (D) $8.66m$

9. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively, then the distances of the point from the poles is

(A) $30m$ (B) $60\ m$ (C) $20\sqrt{3}\,m$ (D) $20m$

10. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 *m* away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the tower is 30° as shown in the Fig, then the width of the canal is



(A) $10\sqrt{3}\,m$ (B) $20\sqrt{3}\,m$ (C) $10m$ (D) $20m$

11. From the top of a 7 *m* high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. The height of the tower is

(A) $14(\sqrt{3}+1)m$ (B) $7(\sqrt{3}-1)m$ (C) $7(\sqrt{3}+1)m$ (D) $14(3-\sqrt{3})m$

12. As observed from the top of a 75 *m* high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, then the distance between the two ships is

(A) $75(\sqrt{3}-1)m$ (B) $75(\sqrt{3}+1)m$ (C) $75(\sqrt{3}+3)m$ (D) $25(3-\sqrt{3})m$

13. A boy observed the top of an electric pole at an angle of elevation of when the observation point is 8m from the foot of the pole then the height of the pole is

(A) $4\sqrt{3}\ m$ (B) $\sqrt{3}\ m$ (C) $8\sqrt{3}\ m$ (D) $2\sqrt{3}\ m$

14. Rajender observes a person standing on the ground from a helicopter at angle of depression 45°. If the helicopter flies at a height of 50 *m* from the ground, then the distance of the person from Rajender is

(A) $50\sqrt{2}\ m$ (B) $25\sqrt{2}\ m$ (C) $\sqrt{2}\ m$ (D) None

15. Length of the shadow of a 15 *m* high pole is m at 7° clock in the morning. Then what is the angle of elevation of the sun rays with the ground at the time?

(A) 45^0 (B) 30^0 (C) 60^0 (D) 90^0

16. The ratio of the length of a rod and its shadow is $1:\sqrt{3}$. The angle of elevation of the sun is

(A) 30^0 (B) 45^0 (C) 60^0 (D) 90^0

17. If the angle of elevation of a tower from a distance of 100 *m* from its foot is 60^0, then the height of the tower is

(A) $100\sqrt{3}\ m$ (B) $\frac{100}{\sqrt{3}}\ m$ (C) $50\sqrt{3}\ m$ (D) $\frac{200}{\sqrt{3}}\ m$

18. If the altitude of the sun is at 60^0, then the height of the vertical tower that will cast a shadow of length, 30 *m* is

(A) $30\sqrt{3}\ m$ (B) $15\ m$ (C) $\frac{30}{\sqrt{3}}\ m$ (D) $15\sqrt{2}\ m$

19. A statue stands on the top of a 2 *m* tall pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60^0 and from the same point, the angle of elevation of the top of the pedestal is 45^0. Then the height of the statue is

(A) $1.5\ m$ (B) $1.3\ m$ (C) $1.2\ m$ (D) None

20. From the top of a building, the angle of elevation of the top of a cell tower is and the angle of depression to its foot is 45^0. If distance of the building from the tower is 7 *m*, then the height of the tower is

(A) $17\ m$ (B) $18\ m$ (C) $19\ m$ (D) $20\ m$

21. A wire of length 18m had been tied with an electric pole at angle of elevation with the ground. As it is covering a long distance, it was cut and tied at angle of elevation 60^0 with the ground. Then the length of the wire that has to cut

(A) 9.6 m (B) 7.6 m (C) 86 m (D) None

22. The angles of elevation of the top of a tower from two points at a distance of 4 *m* and 9 *m*, then the height of the tower from the base of the tower and in the same straight line with it are complementary

(A) 8 m (B) 7 m (C) 6 m (D) 5 m

23. From a light house the angles of depression of two ships on opposite sides of the light house are observed to be 30^0 and 45^0. If the height of the light house is h meters. Then the distance between the ship is

(A) $(\sqrt{3}+1)h\ m$ (B) $(\sqrt{3}-1)h\ m$ (C) $\sqrt{3}\ h\ m$ (D) $1+\left(1+\frac{1}{\sqrt{3}}\right)h\,m$

24. The tops of two poles of height 20 *m* and 14 *m* are connected by a wire. If the wire makes an angle of 30^0 with the horizontal, then the length of the wire is

(A) 12 m (B) 10 m (C) 8 m (D) 6 m

25. From the top of a cliff 25 *m* high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is

(A) 25 m (B) 5 m (C) 75m (D) 100m

26. If the angle of elevation of a cloud from a point 200 *m* above the lake is 30^0 and the angle of depression of its reflection in the lake is 60^0, then the height of the cloud above lake, is

(A) 200m (B) 500m (C) 300m (D) None

27. The height of a tower is 100 *m*. When the angle of elevation of the sun changes from from 30^0 to 45^0, the shadow of the tower becomes x meters less. The value of x is

(A) 100 m (B) $100\sqrt{3}\ m$ (C) $100(\sqrt{3}-1)\ m$ (D) $\frac{100}{\sqrt{3}}\ m$

28. Two persons are *a* meter apart and the height of one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevation of their tops to be complementary, then the height of the shorter person is

(A) $\frac{a}{4}$ (B) $\frac{a}{\sqrt{2}}$ (C) $a\sqrt{2}$ (D) $\frac{a}{2\sqrt{2}}$

29. A tree breaks due to storm and the broken pat bends so that the top of the tree touches the ground by making angle with the ground. The instance between the foot of the tree and the top of the tree on the ground is *m*. then the height of the tree before falling down is

(A) $6\sqrt{3}\ m$ (B) $\sqrt{3}\ m$ (C) 12 m (D) $\frac{6}{\sqrt{3}}\ m$

30. Two men on either side of a temple of 30 meters high observe its top at the angles of elevation 30^0 and 60^0 respectively. Then the distance between the two men is

(A) $20\sqrt{3}\ m$ (B) $40\sqrt{3}\ m$ (C) $\frac{20}{\sqrt{3}}\ m$ (D) $\frac{40}{\sqrt{3}}\ m$

ANSWERS

1	C	2	A	3	D	4	A	5	D	6	B	7	A	8	C	9	B	10	C
11	C	12	A	13	C	14	A	15	C	16	A	17	B	18	A	19	A	20	C
21	B	22	C	23	A	24	A	25	B	26	D	27	C	28	D	29	A	30	B

* * *

10. CIRCLES

SYNOPSIS

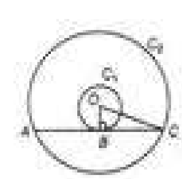
1. A **tangent** to a circle is a line that intersects the circle at only one point.
2. A **secant** to a circle is a line that intersects the circle at two points.
3. There is only one tangent at a point of the circle.
4. The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide.
5. The common point of the tangent and the circle is called the **point of contact** and the tangent is said to **touch** the circle at the common point.
6. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
7. At any point on a circle there can be one and only one tangent.
8. The line containing the radius through the point of contact is also sometimes called the '**normal**' to the circle at the point.
9. There is no tangent to a circle passing through a point lying inside the circle.
10. There is one and only one tangent to a circle passing through a point lying on the circle.
11. There are exactly two tangents to a circle through a point lying outside the circle.
12. The length of the segment of the tangent from the external point P and the point of contact with the circle is called the **length of the tangent** from the point P to the circle.
13. The lengths of tangents drawn from an external point to a circle are equal.
14. In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact

SOLVED PROBLEMS

Question 1: If radii of two concentric circles are 4 *cm* and 5 *cm*, then length of each chord of one circle which is tangent to the other circle, is

(A) 3 *cm* (B) 6 *cm* (C) 9 *cm* (D) 1 *cm*

Solution: (B) Let O be the centre of two concentric circles C_1 and C_2,whose radii are $r_1 = 4$ *cm* and $r_2 = 5$ *cm*.



Now, we draw a chord AC of circle C_2, which touches the circle C_1 at B.

Also, join OB, which is perpendicular to AC. [Tangent at any point of Circle is perpendicular to radius thorough the point of contact]

Now, in right angled Δ OBC, by using Pythagoras theorem, $OC^2 = BC^2 + BO^2$

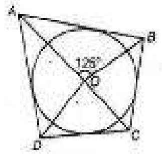
$\Rightarrow 5^2 = BC^2 + 4^2$

$\Rightarrow BC^2 = 5^2 - 4^2 = 25 - 16 = 9 = 3^2$

$\Rightarrow BC = 3cm$

$\therefore$Length of chord $AC = 2 \times BC = 2 \times 3 = 6cm$

Question 2: In figure, if $\angle$AOB = 125°, then $\angle$COD is equal to

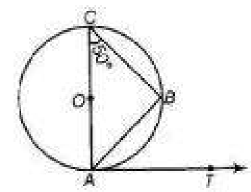


(A) 62.5° (B) 45° (C) 35° (D) 55°

Solution: (D) We know that, the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

i.e., $\angle AOB + \angle COD = 180°$

$\Rightarrow \angle COD = 180° - \angle AOB = 180° - 125° = 55°$



Question 3: In figure, AB is a chord of the circle and AOC is its diameter suchthat $\angle ACB = 50°$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to

(A) 45° (B) 60° (C) 50° (D) 55°

Solution: (C) In figure, AOC is a diameter of the circle. We know that, diameter subtends an angle 90° at the circle.

So, $\angle ABC = 90°$

In ΔABC, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + 90° + 50° = 180°$

$\Rightarrow \angle A + 140° = 180°$

$\Rightarrow \angle A = 180° - 140° = 40°$

Now, AT is the tangent to the circle at a point A. So, OA is perpendicular to AT.

$\therefore \angle OAT = 90°$

We have, $\angle OAB + \angle BAT = 90°$

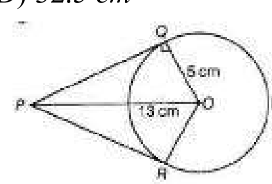
On putting $\angle OAB = 40°$, we get

$\Rightarrow \angle BAT = 90° - 40° = 50°$

Hence, the value of $\angle BAT$ is 50°

Question 4: From a point P which is at a distance of 13 *cm* from the centre O of a circle of radius 5 *cm*, the pair of tangents PQ and PR to the circle is drawn. Then, the area of the quadrilateral PQOR is

(A) 60 cm^2 (B) 65 cm^2 (C) 30 cm^2 (D) 32.5 cm^2



Solution: (A) Firstly, draw a circle of radius 5 *cm* having centre O. P is a point at a distance of 13 *cm* from O. A pair of tangents PQ and PR are drawn.

Thus, quadrilateral PQOR is formed.

Since, $OQ \perp QP$

Now, in right angled Δ PQO, by using Pythagoras theorem, $OP^2 = OQ^2 + QP^2$

$\Rightarrow 13^2 = 5^2 + QP^2$

$\Rightarrow QP^2 = 13^2 - 5^2 = 169 - 25 = 144 = 12^2$

$\Rightarrow QP = 12\,cm$

Now, area of Δ OQP $= \frac{1}{2} \times QP \times QO = \frac{1}{2} \times 12 \times 5 = 30\,cm^2$

$\therefore$ Area of the quadrilateral PQOR $= 2 \times \Delta OQP = 2 \times 30 = 60\,cm^2$

Question 5: At one end A of a diameter AB of a circle of radius 5 *cm*, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 *cm* from A, is

(A) 4 *cm* (B) 5*cm* (C) 6 *cm* (D) 8 *cm*

Solution: (D)

First, draw a circle of radius 5 *cm* having centre O. A tangent XY is drawn at point A.

A chord CD is drawn which is parallel to XY and at a distance of 8 *cm* from A.

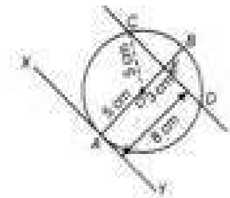
Now, $\angle OAY = 90°$.

We have, $\angle OAY + \angle OED = 180°$

$\Rightarrow \angle OED = 180° - 90° = 90°$

Also, $AE = 8\ cm$ and join OC

Now, in right angled Δ OEC, by using Pythagoras theorem,



$OC^2 = OE^2 + EC^2$

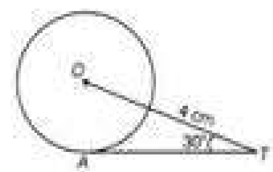
$\Rightarrow 5^2 = 3^2 + EC^2$

$\Rightarrow EC^2 = 5^2 - 3^2 = 25 - 9 = 16 = 4^2$

$\Rightarrow EC = 4\,cm$

$\therefore$ Length of chord $CD = 2 \times CE = 2 \times 4 = 8\,cm$

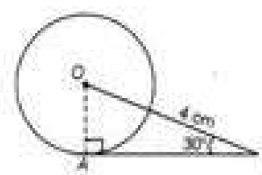
Question 6: In figure, AT is a tangent to the circle with centre O suchthat OT = 4 *cm* and $\angle OTA = 30°$. Then, AT is equal to



(A) 4 *cm* (B) 2 *cm*

(C) 2√3 *cm* (D) 4√3 *cm*

Solution: (C) Join OA. We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.



$\angle OAT = 90°$.

In $\Delta OAT,\ \cos 30^0 = \frac{AT}{OT} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4} \Rightarrow AT = 2\sqrt{3}\ cm$

Question 7: If O is the centre of a circle, PQ is a chord and the tangent PR at P, makes an angle of 50° with PQ, then $\angle POQ$ is equal to

(A) 100° (B) 80° (C) 90° (D) 75°

Solution: (A) Given, $\angle QPR = 50°$

We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore \angle OQR = 90°$.

We have, $\angle OPQ + \angle QPR = 90°$

$\Rightarrow \angle OPQ = 90° - 50° = 40°$

Now, OP = OQ = radius of circle

$\therefore \angle OQP = \angle OPQ = 40°$ [since, angles opposite to sides are equal]

In $\Delta OPQ,\ \angle O + \angle P + \angle Q = 180°$

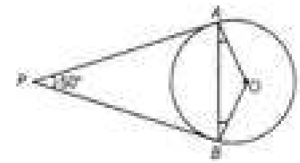
$\Rightarrow \angle O + 40° + 40° = 180°$

$\Rightarrow \angle O + 80° = 180°$

$\Rightarrow \angle O = 180° - 80° = 100°$

Question 8:

In figure, if PA and PB are tangents to the circle with centre O suchthat $\angle APB = 50°$, then $\angle OAB$ is equal to



(A) 25° (B) 30°

(C) 40° (D) 50°

Solution: (A) Given, PA and PB are tangent lines.

$\therefore$PA = PB [since, the length of tangents drawn from an external point to a circle is equal]

$\Rightarrow \angle PBA = \angle PAB = \theta$

In Δ PAB, $\angle P + \angle A + \angle B = 180°$

$\Rightarrow 50° + \theta + \theta = 180°$

$\Rightarrow 2\theta = 180° - 50° = 130°$

$\therefore \theta = 65°$

Also, $OA \perp PA$ [since, tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore \angle PAO = 90°$.

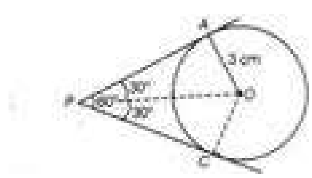
We have, $\angle PAB + \angle BAO = 90°$

$\Rightarrow \angle BAO = 90° - 65° = 25°$

Question 9: If two tangents inclined at an angle 60° are drawn to a circle of radius 3 *cm*, then the length of each tangent is

(A) $\frac{3}{2}\sqrt{3}$ *cm* (B) 6 *cm* (C) 3 *cm* (D) $3\sqrt{3}$ *cm*

Solution: (D) Let P be an external point and a pair of tangents is drawn from point P and angle between these two tangents is 60°.



Join OA and OP.

Also, OP is a bisector line of $\angle APC$.

$\therefore \angle APO = \angle CPO = 30°$

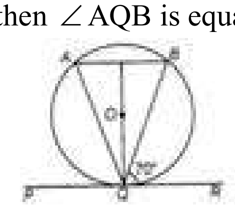
Tangent at any point of a circle is perpendicular to the radius through the point of contact.

In right angled Δ OAP, $\tan 30^0 = \frac{OA}{AP} = \frac{3}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AP} \Rightarrow AP = 3\sqrt{3}$ *cm*

Hence, the length of each tangent is $3\sqrt{3}$ *cm*.

Question 10: In figure, if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70°$, then $\angle AQB$ is equal to



(A) 20° (B) 40° (C) 35° (D) 45°

Solution: (C) Given, AB is a chord parallel to PR.

$\therefore \angle ABQ = \angle BQR = 70°$ [alternate angles]

Also QD is perpendicular to AB and QD bisects AB.

In Δ QDA and Δ QDB, $\angle$ QDA = $\angle$ QDB [each 90°]

Also, AD = BD, QD = QD

$\therefore \Delta ADQ \cong \Delta BDQ$

Then, $\angle$ QAD = $\angle$ QBD, also $\angle$ ABQ = $\angle$ BQR

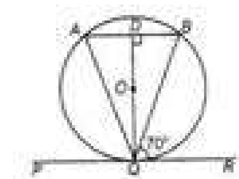
$\therefore \angle ABQ = 70°$.

Hence, $\angle QAB = 70°$.

In Δ ABQ, $\angle A + \angle B + \angle Q = 180°$

$\Rightarrow 70° + 70° + \angle Q = 180°$

$\Rightarrow \angle Q = 180° - 140° = 40°$



MULTIPLE CHOICE QUESTIONS

1. **A circle can have**

   (A) only one tangent (B) exactly two tangents
   (C) three tangents (D) infinitely many tangents

2. **A tangent to a circle intersects it in**

   (A) one point (B) two points (C) three points (D) infinitely many points

3. **If a line intersecting a circle in two points, then it is called a**

   (A) secant (B) tangent (C) normal (D) none

4. **At most number of parallel tangents to a circle is**

   (A) 2 (B) 4 (C) 6 (D) infinite

5. **The number of pairs of parallel tangents to a circle is**

   (A) 1 (B) 2 (C) 4 (D) infinite

6. **The common point of a tangent to a circle and the circle is called**

   (A) secant (B) tangent (C) normal (D) point of contact

7. **Number of tangents can be drawn from a point inside a circle is**

   (A) 0 (B) 1 (C) 2 (D) infinite

8. **Number of tangents can be drawn from a point outside a circle is**

   (A) 0 (B) 1 (C) 2 (D) infinite

9. **Number of tangents can be drawn from a point on the circle is**

   (A) 0 (B) 1 (C) 2 (D) infinite

10. **A tangent PQ at a point P of a circle of radius 5 *cm* meets a line through the centre O at a point Q so that OQ = 12 *cm*, then length PQ is**

    (A) 12 *cm* (B) 13 *cm* (C) 8.5 *cm* (D) $\sqrt{119}$ *cm*

11. From a point Q, the length of the tangent to a circle is 24 *cm* and the distance of Q from the centre is 25 *cm*. The radius of the circle is

(A) 7 *cm* (B) 12 *cm* (C) 15 *cm* (D) 24.5 *cm*

12. If TP and TQ are the two tangents to a circle with centre O so that $\angle$ POQ = 110°, then $\angle$PTQ is equal to

(A) 60° (B) 70° (C) 80° (D) 90°

13. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then $\angle$ POA is equal to

(A) 50° (B) 60° (C) 70° (D) 80°

14. The length of a tangent from a point A at distance 5 *cm* from the centre of the circle is 4 *cm*, then the radius of the circle is

(A) 7 *cm* (B) 6 *cm* (C) 10 *cm* (D) 3 *cm*

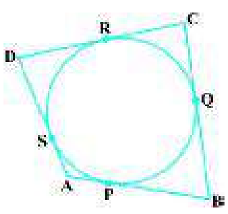
15. Two concentric circles are of radii 5 *cm* and 3 *cm*. The length of the chord of the larger circle which touches the smaller circle is

(A) 10 *cm* (B) 8 *cm* (C) 6 *cm* (D) 4 *cm*

16. The length of a tangent from a point A at distance 5 *cm* from the centre of the circle is 4 *cm*, then the radius of the circle is

(A) 7 *cm* (B) 8 *cm* (C) 10 *cm* (D) 12 *cm*

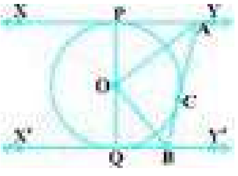
17. A quadrilateral ABCD is drawn to circumscribe a circle as shown in the fig. Then



(A) AB + BC = AD + BC (B) AB + CD = BD + BC

(C) AD + CD = BC + AC (D) AB + CD = AD + BC

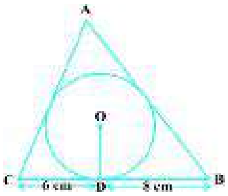
18. If XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and $X'Y'$ at B as shown in the fig, then $\angle$AOB =



(A) 80° (B) 60°

(C) 70° (D) 90°

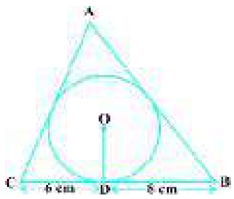
19. A triangle ABC is drawn to circumscribe a circle of radius 4 *cm* such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 *cm* and 6 *cm* respectively as shown in the Fig. Then sides AB =



(A) 11 *cm* (B) 12 *cm*

(C) 10 *cm* (D) 15 *cm*

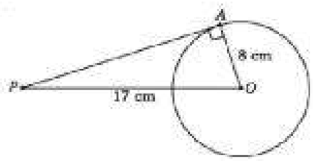
20. A triangle ABC is drawn to circumscribe a circle of radius 4 *cm* such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 *cm* and 6 *cm* respectively as shown in the Fig. Then sides AC =



(A) 11 *cm* (B) 12 *cm*

(C) 13 *cm* (D) 14 *cm*

21. If a point P is 17 *cm* from the centre of a circle, of radius 8 *cm* then the length of the tangent drawn to the circle from the point P is



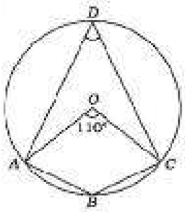
(A) 10 *cm* (B) 12 *cm*

(C) 15 *cm* (D) 14 *cm*

22. The length of a tangent from a point 15 *cm* away from the centre of a circle of radius 9 *cm* then drawn to the circle from the point P is

(A) 12 *cm* (B) 9 *cm* (C) 10 *cm* (D) 15 *cm*

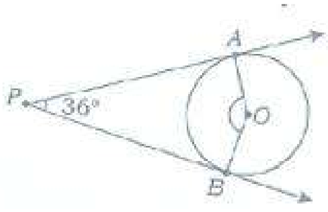
23. In the given figure, O is the centre of the circle and ∠AOC = 110°, then ∠ADC is equal to



(A) 110° (B) 55°

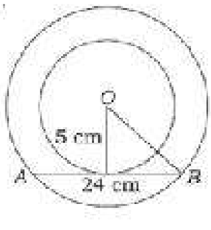
(C) 70° (D) 125°

24. In the given figure PA and PB are the tangents to the circle with centre at O and ∠APB = 36°, then ∠AOB is equal to



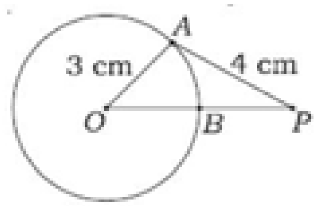
(A) 72° (B) 134° (C) 144° (D) 154°

25. In two concentric circles, a chord of length 24 *cm* of larger circle becomes a tangent to the smaller circle whose radius 5 *cm*, then the radius of the larger circle is



(A) 8 *cm* (B) 10 *cm* (C) 12 *cm* (D) 13 *cm*

26. In the given figure PA is the tangent drawn from an external point P to the circle with centre O. If the radius of the circle is 3 *cm* and PA = 4 *cm,* then the length of PB is



(A) 3 *cm* (B) 4 *cm* (C) 5 *cm* (D) 2 *cm*

27. Two concentric circles are of radii 12 *cm* and 5 *cm* are drawn. A chord of the larger circle becomes a tangent of the smaller circle. The length of the chord is

(A) 26 *cm* (B) 17 *cm* (C) 13 *cm* (D) 7 *cm*

28. A tangent is drawn from an external point P to the circle of 8 *cm* radius. If the length of the tangent is 15 *cm*, then the distance between the centre of the circle and point P is

(A) 23 *cm* (B) 20 *cm* (C) 17 *cm* (D) None

29. If PA and PB are the lengths of tangents drawn from an external point P to the circle, then

(A) $PA \neq PB$ (B) $PA = PB$ (C) $PA < PB$ (D) $PA > PB$

30. If TP and TQ are two tangents to a circle with centre O so that $\angle POQ = 110°$, then $\angle PTQ$ is equal to

(A) 60° (B) 70° (C) 80° (D) 90°

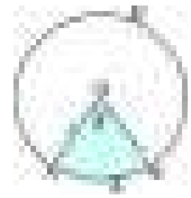
ANSWERS

1	D	2	A	3	A	4	A	5	D	6	D	7	A	8	C	9	B	10	D
11	A	12	B	13	A	14	D	15	B	16	B	17	D	18	D	19	D	20	C
21	C	22	A	23	B	24	C	25	D	26	D	27	C	28	C	29	B	30	C

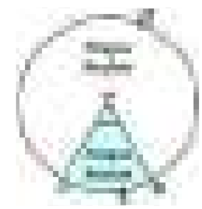
* * *

11. AREAS RELATED TO CIRCLES

SYNOPSIS



1. Length of an arc of a sector of a circle with radius *r* and angle with degree measure θ is $\frac{\theta}{360}\times 2\pi r$



2. Area of a sector of a circle with radius *r* and angle with degree measure θ is $\frac{\theta}{360}\times \pi r^2$

3. In the following Fig, shaded region OAPB is a *sector* of the circle with centre O. $\angle$AOB is called the *angle* of the sector. Note that in this figure, unshaded region OAQB is also a sector of the circle. OAPB is called the *minor sector* and OAQB is called the *major sector*. Also angle of the major sector is 360° – $\angle$AOB.

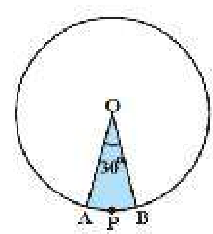
4. Area of segment of a circle = Area of the corresponding sector OAPB – Area of the corresponding triangle OAB.

SOLVED PROBLEMS

Question 1: The area of the sector of a circle with radius 4 *cm* and of angle 30°is (Use $\pi = 3.14$)

(A) 4.19 cm^2 (B) 46.1 cm^2 (C) 50.29 cm^2 (D) 42.3 cm^2

Solution: (A) Given, radius *r* is 4 *cm* and angle of the sector is $\theta = 30^0$



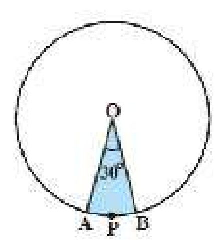
Area of the sector OAPB = $\frac{\theta}{360}\times \pi r^2 = \frac{30}{360}\times\frac{22}{7}\times(4)^2 = \frac{30}{360}\times\frac{22}{7}\times(4)^2$

$= 4.19\ cm^2$ (approx.)

Question 2: The area of the corresponding major sector of a circle with radius 4 *cm* and of angle 30°is (Use $\pi = 3.14$)

(A) 4.19 cm^2 (B) 46.1 cm^2 (C) 50.29 cm^2 (D) 42.3 cm^2

Solution: (B) Given, radius *r* is 4 *cm* and angle of the sector is $\theta = 30^0$



Area of the sector OAPB = $\frac{\theta}{360}\times \pi r^2 = \frac{30}{360}\times\frac{22}{7}\times(4)^2$

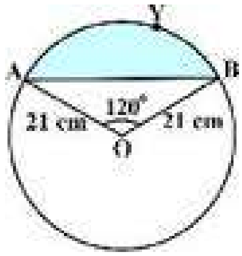
$= \frac{30}{360}\times\frac{22}{7}\times(4)^2 = 4.19\ cm^2$ (approx.)

Area of the corresponding major sector $= \pi r^2 -$ area of sector OAPB

$= (3.14 \times 16 - 4.19)\ cm^2 = 46.05\ cm^2$

$= 46.1\ cm^2$ (approx.)

Question 3: The area of the segment AYB as shown in the following Fig. if radius of the circle is 21 *cm* and $\angle$AOB = 120°. (Use $\pi = \frac{22}{7}$)



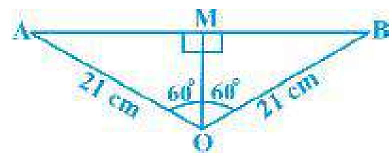
(A) $\frac{21}{4}\left(88+21\sqrt{3}\right)cm^2$ (B) $\frac{21}{4}\left(88-21\sqrt{3}\right)cm^2$

(C) $\frac{21}{4}\left(22-21\sqrt{3}\right)cm^2$ (D) $\frac{21}{4}\left(21+88\sqrt{3}\right)cm^2$

Solution: (B) Area of the segment AYB = Area of sector OAYB – Area of ΔOAB … (1)

Now, area of the sector OAYB = $\frac{\theta}{360}\times\pi r^2$

$=\frac{120}{360}\times\frac{22}{7}\times(21)^2=462\ cm^2$ … (2)



For finding the area of ΔOAB, draw OM ⊥ AB as shown in Fig.

Note that OA = OB. Therefore, by RHS congruence, ΔAOM ≅ Δ BOM.

So, M is the mid-point of AB and ∠AOM = ∠ BOM = $\frac{1}{2}\times120^0=60^0$.

Let OM = x cm

So, from ΔOMA, $\frac{OM}{OA}=\cos60^0 \Rightarrow\frac{x}{21}=\frac{1}{2}\Rightarrow x=\frac{21}{2}$

So, OM $=\frac{21}{2}cm$

Also, $\frac{AM}{OA}=\sin60^0=\frac{\sqrt{3}}{2}\Rightarrow\frac{AM}{21}=\frac{\sqrt{3}}{2}\Rightarrow AM=\frac{21\sqrt{3}}{2}cm$

Therefore, AB = 2 AM = $2\times\frac{21\sqrt{3}}{2}=21\sqrt{3}\,cm$

So, area of ΔOAB = $\frac{1}{2}\times$ AB × OM $=\frac{1}{2}\times21\sqrt{3}\times\frac{21}{2}=\frac{441\sqrt{3}}{4}\ cm^2$ … (3)

Therefore, area of the segment AYB $=\left(462-\frac{441\sqrt{3}}{4}\right)=\frac{21}{4}\left(88-21\sqrt{3}\right)cm^2$

MULTIPLE CHOICE QUESTIONS

1. The area of a sector of a circle with radius 6 *cm* if angle of the sector is 60°is

(A) $\frac{136}{7}cm^2$ (B) $\frac{132}{7}cm^2$ (C) $\frac{135}{7}cm^2$ (D) $\frac{134}{7}cm^2$

2. The area of a quadrant of a circle whose circumference is 22 is

(A) $\frac{47}{8}cm^2$ (B) $\frac{57}{8}cm^2$ (C) $\frac{67}{8}cm^2$ (D) $\frac{77}{8}cm^2$

3. The length of the minute hand of a clock is 14 *cm*, then the area swept by the minute hand in 5 minutes is

(A) $\frac{124}{3}cm^2$ (B) $\frac{134}{3}cm^2$ (C) $\frac{164}{3}cm^2$ (D) $\frac{154}{3}cm^2$

4. A chord of a circle of radius 10 *cm* subtends a right angle at the centre, then the area of the corresponding minor segment is(Use $\pi = 3.14$)

(A) $28.5\,cm^2$ (B) $235.5\,cm^2$ (C) $245.5\,cm^2$ (D) $29.5\,cm^2$

5. In a circle of radius 21 *cm*, an arc subtends an angle of 60° at the centre, then the length of the arc is

(A) $22cm$ (B) $23cm$ (C) $24cm$ (D) $29cm$

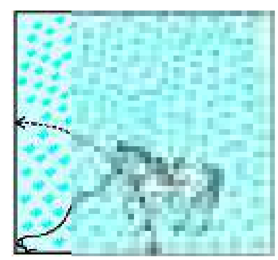
6. A chord of a circle of radius 15 *cm* subtends an angle of 60° at the centre. Find the areas of the corresponding minor segment of the circle is (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

(A) $685.0675\,cm^2$ (B) $20.4375\,cm^2$ (C) $20.5475\,cm^2$ (D) $686.0625\,cm^2$

7. A chord of a circle of radius 12 *cm* subtends an angle of 120° at the centre, then the area of the corresponding segment of the circle is (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)

(A) $88.44\,cm^2$ (B) $66.33\,cm^2$ (C) $22.11\,cm^2$ (D) $44.22\,cm^2$

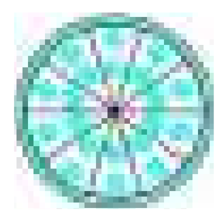
8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 *m* by means of a 5 *m* long rope as shown in the Fig. The area of that part of the field in which the horse can graze is (Use $\pi = 3.14$)



(A) $19.675\,cm^2$ (B) $58.675\,cm^2$

(C) $19.625\,cm^2$ (D) $58.875\,cm^2$

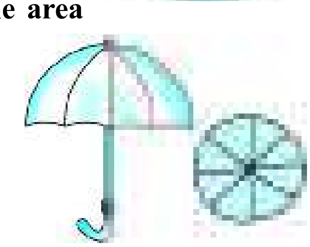
9. A brooch is made with silver wire in the form of a circle with diameter 35 *mm*. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the Fig. The total length of the silver wire required is



(A) $288mm$ (B) $315mm$

(C) $285mm$ (D) $275mm$

10. An umbrella has 8 ribs which are equally spaced as shown in the Fig. Assuming umbrella to be a flat circle of radius 45 *cm*, then the area between the two consecutive ribs of the umbrella is



(A) $\frac{22255}{28}cm^2$ (B) $\frac{22275}{28}cm^2$

(C) $\frac{22225}{28}cm^2$ (D) $\frac{22265}{28}cm^2$

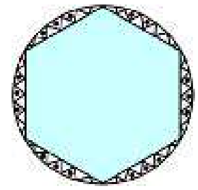
11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 *cm* sweeping through an angle of 115°. The total area cleaned at each sweep of the blades is

(A) $\frac{158525}{126}cm^2$ (B) $\frac{158725}{126}cm^2$ (C) $\frac{158125}{126}cm^2$ (D) $\frac{158625}{126}cm^2$

12. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 *km*. The area of the sea over which the ships are warned is (Use $\pi = 3.14$)

(A) $189.67\,km^2$ (B) $189.77\,km^2$ (C) $189.97\,km^2$ (D) $189.87\,km^2$

13. A round table cover has six equal designs as shown in the following Fig. If the radius of the cover is 28 *cm*, then the cost of making the designs at the rate of Rs 0.35 per cm^2.



(Use $\sqrt{3}=1.73$)

(A) Rs165.68 (B) Rs167.68

(C) Rs162.68 (D) Rs168.68

14. Area of a sector of angle θ (in degrees) of a circle with radius *r* is

(A) $\frac{\theta}{180}\times 2\pi r$ (B) $\frac{\theta}{180}\times \pi r^2$ (C) $\frac{\theta}{360}\times 2\pi r$ (D) $\frac{\theta}{720}\times 2\pi r^2$

15. A chord of a circle of radius 10 *cm* subtends a right angle at the centre, then the area of the corresponding major sector is(Use $\pi=3.14$)

(A) $28.5\,cm^2$ (B) $235.5\,cm^2$ (C) $245.5\,cm^2$ (D) $29.5\,cm^2$

16. In a circle of radius 21 *cm*, an arc subtends an angle of 60° at the centre, then area of the sector formed by the arc is

(A) $233\,cm^2$ (B) $231\,cm^2$ (C) $\frac{441\sqrt{3}}{4}cm^2$ (D) $\left(231-\frac{441\sqrt{3}}{4}\right)cm^2$

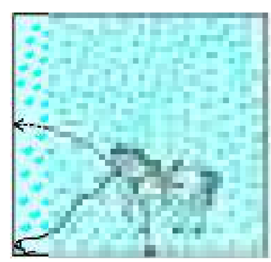
17. In a circle of radius 21 *cm*, an arc subtends an angle of 60° at the centre, then the area of the segment formed by the corresponding chord is

(A) $233\,cm^2$ (B) $231\,cm^2$ (C) $\frac{441\sqrt{3}}{4}cm^2$ (D) $\left(231-\frac{441\sqrt{3}}{4}\right)cm^2$

18. A chord of a circle of radius 15 *cm* subtends an angle of 60° at the centre. Find the areas of the corresponding major segment of the circle is (Use $\pi=3.14$ and $\sqrt{3}=3.14$)

(A) $685.0675\,cm^2$ (B) $20.4375\,cm^2$ (C) $20.5475\,cm^2$ (D) $686.0625\,cm^2$

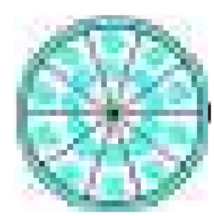
19. A horse is tied to a peg at one corner of a square shaped grass field of side 15 *m* by means of a 5 *m* long rope as shown in the Fig. The the increase in the grazing area if the rope were 10 *m* long instead of 5 *m* is (Use $\pi=3.14$)



(A) $19.675\,cm^2$ (B) $58.675\,cm^2$

(C) $19.625\,cm^2$ (D) $58.875\,cm^2$

20. A brooch is made with silver wire in the form of a circle with diameter 35 *mm*. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in the Fig. The area of each sector of the brooch is



(A) $\frac{355}{4}mm^2$ (B) $\frac{365}{4}mm^2$

(C) $\frac{375}{4}mm^2$ (D) $\frac{385}{4}mm^2$

ANSWERS

1	B	2	D	3	D	4	A	5	A	6	B	7	A	8	C	9	C	10	B
11	C	12	C	13	C	14	D	15	B	16	B	17	D	18	D	19	D	20	D

* * *

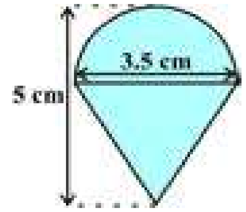
12. SURFACE AREAS AND VOLUMES

SYNOPSIS

1. The total surface area of a cuboid of length $l\,cm$, breadth $b\,cm$ and height $h\,cm$ is $2(lb+bh+hl)\ cm^2$.
2. The volume of a cuboid of length $l\,cm$, breadth $b\,cm$ and height $h\,cm$ is $lbh\ cm^3$.
3. The total surface area of a cube having edge $a\,cm$ is $6a^2\ cm^2$.
4. The volume of a cube having edge $a\,cm$ is $a^3\ cm^3$.
5. The total surface area of a cone of base radius $r\ cm$ and height $h\,cm$ is $\pi r(r+l)\ cm^2$, where $l=\sqrt{r^2+h^2}$ is slant height.
6. The volume of a cone of base radius $r\ cm$ and height $h\,cm$ is $\frac{1}{3}\pi r^2 h\ cm^3$.
7. The total surface area of a cyllinder of base radius $r\ cm$ and height $h\,cm$ is $2\pi r(r+h)\ cm^2$
8. The volume of a cyllinder of base radius $r\ cm$ and height $h\,cm$ is $\pi r^2 h\ cm^3$.
9. The total surface area of a sphere of radius $r\ cm$ is $4\pi r^2\ cm^2$
10. The volume of a sphere of radius $r\ cm$ is $\frac{4}{3}\pi r^3\ cm^3$.
11. The total surface area of a hemisphere of radius $r\ cm$ is $3\pi r^2\ cm^2$
12. The volume of a hemi sphere of radius $r\ cm$ is $\frac{2}{3}\pi r^3\ cm^3$.
13. We can find the surface area of objects formed by combining any two of the basic solids, namely, cuboid, cone, cylinder, sphere and hemisphere.
14. We can find the volume of objects formed by combining any two of a cuboid, cone, cylinder, sphere and hemisphere.

SOLVED PROBLEMS

Question 1: Rasheed got a playing top (*lattu*) as his birthday present, which surprisingly had no colour on it. He wanted to colour it with his crayons. The top is shaped like a cone surmounted by a hemisphere as shown in the following Fig. The entire top is 5 cm in height and the diameter of the top is 3.5 cm, then the area he has to colour is (Take $\pi=\frac{22}{7}$)



(A) $36.6\,cm^2$ (B) $37.6\,cm^2$ (C) $38.6\,cm^2$ (D) $39.6\,cm^2$

Solution: (D) We have, TSA of the toy = CSA of hemisphere + CSA of cone

Now, the curved surface area of the hemisphere = $\frac{1}{2}(4\pi r^2)=2\pi r^2=2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2}\,cm^2$

Also, the height of the cone = height of the top – height (radius) of the hemispherical part

$=\left(5-\frac{3.5}{2}\right)=3.25\,cm$

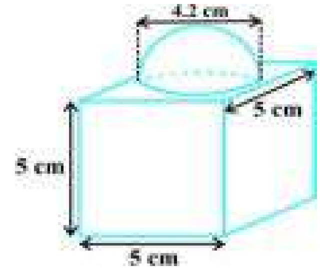
So, the slant height of the cone $(l)=\sqrt{r^2+h^2}=\sqrt{\left(\frac{3.5}{2}\right)^2+(3.25)^2}=3.7\,cm$ (approx.)

Therefore, CSA of cone $\pi rl = \left(\frac{22}{7}\times\frac{3.5}{2}\times 3.7\right)\ cm^2$

This gives the surface area of the top as $= 2\times\frac{22}{7}\times\frac{3.5}{2}\times\frac{3.5}{2} + \frac{22}{7}\times\frac{3.5}{2}\times 3.7$

$$= \frac{22}{7}\times\frac{3.5}{2}(3.5+3.7) = \frac{22}{7}\times\frac{3.5}{2}\times 7.2 = 39.6\ cm^2 \text{(approx)}$$

Question 2: The decorative block shown in the following Fig. is made of two solids, a cube and a hemisphere. The base of the block is a cube with edge 5 *cm*, and the hemisphere fixed on the top has a diameter of 4.2 *cm*, then the total surface area of the block is (Take $\pi = \frac{22}{7}$)

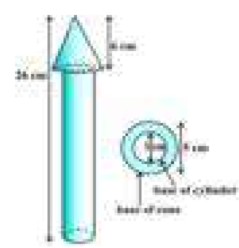


(A) $153.86\,cm^2$ (B) $163.86\,cm^2$
(C) $143.86\,cm^2$ (D) $173.86\,cm^2$

Solution: (B) The total surface area of the cube = 6 × (edge)^2 = 6 × 5 × 5 cm^2 = 150 cm^2.
Note that the part of the cube where the hemisphere is attached is not included in the surface area.
So, the surface area of the block = TSA of cube – base area of hemisphere + CSA of hemisphere

$$= 150 - \pi r^2 + 2\pi r^2 = 150 + \pi r^2 = 150 + \frac{22}{7}\times\frac{4.2}{2}\times\frac{4.2}{2} = 150 + 13.86 = 163.86\ cm^2$$

Question 3: A wooden toy rocket is in the shape of a cone mounted on a cylinder as in the following Fig. The height of the entire rocket is 26 *cm*, while height of the conical part is 6 *cm*. The base of the conical portion has a diameter of 5 *cm*, while the base diameter of the cylindrical portion is 3 *cm*. If the conical portion is to be painted orange and the cylindrical portion yellow, find the area of the rocket painted with each of these colours. (Take $\pi = 3.14$)



(A) $195.465\,cm^2$ (B) $194.465\,cm^2$ (C) $193.465\,cm^2$ (D) $192.465\,cm^2$

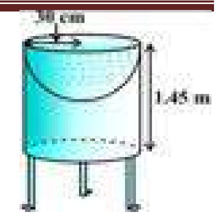
Solution: (A) Denote radius of cone by *r*, slant height of cone by *l*, height of cone by *h*, radius of cylinder by r' and height of cylinder by h'. Then $r = 2.5\ cm$,
$h = 6\ cm,\ r' = 1.5\ cm,\ h' = 26 - 6 = 20\ cm$ and $l = \sqrt{r^2+h^2} = \sqrt{2.5^2+6^2} = 6.5\,cm$
Here, the conical portion has its circular base resting on the base of the cylinder, but the base of the cone is larger than the base of the cylinder. So, a part of the base of the cone (a ring) is to be painted.
So, the area to be painted orange

$$= \text{CSA of the cone + base area of the cone – base area of the cylinder}$$
$$= \pi rl + \pi r^2 - \pi (r')^2$$
$$= \pi[(2.5\times 6.5) + (2.5)^2 - (1.5)^2]$$
$$= \pi[16.25 + 6.25 - 2.25] = 3.14\times 20.25 = 63.585\ cm^2$$

Now, the area to be painted yellow = CSA of the cylinder + area of one base of the cylinder

$$= 2\pi r'h' + \pi (r')^2$$
$$= \pi r'(2h' + r')$$
$$= (3.14\times 1.5)(2\times 20 + 1.5) = 4.71\times 41.5 = 195.465\ cm^2$$

Question 4: Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end as shown in the following Fig. The height of the cylinder is 1.45 *m* and its radius is 30 *cm*, then the total surface area of the bird-bath is (Take $\pi = \frac{22}{7}$)



Solution: (B) Let *h* be height of the cylinder, and *r* the common radius of the cylinder and hemisphere.
Then, the total surface area of the bird-bath = CSA of cylinder + CSA of hemisphere

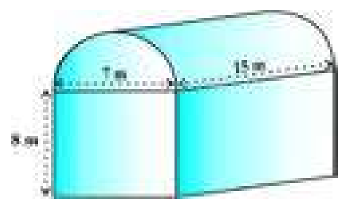
$$= 2\pi rh + 2\pi r^2 = 2\pi r(h+r)$$

$$= 2\times\frac{22}{7}\times 30(145+30)$$

$$= 2\times\frac{22}{7}\times 30(175) = 33000\ cm^2$$

(A) $33300\ cm^2$ (B) $33000\ cm^2$ (C) $33330\ cm^2$ (D) $33333\ cm^2$

Question 5: Shanta runs an industry in a shed which is in the shape of a cuboid surmounted by a half cylinder as shown in the following Fig. If the base of the shed is of dimension $7\,m\times 15\,m$, and the height of the cuboidal portion is 8 *m*, then the volume of air that the shed can hold is



(A) $1128.75\,m^3$ (B) $1228.75\,m^3$ (C) $1188.75\,m^3$ (D) $1228.25\,m^3$

Solution: (A) The volume of air inside the shed (when there are no people or machinery) is given by the volume of air inside the cuboid and inside the half cylinder, taken together.
Now, the length, breadth and height of the cuboid are 15 *m*, 7 *m* and 8 *m*, respectively.
Also, the diameter of the half cylinder is 7 *m* and its height is 15 *m*.

So, the required volume = volume of the cuboid + $\frac{1}{2}$ volume of the cylinder

$$= 15\times 7\times 8 + \frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 15$$

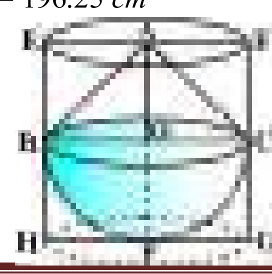
$$= 840 + 288.75 = 1128.75\ m^3$$

Question 6: A juice seller was serving his customers using glasses as shown in the following Fig. The inner diameter of the cylindrical glass was 5 *cm*, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 *cm*, then the apparent capacity of the glass is (Use $\pi = 3.14$.)

(A) $196.25\,cm^3$ (B) $163.54\,cm^3$
(C) $163.35\,cm^3$ (D) $196.65\,cm^3$

Solution: (A) Since the inner diameter of the glass = 5 *cm* and height = 10 *cm*, the apparent capacity of the glass = $\pi r^2 h$ = 3.14 × 2.5 × 2.5 × 10 = 196.25 cm^3

Question 7: A solid toy is in the form of a hemisphere surmounted by a right circular cone as shown in the Fig. The height of the cone is 2 *cm* and the diameter of the base is 4 *cm*, then the volume of the toy is (Take $\pi = 3.14$)



(A) $25.12\ cm^3$ (B) $22.12\ cm^3$
(C) $22.22\ cm^3$ (D) $24.12\ cm^3$

Solution: (A) Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere. The radius BO of the hemisphere (as well as of the cone) = $\frac{1}{2}$ × 4 *cm* = 2 *cm*.

So, volume of the toy = $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{2}{3}\times 3.14\times(2)^3 + \frac{1}{3}\times 3.14\times(2)^2\times 2 = 25.12\ cm^3$

Question 8: If a hollow cube of internal edge 22 *cm* is filled with spherical marbles of diameter 0.5 *cm* and it is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then, the number of marbles that the cube can accommodate is

(A) 142244 (B) 142344 (C) 142444 (D) 142544

Solution: (A) Given, edge of the cube is $a = 22\ cm$

$\therefore$ Volume of the cube is $V = (22)^3 = 10648\, cm^3$ [$\because V = a^3$]

Also, given, diameter of marble is = 0.5 *cm*

$\therefore$ Radius of the marble is $r = \frac{0.5}{2} = 0.25\, cm$ [$\because d = 2r$]

$\therefore$ Volume of one marble is $V = \frac{4}{3}\times\frac{22}{7}\times(0.25)^3$ $\left[\because V = \frac{4}{3}\pi r^3\right]$

$$= \frac{1.375}{21} = 0.0655\, cm^3$$

Filled space of the cube = Volume of the cube $-\frac{1}{8}\times$ Volume of the cube

$$= \frac{7}{8}\times \text{ Volume of the cube} = \frac{7}{8}\times 10648 = 9317\ cm^3$$

$\therefore$ Required number of marbles = $\dfrac{\text{total space filled by marbles in a cube}}{\text{Volume of one marble}}$

$$= \frac{9317}{0.0655} = 142244 \text{ (approx)}$$

Question 9: A metallic spherical shell of internal and external diameters 4 *cm* and 8 *cm*, respectively is melted and recast into the form a cone of base diameter 8 *cm*. The height of the cone is

(A) 12 *cm* (B) 14 *cm* (C) 15 *cm* (D) 18 *cm*

Solution: (B) Given, internal diameter of spherical shell = 4 *cm* and external diameter of shell = 8 *cm*

$\therefore$ Internal radius of the spherical shell is $r_1 = \frac{4}{2} = 2\, cm$ and External radius of the spherical shell is $r_2 = \frac{8}{2} = 4\, cm$ [$\because diameter = 2(radius)$]

Now, volume of the spherical shell is $V = \frac{4}{3}\pi(r_2^3 - r_1^3)$

$$= \frac{4\pi}{3}\times(4^3 - 2^3) = \frac{4\pi}{3}\times(64-8) = \frac{4\pi}{3}\times 56 = \frac{224\pi}{3}\ cm^3$$

Let height of the cone = *h cm*

$\therefore$ Diameter of the base of cone = 8 *cm*

$\therefore$ Radius of the base of cone is $r = \frac{8}{2} = 4\, cm$ [$\because diameter = 2(radius)$]

$\therefore$ Volume of cone = Volume of the spherical shell

$\Rightarrow \frac{1}{3}\pi(4^2)h = \frac{224\pi}{3}$

$\Rightarrow 16h = 224$
$\Rightarrow h = 14$
Hence, the height of the cone is 14 *cm*.

Question 10: If a solid piece of iron in the form of a cuboid of dimensions $49cm \times 33cm \times 24cm$, is moulded to form a solid sphere. Then, radius of the sphere is

(A) 21 *cm* (B) 23 *cm* (C) 25 *cm* (D) 19 *cm*

Solution: (A) Given, dimensions of the cuboid = $49cm \times 33cm \times 24cm$.

∴ Volume of the cuboid = $49 \times 33 \times 24 = 38808 \ cm^3$ [$\because V = lbh$]

Let the radius of the sphere = *r cm*

Volume of the Sphere is $V = \frac{4}{3}\pi r^3$

∴ Volume of the Sphere = Volume of the cuboid

$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = 38808$

$\Rightarrow r^3 = 38808 \times \frac{7}{22} \times \frac{3}{4}$

$\Rightarrow r^3 = 441 \times 21 = (21)^3$

$\therefore r = 21$

Hence, the radius of the sphere is 21 *cm*.

Question11: A mason constructs a wall of dimensions $270cm \times 300cm \times 350cm$, with the bricks each of size $22.5cm \times 11.25cm \times 8.75cm$ and it is assumed that $\frac{1}{8}$ space is covered by the mortar. Then, the number of bricks used to construct the wall is

(A) 11100 (B) 11200 (C) 11000 (D) 11300

Solution: (B) Volume of the wall = $270 \times 300 \times 350 = 28350000 \ cm^3$ [$\because V = lbh$]

Since, $\frac{1}{8}$ space is covered by the mortar.

So, remaining space of the wall = Volume of the wall − Volume of the mortar

$= 28350000 - \frac{1}{8} \times 28350000 = 28350000 \times \frac{7}{8} = 24806250 \ cm^3$

Now, volume of one brick $= 22.5 \times 11.25 \times 8.75 = 2214.844 \ cm^3$ [$\because V = lbh$]

$\therefore$ Required number of bricks $= \frac{24806250}{2214.844} = 11200$ (approx)

Hence, the number of bricks used to construct the wall is 11200

Question 12: Twelve solid spheres of the same size are made by melting a solid metallic cylinder of base diameter 2 *cm* and height 16 *cm*. The diameter of each sphere is

(A) 4 *cm* (B) 3*cm* (C) 2 *cm* (D) 6 *cm*

Solution: (C) Given, diameter of the cylinder = 2 *cm*

∴ Radius r =1 cm and height of the cylinder h = 16 *cm* [$\because diameter = 2(radius)$]

∴ Volume of the cylinder is $V = \pi r^2 h = \pi (1)^2 16 = 16\pi \ cm^3$

Let the radius of the sphere = *r cm*

Volume of the Sphere is $V = \frac{4}{3}\pi r^3$

Let the radius of the sphere = *r cm*

Given, Volume of twelve Spheres is equal to Volume of the cylinder

i.e., $12\left(\frac{4}{3}\pi r^3\right) = 16\pi$

$\Rightarrow r^3 = 1$

$\Rightarrow r = 1cm$

$\therefore$ Diameter of each sphere, $d = 2r = 2\times1 = 2cm$

Hence, the required diameter of each sphere is 2 *cm*.

Question 13: The radii of the top and bottom of a bucket of slant height 45 *cm* are 28 *cm* and 7 *cm*, respectively. The curved surface area of the bucket is

(A) 4950 cm^2 (B) 4951 cm^2 (C) 4952 cm^2 (D) 4953 cm^2

Solution: (A) Given, the radius of the top of the bucket, R = 28 *cm* and the radius of the bottom of the bucket, r = 7 *cm*

Slant height of the bucket, l = 45 *cm*

Since, bucket is in the form of frustum of a cone.

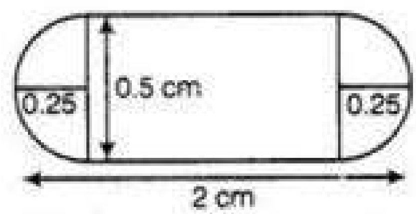
$\therefore$ Curved surface area of the bucket $= \pi l(\mathrm{R}+r) = \frac{22}{7}\times45\times(28+7) = \frac{22}{7}\times45\times35 = 4950$ cm^2

Question 14: A medicine-capsule is in the shape of a cylinder of diameter 0.5 *cm* with two Hemispheres stuck to each of its ends. The length of entire capsule is 2 *cm*. The Capacity of the capsule is

(A) 0.36 cm^3 (B) 0.35 cm^3 (C) 0.34 cm^3 (D) 0.33 cm^3

Solution: (A) Given, diameter of cylinder = Diameter of hemisphere = 0.5 *cm*

$\therefore$ Radius of the cylinder r = Radius of the hemisphere $r = \frac{0.5}{2} = 0.25$ *cm* and height of the capsule is $= 2$ *cm*



$\therefore$ Length of the cylindrical part of the capsule, h

h = Length of the capsule − Radius of the both hemispheres

$\therefore h = 2-(0.25+0.25) = 2-0.5 = 1.5$ *cm*

Now, capacity of capsule = Volume of cylindrical part + $2\times$ Volume of hemisphere

$$= \pi r^2 h + 2\times\frac{2}{3}\pi r^3 = \pi r^2\left[h+\frac{4}{3}r\right] = \frac{22}{7}\times(0.25)^2\times\left[1.5+\frac{4}{3}(0.25)\right]$$

$$= \frac{22}{7}\times(0.25)^2\times\left[1.5+0.33\right] = \frac{22}{7}\times0.0625\times1.83 = 0.36\ cm^3$$

Hence, the capacity of capsule is 0.36 cm^3

Question 15: If two solid hemispheres of same base radius r are joined together along their bases, then curved surface area of this new solid is

(A) $4\pi r^2$ (B) $6\pi r^2$ (C) $3\pi r^2$ (D) $8\pi r^2$

Solution: (A) Because curved surface area of a hemisphere is 2w2 and here, we join two solid Hemispheres along their bases of radius r, from which we get a solid sphere.

Hence, the curved surface area of new solid = $2\pi r^2 + 2\pi r^2 = 4\pi r^2$

Question 16: A right circular cylinder of radius r *cm* and height h *cm* (where, $h > 2r$) just encloses a sphere of diameter

(A) r *cm* (B) 2 r *cm* (C) h *cm* (D) 2h *cm*

Solution: (B) Because the sphere encloses in the cylinder, therefore the diameter of sphere is equal to diameter of cylinder which is 2 r *cm*.

Question 17: The diameters of the two circular ends of the bucket are 44 *cm* and 24 *cm*. The height of the bucket is 35 *cm*. The capacity of the bucket is

(A) 32.7L (B) 33.7L (C) 34.7L (D) 31.7L

Solution: (A) Given, diameter of one end of the bucket is 44 *cm i.e.*, 2 R = 44 *cm* $\Rightarrow$ R = 22 *cm*

And diameter of another end of the bucket is 44 *cm i.e.*, 2 r = 24 *cm* $\Rightarrow r$ = 12 *cm.*

Height of the bucket is h = 35 *cm.*

Since, the shape of a bucket is look like as frustum of a cone.
Therefore, capacity of the bucket = Volume of frustum of a cone

$$= \frac{1}{3}\pi h[R^2 + r^2 + Rr] = \frac{1}{3}\times\frac{22}{7}\times 35[22^2 + 12^2 + 22\times 12]$$

$$= \frac{1}{3}\times\frac{22}{7}\times 35[484 + 144 + 264] = \frac{1}{3}\times\frac{22}{7}\times 35\times 892$$

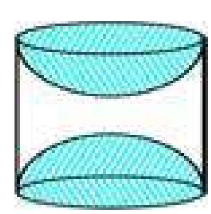
$$= 32706.6\ cm^3 = 32.7\,L[\because 1\,L = 1000\ cm^3]$$

Hence, the capacity of bucket is 32.7 L.

MULTIPLE CHOICE QUESTIONS

1. **2 cubes each of volume 64 cm^3 are joined end to end, then the surface area of the resulting Cuboid is**
   (A) $240\,cm^2$ (B) $160\,cm^2$ (C) $216\,cm^2$ (D) $96\,cm^2$
2. **A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 *cm* and the total height of the vessel is 13 *cm*, then the inner surface area of the vessel is**
   (A) $270\,cm^2$ (B) $216\,cm^2$ (C) $286\,cm^2$ (D) $572\,cm^2$
3. **A toy is in the form of a cone of radius 3.5 *cm* mounted on a hemisphere of same radius. The total height of the toy is 15.5 *cm*, then the total surface area of the toy is**
   (A) $213.5\,cm^2$ (B) $216.5\,cm^2$ (C) $215.5\,cm^2$ (D) $214.5\,cm^2$
4. **A cubical block of side 7 *cm* is surmounted by a hemisphere, then the surface area of the solid is**
   (A) $333.5\,cm^2$ (B) $332.5\,cm^2$ (C) $323.5\,cm^2$ (D) $316.5\,cm^2$
5. **A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter *l* of the hemisphere is equal to the edge of the cube, then the surface area of the remaining solid is**
   (A) $\frac{l^2}{4}(\pi + 12)\,cm^2$ (B) $\frac{l^2}{4}(\pi + 8)\,cm^2$ (C) $\frac{l^2}{4}(\pi + 16)\,cm^2$ (D) $\frac{l^2}{4}(\pi + 24)\,cm^2$
6. **A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends as shown in the following Fig. The length of the entire capsule is 14 *mm* and the diameter of the capsule is 5 *mm*, then its surface area is**

   (A) $210\,mm^2$ (B) $220\,mm^2$ (C) $215\,mm^2$ (D) $222\,mm^2$
7. **A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 *m* and 4 *m* respectively, and the slant height of the top is 2.8 *m*, then the area of the canvas used for making the tent is**
   (A) $44\ m^2$ (B) $22\,m^2$ (C) $21\ m^2$ (D) $26\,m^2$
8. **From a solid cylinder whose height is 2.4 *cm* and diameter 1.4 *cm*, a conical cavity of the same height and same diameter is hollowed out, then the total surface area of the remaining solid to the nearest cm^2 is**
   (A) $16\ cm^2$ (B) $12\,cm^2$ (C) $18\ cm^2$ (D) $14\,cm^2$
9. **A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the following Fig. If the height of the cylinder is 10 *cm*, and its base is of radius 3.5 *cm*, then the total surface area of the article is**



   (A) $384\ cm^2$ (B) $324\,cm^2$
   (C) $374\ cm^2$ (D) $354\,cm^2$

10. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 *m* and 4 *m* respectively, and the slant height of the top is 2.8 *m*, the cost of the canvas of the tent at the rate of Rs 500 per m^2, then the total cost is (Note that the base of the tent will not be covered with canvas.)

(A) Rs 20,000 (B) Rs 22,000 (C) Rs 21,000 (D) Rs 24,000

11. A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 *cm* and the height of the cone is equal to its radius, then the volume of the solid in terms of π is

(A) $2\pi\ cm^3$ (B) $\pi\ cm^3$ (C) $3\pi\ cm^3$ (D) $4\pi\ cm^3$

12. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 *cm* and its length is 12 *cm*. If each cone has a height of 2 *cm*, then the volume of air contained in the model that Rachel made is (Assume the outer and inner dimensions of the model to be nearly the same.)

(A) 22 cm^3 (B) 33 cm^3 (C) 66 cm^3 (D) 44 cm^3

13. A *gulab jamun,* contains sugar syrup up to about 30% of its volume. How much syrup would be found approximately in 45 *gulab jamuns,* each shaped like a cylinder as shown in the Fig. with two hemispherical ends with length 5 *cm* and diameter 2.8 *cm*

(A) 318 cm^3 (B) 328 cm^3

(C) 338 cm^3 (D) 368 cm^3

14. A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are $15cm \times 10cm \times 3.5cm$. The radius of each of the depressions is 0.5 *cm* and the depth is 1.4 *cm*, then the volume of wood in the entire stand is

(A) 532.33 cm^3 (B) 523.53 cm^3

(C) 543.53 cm^3 (D) 563.53 cm^3

15. A vessel is in the form of an inverted cone. Its height is 8 *cm* and the radius of its top, which is open, is 5 *cm*. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 *cm* are dropped into the vessel, one fourth of the water flows out, then the number of lead shots dropped in the vessel is

(A) 90 (B) 120 (C) 100 (D) 110

16. A solid iron pole consists of a cylinder of height 220 *cm* and base diameter 24 *cm*, which is surmounted by another cylinder of height 60 *cm* and radius 8 *cm*. Given that 1 cm^3 of iron has approximately 8*g* mass, then the mass of the pole is (Use π = 3.14)

(A) 896.26 *kg* (B) 893.26 *kg* (C) 892.26 *kg* (D) 894.26 *kg*

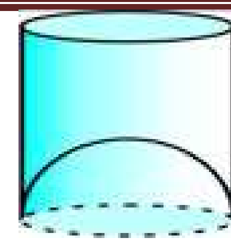
17. A solid consisting of a right circular cone of height 120 *cm* and radius 60 *cm* standing on a hemisphere of radius 60 *cm* is placed upright in a right circular cylinder full of water such that it touches the bottom. If the radius of the cylinder is 60 *cm* and its height is 180 *cm*, then the volume of water left in the cylinder is (approximately)

(A) 1.231 m^3 (B) 1.131 m^3 (C) 1.331 m^3 (D) 1.333 m^3

18. A spherical glass vessel has a cylindrical neck 8 *cm* long, 2 *cm* in diameter; the diameter of the spherical part is 8.5 *cm*, then the amount of water it holds is (Use π = 3.14).

(A) 346.64 cm^3 (B) 346.54 cm^3 (C) 346.56 cm^3 (D) 346.51 cm^3

19. A juice seller was serving his customers using glasses as shown in the following Fig. The inner diameter of the cylindrical glass was 5 *cm*, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 *cm*, then the actual capacity of the glass is (Use $\pi = 3.14$.)



(A) $196.25\,cm^3$ (B) $163.54\,cm^3$
(C) $163.35\,cm^3$ (D) $196.65\,cm^3$

20. A cylindrical pencil sharpened at one edge is the combination of
(A) a cone and a cylinder (B)frustum of a cone and a cylinder
(C) a hemi sphere and a cylinder (D) two cylinders

21. A surahi is the combination of
(A) a sphere and a cylinder (B) a hemi sphere and a cylinder
(C) two hemi spheres (D) a cylinder and a cone

22. A plumb line (sahul) is the combination of
(A) a cone and a cylinder (B) a hemi sphere and a cone
(C) frustum of a cone and a cylinder (D) sphere and cylinder

23. The shape of a glass (tumbler) is usually in the form of
(A) a cone (B) frustum of a cone (C) a cylinder (D) a sphere

24. The shape of a gilli, in the gilli-danda game is a combination of
(A) two cylinders (B) a cone and a cylinder
(C) two cones and a cylinder (D) two cylinders and a cone

25. A shuttle cock used for playing badminton has the shape of the combination of
(A) a cylinder and a sphere (B) a cylinder and a hemisphere
(C) a sphere and a cone (D) frustum of a cone and a hemi sphere

26. A cone is cut through a plane parallel to its base and then the cone that is formed on one side of that plane is removed. The new part that is left over on the other side of the Plane is called
(A) a frustum of a cone (B) cone (C) cylinder (D) sphere

27. If volumes of two spheres are in the ratio 64:27, then the ratio of their surface areas is
(A) 3:4 (B) 4:3 (C) 9:16 (D) 16:9

28. In a right circular cone, the cross-section made by a plane parallel to the base is a
(A) circle (B) frustum of a cone (C) sphere (D) hemi sphere

29. During conversion of a solid from one shape to another, the volume of the new shape will
(A) increase (B)decrease (C) remain unaltered (D)be doubled

30. The sharpened edge of the pencil gives an idea about the
(A) circle (B) cone (C) rectangle (D) None

31. The diameter of a sphere is 6 *cm*. It is melted and drawn into a wire of diameter 2 *mm*. The length of the wire is
(A) 12 *m* (B)18 *m* (C) 36 *m* (D) 66 *m*

32. A solid sphere of radius "*r*" is melted and cast into the shape of a solid cone of height *r*, then the radius of the base of the cone is
(A) $2r$ (B) $3r$ (C) r (D) $4r$

33. The curved surface area of a cylinder is $264\,m^2$ and its volume is $924\,m^3$. The ratio of its diameter to its height is
(A) 3:7 (B) 7:3 (C) 6:7 (D) 7:6

34. A cylinder with base radius 8 *cm* and height 2 *cm* is melted to form a cone of height 6 *cm*, then the radius of the cone is
(A) 4 *cm* (B) 5 *cm* (C) 6 *cm* (D) 8 *cm*

35. If three metallic spheres of radii 6 *cm*, 8 *cm* and 10 *cm* are melted to form a single sphere, the diameter of the sphere is
(A)12 *cm* (B) 24 *cm* (C) 30 *cm* (D) 36 *cm*

36. The volumes of two hemi spheres are in the ratio 64:27. The ratio of their surface areas is

(A) 1:2 (B) 2:3 (C) 9:16 (D) 16:9

37. The surface area of a sphere is same as the curved surface area of a right circular cylinder whose height and diameter are 12 *cm* each. The radius of the sphere is

(A) 3 *cm* (B) 4 *cm* (C) 6 *cm* (D) 12 *cm*

38. 12 spheres of the same size are made from melting a solid cylinder of 16 *cm* diameter and 2 *cm* height. The diameter of each sphere is

(A) $\sqrt{3}\,cm$ (B) 2 *cm* (C) 3 *cm* (D) 4 *cm*

39. A solid piece of iron of dimensions $49cm \times 33cm \times 24cm$ is molded into a sphere. The radius of the sphere is

(A) 21 *cm* (B) 28 *cm* (C) 35 *cm* (D) None

40. The maximum volume of a cone that can be carved out of a solid hemisphere of radius *r* is

(A) $3\pi r^2$ (B) $\frac{\pi r^3}{3}$ (C) $\frac{\pi r^2}{3}$ (D) $3\pi r^3$

41. The radii of two cylinders are in the ratio 3:5. If their heights are in the ratio 2:3, then the ratio of their curved surface areas is

(A) 2:5 (B) 5:2 (C) 2:3 (D) 3:5

42. The curved surface area of right circular cone of height is 15 *cm* and its diameter is 16 *cm* is

(A) $60\pi\ cm^2$ (B) $68\pi\ cm^2$ (C) $120\pi\ cm^2$ (D) $136\pi\ cm^2$

43. A right circular cylinder of radius r and height $h(h = 2r)$ just encloses sphere of diameter

(A) h (B) r (C) $2r$ (D) $2h$

44. If four times the sum of the areas of two circular faces of a cylinder of height 8 *cm* is equal to twice the curved surface area, then diameter of the cylinder is

(A) 4 *cm* (B) 8 *cm* (C) 2 *cm* (D) 6 *cm*

45. A metallic sphere of radius 10.5 *cm* is melted and then recast into small cones, each of radius 3.5 *cm* and height 3 *cm*. The number of such cones is

(A) 63 (B) 126 (C) 21 (D) 130

46. An oil drum is in the shape of a cylinder having the following dimensions; diameter 2 *m* and height is 7 *m*. The painter charges Rs.3/- per m^2 to paint the drum. Then the total charges to be paid to the painter for 10 drums

(A) Rs.1508/- (B) Rs.1507/- (C) Rs.1506/- (D) None

47. A sphere, a cylinder and a cone are of the same radius and same height. Then ratio of their curved surface areas is

(A) 1:2:3 (B) 2:2: $\sqrt{5}$ (C) 4:4: $\sqrt{5}$ (D) None

48. A company wanted to manufacture 100 hemispherical basins from thin steel sheet. If the radius of hemispherical basin is 21 *cm*, then the required area of steel sheet to manufacture the basins.

(A) $28.32\,m^2$ (B) $27.72\,m^2$ (C) $28.45\,m^2$ (D) None

49. A joker's cap is in the form of right circular cone whose base radius is 7 *cm* and height is 24 *cm*. Then the area of the sheet required to make 20 such caps.

(A) 500 cm^2 (B) 520 cm^2 (C) 510 cm^2 (D) 550 cm^2

50. A cylinder and cone have bases of equal radii and are of equal heights. Then their volumes are in the ratio is

(A) 1:1 (B) 1:2 (C) 3:1 (D) None

51. A toy is in the form of a cone mounted on a hemisphere. The diameter of the base and the height of the cone are 6 *cm* and 4 *cm* respectively. Then the surface area of the toy is

(A) 105 cm^2 (B) 103.62 cm^2 (C) $104.12\,cm^2$ (D) None

52. Two cubes each of volume 32 cm^3 are joined end to end together. Then the total surface area of the resulting cuboid is

(A) 100 cm^2 (B) 120 cm^2 (C) 40 cm^2 (D) 20 cm^2

53. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The length of the capsule is 14 *mm* and the width is 5 *mm*. Then its surface area is

(A) $220\ mm^2$ (B) $120\ mm^2$ (C) $70\ mm^2$ (D) $76\ mm^2$

54. The volume of the largest right circular cone that can be cut out of a cube whose edge is 7 *cm*

(A) $88.62\ cm^3$ (B) $89.83\ cm^3$ (C) $90\ cm^3$ (D) None

55. A cone of height 24 *cm* and radius of base 6 *cm* is made up of modelling clay. A child reshapes it in the form of a sphere, then the radius of the sphere is

(A) 4 *cm* (B) 5 *cm* (C) 6 *cm* (D) 2 *cm*

56. The number of silver coins, 1.75 *cm* in diameter and thickness 2 *mm*, need to be melted to form a cuboid of dimensions $5.5cm \times 10cm \times 3.5cm$

(A) 150 (B) 120 (C) 140 (D) 130

57. A solid metallic sphere of diameter 28 *cm* is melted and recast into a number of smaller cones, each of diameter $4\frac{2}{3}cm$ and height 3 *cm*. Then the number of cones so formed.

(A) 675 (B) 674 (C) 671 (D) 672

58. Three metal cubes with edges 15 *cm*, 12 *cm* and 9 *cm* respectively are melted together and formed in a simple cube. Then the edge of this cube is

(A) 14 *cm* (B) 18 *cm* (C) 12 *cm* (D) 14 *cm*

59. A golf ball has diameter equals to 4.1 *cm*. Its surface has 150 dimples which are in hemispherical shape of radius 2 *mm*. Then the total surface area which is exposed to the surroundings.

(A) $40.85\ cm^2$ (B) $43.40\,cm^2$ (C) $84.21\,cm^2$ (D) None

60. The number of spherical balls can be made out of a solid cube of lead whose edge measures 44 *cm* and each ball being 4 *cm* in diameter.

(A) 2500 (B) 2501 (C) 2541 (D) None

61. If the length, breadth and height of a cuboid are 8 *cm*, 3 *cm* and 4 *cm* respectively, then the total surface area of the cuboid is

(A) $48\ cm^2$ (B) $72\ cm^2$ (C) $136\ cm^2$ (D) $108\ cm^2$

62. If the volume of a cylinder is $500\ m^3$ and the area of its base is $25\ m^2$, then its height is

(A) $20m$ (B) $15\ m$ (C) $50\ m$ (D) $30\ m$

63. If the height of a conical tent is $3\ m$ and the radius of its base is $4\ m$, then its slant height is

(A) $3m$ (B) $4\ m$ (C) $5\ m$ (D) $7\ m$

64. If the radius of the base of a right circular cylinder is halved, keeping the height same, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is

(A) 1:4 (B) 2:1 (C) 1:2 (D) 4:1

65. A joker's cap is in the form of right circular cone whose base radius is 7 *cm* and height is 24 *cm*. Then the area of the sheet required to make 10 such caps.

(A) $550\ cm^2$ (B) $5500\ cm^2$ (C) $55000\ cm^2$ (D) None

66. A right circular cylinder has base radius 14 *cm* and height 21 *cm*. The curved surface area is

(A) $1848\ cm^2$ (B) $616\ cm^2$ (C) $3080\ cm^2$ (D) $12936\ cm^2$

67. The volume of the sphere of radius 21 *cm* is.

(A) $5544\ cm^3$ (B) $38808\ cm^3$ (C) $1155\ cm^3$ (D) $8983\ cm^3$

68. If the curved surface area of a cone is $4070\ cm^2$ and its diameter is $70\ cm$, then its slant height is

(A) $27cm$ (B) $37cm$ (C) $47cm$ (D) $57cm$

69. Under the usual notation, the surface area of a cuboid is

(A) $lb+bh+hl$ (B) $\frac{lb+bh+hl}{2}$ (C) $2(lb+bh+hl)$ (D) None

70. An iron spherical ball of volume $232848\ cm^3$ has been melted and converted into a cone with vertical angle of 120^0, then the height of the cone is
(A) $42\sqrt{3}\,cm$ (B) $42\,cm$ (C) $21\,cm$ (D) None

71. Three metallic spheres of radii $3\ cm, 4\ cm$ and $5\ cm$ are melted to form a single sphere, then the radius of the resulting sphere is
(A) $12\,cm$ (B) $6\,cm$ (C) $7\,cm$ (D) $9\,cm$

72. A hemispherical bowl of internal diameter $36\ cm$ contains a liquid. How many cylindrical bottles of radius $3\,cm$ are required to empty the bowl?
(A) 1000 (B) 1078 (C) 1152 (D) None

73. The diameter of a sphere is equal to the height of the cone of equal volume. If *r* and R are the radii of cone and sphere respectively, then $r^2 =$
(A) $2R^2$ (B) $\frac{R^2}{2}$ (C) $4R^2$ (D) R^2

74. A solid sphere of diameter $18\ cm$ is melted and recast into small identical cones of height $3\ cm$ and radius $6\ cm$, then the number of cones formed is
(A) 27 (B) 32 (C) 12 (D) 18

75. From a wooden log of dimensions $6\ cm, 8cm, 10cm$, a right circular cone and cylinder of same base diameter $6\ cm$ and equal height $7\ cm$ are formed. The quantity of wooden lost in this process is
(A) $216\,cc$ (B) $250\,cc$ (C) $175\,cc$ (D) $300\,cc$

76. From a wooden log of dimensions $6\ cm, 8cm, 10cm$, a right circular cone and cylinder of same base diameter $6\ cm$ and equal height $7\ cm$ are formed. The quantity of wooden lost in this process is
(A) $216\,cc$ (B) $250\,cc$ (C) $175\,cc$ (D) $300\,cc$

77. The length, breadth and height of a room are $10m, 10\sqrt{2}m$ and $10m$ respectively. The angle of elevation of a top corner of room from any point on a diagonal of the base of the room is
(A) 45^0 (B) 60^0 (C) 30^0 (D) None

78. The total surface area of a cone with slant height $21m$ and diameter of its base $24m$ is
(A) $252\pi sq.m$ (B) $504\pi sq.m$ (C) $396\pi sq.m$ (D) $1080\pi sq.m$

79. The total surface area of a cuboid of length l, breadth b and height h is
(A) lbh (B) $2h(l+b)$ (C) $2(lb+bh+hl)$ (D) $2(l+b)$

80. With the usual notation, if $r = 7\,cm$ and $h = 10\,cm$ in a cone, then its lateral height is (approximately)
(A) $13.4\,cm$ (B) $10.3\,cm$ (C) $18.2\,cm$ (D) $12.2\,cm$

ANSWERS

1	B	2	D	3	D	4	B	5	D	6	B	7	A	8	C	9	C	10	B
11	C	12	C	13	C	14	B	15	C	16	C	17	B	18	D	19	B	20	A
21	A	22	B	23	B	24	C	25	D	26	A	27	D	28	B	29	C	30	B
31	C	32	A	33	B	34	D	35	B	36	D	37	C	38	D	39	A	40	B
41	A	42	D	43	C	44	B	45	B	46	A	47	C	48	B	49	D	50	C
51	B	52	C	53	A	54	B	55	C	56	A	57	D	58	B	59	A	60	C
61	C	62	A	63	C	64	D	65	D	66	A	67	B	68	B	69	C	70	A
71	B	72	D	73	C	74	A	75	A	76	A	77	A	78	C	79	C	80	D

13. STATISTICS

SYNOPSIS

1. The mean for grouped data can be found by:

(i) the direct method : If $x_1, x_2, ..., x_n$ are observations with respective frequencies $f_1, f_2, ..., f_n$, then this means observation x_1 occurs f_1 times, x_2 occurs f_2 times, and so on.

The mean $\bar{x}$ of the data is given by $\bar{x} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + ... + f_n} = \frac{\sum f_ix_i}{\sum f_i}$

$$\therefore \bar{x} = \frac{\sum f_ix_i}{\sum f_i}$$

(ii) the assumed mean method : $\bar{x} = a + \frac{\sum f_id_i}{\sum f_i}$

where a is the assumed mean and **Deviation** of 'a' from each of the x_i's $i.e., d_i = x_i - a$,

(iii) the step deviation method : $\bar{x} = a + \left(\frac{\sum f_iu_i}{\sum f_i}\right) \times h$,

Where $u_i = \frac{x_i - a}{h}$, a is the assumed mean and h is the class size.

2. The step-deviation method will be convenient to apply if all the d_is have a common factor.

3. The mean obtained by all the three methods is the same.

4. The assumed mean method and step-deviation method are just simplified forms of the direct method

5. **Mode:** A mode is that value among the observations which occurs most frequently.

6. The mode for grouped data can be found by using the formula:

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$,

Where, $l =$ lower limit of the modal class,

$h =$ size of the class interval (assuming all class sizes to be equal),

$f_1 =$ frequency of the modal class,

$f_0 =$ frequency of the class preceding the modal class,

$f_2 =$ frequency of the class succeeding the modal class.

7. A class with the maximum frequency, called the **modal class**.

8. The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class.

9. If "n" is odd the Median is $\left(\frac{n+1}{2}\right)^{th}$ observation and if "n" is even then the Median will be average of the $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

10. The median for grouped data is formed by using the formula:

Median $=l+\left(\frac{\frac{n}{2}-cf}{f}\right)\times h,$ where symbols have their usual meanings.

11. There is a empirical relationship between the three measures of central tendency:
3 Median = Mode + 2 Mean

SOLVED PROBLEMS

Question 1: In the formula $\bar{x}=a+\frac{\sum f_i d_i}{\sum f_i}$, for finding the mean of grouped data $d_i's$ are deviation from a of

(A) lower limits of the classes
(B) upper limits of the classes
(C) mid-points of the classes
(D) frequencies of the class

Solution: (C) We know that, $d_i = x_i - a$ $i.e., d_i's$ are the deviation from a of mid-points of the classes.

Question 2: While computing mean of grouped data, we assume that the frequencies are

(A) evenly distributed over all the classes
(B) centred at the class marks of the classes
(C) centred at the upper limits of the classes
(D) centred at the lower limits of the classes

Solution: (B) In computing the mean of grouped data, the frequencies are centred at the classmarks of the classes.

Question 3: If $x_i's$ are mid-points of the class intervals of grouped data, $f_i's$ are corresponding frequencies and $\bar{x}$ is the mean, then $\sum(f_i x_i-\bar{x})$ is equal to

(A) 0
(B) -1
(C) 1
(D) 2

Solution: (A) Since $\bar{x}=\frac{\sum f_i x_i}{n}$

$$\sum(f_i x_i-\bar{x})=\sum f_i x_i-\sum\bar{x}=n\bar{x}-\sum\bar{x}=n\bar{x}-n\bar{x}=0.$$

Question 4: In the formula $\bar{x}=a+h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$, for finding the mean of grouped data frequency distribution u_i is equal to

(A) $\frac{x_i+a}{h}$
(B) $h(x_i-a)$
(C) $\frac{x_i-a}{h}$
(D) $\frac{a-x_i}{h}$

Solution: (C) Given, $\bar{x}=a+h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$.

Which is a step deviation formula and u_i equal to $\frac{x_i-a}{h}$

Question 5: The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its

(A) mean
(B) median
(C) mode
(D) All of these

Solution: (B) Since, the intersection point of less than ogive and more than ogive gives the median on the abscissa.

Question 6: For the following distribution, the sum of lower limits of the median class and modal class is

Class interval	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
Frequency	10	15	12	20	9

(A) 15 (B) 25 (C) 30 (D) 35

Solution: (B) Here,

Class interval	Frequency	Cumulative Frequency
0 - 5	10	10
5 - 10	15	25
10 - 15	12	37
15 - 20	20	57
20 - 25	9	66

Now, $\frac{N}{2}=\frac{66}{2}=33$, which lies in the interval 10-15.

Therefore, lower limit of the median class is10.

The highest frequency is 20, which lies in the interval 15-20.

Therefore, lower limit of modal class is 15.

Hence, required sum is10+15=25.

Question 7: Consider the following frequency distribution

Class interval	0 - 5	6 - 11	12 - 17	18 - 23	24 - 29
Frequency	13	10	15	8	11

The upper limit of the median class is

(A) 7 (B) 17.5 (C) 18 (D) 18.5

Solution:(B)

Class interval	Frequency	Cumulative Frequency
-0.5 – 5.5	13	13
5.5 – 11.5	10	23
11.5 – 17.5	15	38
17.5 – 23.5	8	46
23.5 – 29.5	11	57

Question 8: For the following distribution, the modal class is

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
Number of students	3	12	27	57	75	80

(A) 10-20 (B) 20-30 (C) 30-40 (D) 30- 40

Solution: (C)

Marks	Number of students	Cumulative Frequency
Below 10	3=3	3
10 – 20	(12-3)=9	12
20 – 30	(27-12)=15	27
30 – 40	(57-27)=30	57
40 – 50	(75-87)=18	57
50 – 60	(80-75)=5	57

Here, we see that the highest frequency is 30, which lies in the interval 30-40.

Question 9: Consider the data.

Class interval	65 - 85	85 - 105	105 - 125	125 - 145	145 -165	165 - 185	185 -205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is

(A) 0 (B) 19 (C) 20 (D) 38

Solution: (C)

Class interval	Frequency	Cumulative Frequency
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22
125 - 145	20	42
145 - 165	14	56
165 - 185	7	63
185 - 205	4	67

Here, $\frac{N}{2} = \frac{67}{2} = 33.5$, which lies in the interval 125-145.

Hence, upper limit of median class is 145.

Here, we see that the highest frequency is 20 which lies in 125-145.

Hence, the lower limit of modal class is 125.

Required difference = Upper limit of median class – Lower limit of modal class

= 145 – 125 = 20

Question 10: The times (in seconds) taken by 150 atheletes to run a 100 *m* hurdle race are tabulated below.

Class interval	13.8 - 14	14 – 14.2	14.2 – 14.4	14.4 – 14.6	14.6 -14.8	14.8 - 15
Frequency	2	4	5	71	48	20

The number of atheletes who completed the race in less than 14.6 s is

(A) 11 (B) 71 (C) 82 (D) 130

Solution: (C) The number of atheletes who completed the race in less than 14.6 is 2 + 4 + 5 + 71 = 82

Question 11: Consider the following distribution

Marks	More than or equal to 0	More than or equal to 10	More than or equal to 20	More than or equal to 30	More than or equal to 40	More than or equal to 50
Number of students	63	58	55	51	48	42

The frequency of the class 30-40 is

(A) 3 (B) 4 (C) 3 (D) 4

Solution: Hence, frequency in the class interval 30-40 is 3.

MULTIPLE CHOICE QUESTIONS

1. **The marks obtained by 30 students of Class X of a certain school in a Mathematics paper consisting of 100 marks are presented in table below, then the mean of the marks obtained by the students is**

Marks obtained	10	20	36	40	50	56	60	70	72	80	88	92	95
Number of students	1	1	3	4	3	2	4	4	1	1	2	3	1

(A) 58.3 (B) 56.4 (C) 59.3 (D) 54.4

2. **The table below gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. The mean percentage of female teachers by assumed mean method is**

Percentage of female teachers	15 - 25	25 - 35	35 - 45	45 -55	55 -65	65 - 75	75 -85
Number of States/U.T.	6	11	7	4	4	2	1

(A) 39.71 (B) 39.45 (C) 39.32 (D) 39.65

3. **The distribution below shows the number of wickets taken by bowlers in one day cricket matches. The value of mean by step deviation method is**

Number of wickets	20 - 60	60 - 100	100 - 150	150 -250	250 -350	350 -450
Number of bowlers	7	5	16	12	2	3

(A) 169.71 (B) 159.45 (C) 162.32 (D) 152.89

4. **A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality, then the mean number of plants per house is**

Number of plants	0 - 2	2 - 4	4 - 6	6 -8	8 -10	10 -12	12 -14
Number of houses	1	2	1	5	6	2	3

(A) 8.1 (B) 8.4 (C) 8.7 (D) 8.8

5. Consider the following distribution of daily wages of 50 workers of a factory. The mean daily wages of the workers of the factory is

Daily wages (in Rs.)	**500 - 520**	**520 - 540**	**540 - 560**	**560 -580**	**580 -600**
Number of workers	**12**	**14**	**8**	**6**	**10**

(A) Rs 545.40 (B) Rs 545.60 (C) Rs 545.80 (D) Rs 545.20

6. The following distribution shows the daily pocket allowance of children of a locality. If the mean pocket allowance is Rs 18, then the missing frequency f is

Daily Pocket allowance (in Rs.)	**11 -13**	**13 -15**	**15 -17**	**17 -19**	**19 -21**	**21 -23**	**23 -25**
Number of children	**7**	**6**	**9**	**13**	f	**5**	**4**

(A) 21 (B) 24 (C) 20 (D) 28

7. Thirty women were examined in a hospital by a doctor and the number of heartbeats per minute were recorded and summarised as follows. The mean heartbeats per minute for these women is

No. of heart beats (per minute)	**65 -68**	**68 -71**	**71 -74**	**74 -77**	**77 -80**	**80 -83**	**83 -86**
Number of women	**2**	**4**	**3**	**8**	*7*	**4**	**2**

(A) 75.7 (B) 75.9 (C) 75.6 (D) 75.5

8. In a retail market, fruit vendors were selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes. The mean number of mangoes kept in a packing box is

Number of mangoes	**50 -52**	**53 -55**	**56 -58**	**59 -61**	**62 -64**
Number of boxes	**15**	**110**	**135**	**115**	*25*

(A) 57.69 (B) 57.29 (C) 57.19 (D) 57.39

9. The table below shows the daily expenditure on food of 25 households in a locality. The mean daily expenditure on food is

Daily expenditure (in Rs.)	**100 -150**	**150 -200**	**200 -250**	**250 -300**	**300 -350**
Number of house holds	**4**	**5**	**12**	**2**	**2**

(A) 221 (B) 211 (C) 220 (D) 225

10. To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below. The mean concentration of SO_2 in the air is

Concentration of SO_2 (in ppm)	0.00 -0.04	0.04 -0.08	0.08 -0.12	0.12 -0.16	0.16 -0.20	0.20 -0.24
Frequency	4	9	9	2	4	2

(A) 0.089 ppm (B) 0.079 ppm (C) 0.099 ppm (D) 0.069 ppm

11. A class teacher has the following absentee record of 40 students of a class for the whole term. The mean number of days a student was absent is

Number of days	0 -6	6 -10	10 -14	14 -20	20 -28	28 -38	38 -40
Number of students	11	10	7	4	4	3	1

(A) 12.46 days (B) 12.47 days (C) 12.48 days (D) 12.49 days

12. The following table gives the literacy rate (in %) of 35 cities. The mean literacy rate is

Literacy rate (in %)	45 -55	55 -65	65 -75	75 -85	85 -95
Number of cities	3	10	11	8	3

(A) 69.43% (B) 69.44% (C) 69.45% (D) 69.46%

13. The wickets taken by a bowler in 10 cricket matches are as follows: 2, 6, 4, 5, 0, 2, 1, 3, 2, 3. The mode of the data is

(A) 2 (B) 4 (C) 6 (D) 3

14. A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household. The modal class of the data is

Family size	1 -3	3 -5	5 -7	7 -9	9 -11
Number of families	7	8	2	2	1

(A) 1-3 (B) 9-11 (C) 5-7 (D) 3-5

15. The following table shows the ages of the patients admitted in a hospital during a year. The mode of the data is

Age (in %)	5 -15	15 -25	25 -35	35 -45	45 -55	55 -65
Number of parents	6	11	21	23	14	5

(A) 36.5 years (B) 36.6 years (C) 36.7 years (D) 36.8 years

16. The following data gives the information on the observed lifetimes (in hours) of 225 electrical components. The modal lifetimes of the components is

Life time (in hours)	0 -20	20 -40	40 -60	60 -80	80 -100	100 -120
Frequency	10	35	52	61	38	29

(A) 65.645 hrs (B) 65.625 hrs (C) 65.665 hrs (D) 65.675 hrs

17. The following data gives the distribution of total monthly household expenditure of 200 families of a village. The modal monthly expenditure of the families is (approximately)

Expenditure (in rupeees)	Number of families
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

(A) Rs 1848 (B) Rs 1849 (C) Rs 1850 (D) Rs 1847

18. The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. The mode is

Number of students per teacher	15 - 20	20 - 25	25 - 30	30 -35	35 -40	40 - 45	45 -50	50 -55
Number of States/U.T.	3	8	9	10	3	0	0	2

(A) 30.2 (B) 30.4 (C) 30.6 (D) 30.8

19. The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches. The mode is (approximately)

Runs scored	Number of batsmen
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	7
7000-8000	6
8000-9000	3
9000-1000	1
10000-11000	1

(A) 4607 (B) 4609 (C) 4608 (D) 4607

20. A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarised it in the table given below. The mode of the data is

Number of cars	0 - 10	10 - 20	20 - 30	30 -40	40 -50	50 - 60	60 -70	70 -80
Frequency	7	14	13	12	20	11	15	8

(A) 44.5 (B) 44.6 (C) 44.7 (D) 44.8

21. A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the following data was obtained. The median height is

Height (in *cm*)	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

(A) 147.03 *cm* (B) 148.03 *cm* (C) 149.03 *cm* (D) 150.03 *cm*

22. The median of the following data is 525. If the total frequency is 100, then the values of x and y are

Class intervals	0- 100	100 - 200	200 - 300	300 - 400	400 - 500	500 - 600	600 - 700	700 - 800	800 - 900	900 - 1000
Frequency	2	5	x	12	17	20	y	9	7	4

(A) $x = 18, y = 12$ (B) $x = 12, y = 18$ (C) $x = 15, y = 9$ (D) $x = 9, y = 15$

23. The following frequency distribution gives the monthly consumption of electricity of 68 consumers of a locality. The median is

Monthly consumption (in units)	65 - 85	85 - 105	105 - 125	125 - 145	145-165	165-185	185-205
Number of consumers	4	5	13	20	14	8	4

(A) 136 (B) 137 (C) 138 (D) 139

24. If the median of the distribution of 60 samples given below is 28.5, then the values of x and y are

Class interval	0 - 10	10 - 20	20 – 30	30 - 40	40 - 50	50 - 60
Frequency	5	x	20	15	y	5

(A) $x = 8, y = 7$ (B) $x = 7, y = 8$ (C) $x = 10, y = 5$ (D) $x = 5, y = 10$

25. A life insurance agent found the following data for distribution of ages of 100 policy holders. If policies are given only to persons having age 18 years onwards but less than 60 year, then the median is

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
No. of policy holders	2	6	24	45	78	89	92	98	100

(A) 35.56 (B) 35.46 (C) 35.66 (D) 35.76

26. The lengths of 40 leaves of a plant are measured correct to the nearest millimetre, and the data obtained is represented in the following table. the median length of the leaves is

Length (in mm)	118 -126	126 - 135	136 - 144	145 - 153	154-162	163-171	172-180
Number of leaves	4	5	13	20	14	8	4

(A) $148.75\,mm$ (B) $146.75\,mm$ (C) $149.75\,mm$ (D) $147.75\,mm$

27. The following table gives the distribution of the life time of 400 neon lamps, the median life time of a lamp is

Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48
10000-11000	1

(A) $3409.98\,hrs$ (B) $3408.98\,hrs$ (C) $3407.98\,hrs$ (D) $3406.98\,hrs$

28. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows. The median number of letters in the surnames is

Number of letters	1 - 4	4 - 7	7 - 10	10 -13	13 -16	16 - 19
Number of surnames	6	30	40	16	4	4

(A) 8.08 (B) 8.07 (C) 8.06 (D) 8.05

29. The distribution below gives the weights of 30 students of a class. The median weight of the students is

Weight (in kg)	40 -45	45 - 50	50 - 55	55 - 60	60-65	65-70	70-75
Number of studentss	4	5	13	20	14	8	4

(A) $56.47\,kg$ (B) $56.57\,kg$ (C) $56.67\,kg$ (D) $56.77\,kg$

30. The average of the observations 10, 20, 65, 102, 108, 115 is

(A) 50 (B) 70 (C) 60 (D) 40

31. If 35 is removed from the data 30, 34, 35, 36, 37, 38, 39, 40, then the median increases by

(A) 2 (B) 1.5 (C) 1 (D) 0.5

33. The modal class of the following frequency distribution is

Class interval	0 - 20	20 – 40	40 – 60	60 – 80	80 -100
No. of students	15	18	21	29	17

(A) 80-100 (B) 0-20 (C) 60-80 (D) 40- 60

34. If the mode and mean of a data are 24 and 60 respectively, then the median of the data is

(A) 49 (B) 48 (C) 47 (D) 46

35. The upper limit of the median class of the following frequency distribution is

Class interval	50 - 70	70 - 90	90 - 110	110 - 130	130 -150	150 - 170
Frequency	15	21	32	19	8	5

(A) 110 (B) 90 (C) 130 (D) 70

36. If the mean of the following distribution is 15, then the value of y is

x	5	10	15	20	25
f	6	8	6	y	5

(A) 8 (B) 7 (C) 10 (D) 9

37. If the difference between mode and mean of a data is k times the difference between median and mean, then the value of k is

(A) 2 (B) 3 (C) 1 (D) cannot be determined

38. The median of the first 10 prime numbers is

(A) 11 (B) 12 (C) 13 (D) 14

39. The arithmetic mean of 12, 15, 13, 20, 25 is

(A) 17 (B) 20 (C) 18 (D) None

40. If 5 is added to each and every item of a data, then the arithmetic mean is

(A) 5 times to the first arithmetic mean (B) increased by 5 to the first arithmetic mean

(C) equal to the first arithmetic mean (D) None

41. The median of 24, 20, 32, 18, 28, 16, 25 is

(A) 18 (B) 16 (C) 24 (D) 32

42. The median of the following frequency distribution is

Class interval	0 - 9	10 - 19	20 - 29	30 - 39
Frequency	10	16	24	29

(A) 23.75 (B) 23.25 (C) 25.125 (D) None

43. For the data 9, 8, 7, 7, 6, 7, 2, 1, 7, 9 mode is

(A) 9 (B) 7 (C) 3 (D) 2

44. The modal class of the following distribution is

Class interval	1 - 3	3 - 5	5 - 7	7 - 9
Frequency	7	8	2	1

(A) 1-3 (B) 3-5 (C) 5-7 (D) None

45. The formula for finding the mode for grouped data is

(A) $l-\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$ (B) $l+\left(\frac{f_1-f_0}{f_1-f_0-f_2}\right)\times h$

(C) $l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$ (D) $l-\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right)\times h$

46. Cumulative frequency is used to calculate

(A) Median (B) Mode (C) Mean (D) None

47. For the data 6, 2, 9, 11, 3, 4, 9, 7, 13, 1, which of the following is true?

(A) Median < Mean < Mode (B) Mode < Mean < Median

(C) Mean = Median < Mode (D) Mode = Median < Mean

48. The mean of first n natural numbers is

(A) n (B) $n+1$ (C) $\frac{n+1}{2}$ (D) $\frac{n}{2}$

49. The mode of the following distribution is

Class interval	2 - 4	4 - 6	6 - 8	8 - 10
Frequency	5	7	3	2

(A) 4.66 (B) 4.56 (C) 4.33 (D) 4.46

50. Which of the following is not a measure of central tendency?

(A) Median (B) Mode (C) Mean (D) Range

ANSWERS

1	C	2	A	3	D	4	A	5	D	6	C	7	B	8	C	9	B	10	C
11	C	12	A	13	A	14	D	15	D	16	B	17	A	18	C	19	B	20	C
21	C	22	D	23	B	24	B	25	D	26	B	27	D	28	D	29	C	30	B
31	D	32	B	33	C	34	B	35	A	36	C	37	B	38	B	39	A	40	B
41	C	42	C	43	B	44	B	45	C	46	A	47	C	48	C	49	A	50	D

* * *

14. PROBABILITY

SYNOPSIS

1. We have two types of probability
   (i) Experimental Probability (ii) Theoretical Probability
2. The theoretical Probability of an even E, written as P(E), is defined as
   $P(E) = \frac{\text{number of trials in which the event hapened}}{\text{Total number of trials}}$
   Where we assume that the outcomes of the experiment are equally likely
3. The Probability of a sure event is 1
4. The Probability of an impossible event is 0
5. The probability of an event E is a number P (E) such that $0 \le P(E) \le 1$.
6. An event having only one outcome is called an elementary event. The sum of the probabilities of all elementary events of an experiment is 1.
7. For any event E, $P(E) + P(\overline{E}) = 1$, where $\overline{E}$ stands for "not E".
8. E and $\overline{E}$ are called **complementary** events.
9. **Equally likely events:** Two or more events are said to be equally likely if each one of them has an equal chance of occurrence.
10. **Mutually exclusive events:** Two or more events are mutually exclusive if the occurrence of each event prevents the every other event.
11. **Complementary events:** Consider an event has few outcomes. Event of all other outcomes in the sample survey which are not in the favorable event is called complementary event.
12. **Exhaustive events:** All the events are exhaustive events if their union is the sample space.
13. **Sure events:** The sample space of a random experiment is called sure or certain event as anyone of its elements will surely occur in any trial of the experiment.
14. **Impossible event:** An event which will occur an any account is called an impossible event.

SOLVED PROBLEMS

Question 1: If an event cannot occur, then its probability is

(A) 1 (B) 3/4 (C) 1/2 (D) 0

Solution: (D) The event which cannot occur is said to be impossible event and probability of impossible event is zero.

Question 2: Which of the following cannot be the probability of an event?

(A) 1/2 (B) 0.1 (C) 3 (D) 17/16

Solution: (D) Since, probability of an event always lies between 0 and 1.

Question 3: An event is very unlikely to happen. Its probability is closest to

(A) 0.0001 (B) 0.001 (C) 0.01 (D) 0.1

Solution: (A) The probability of an event which is very unlikely to happen is closest to zero and from the given options 0.0001 is closest to zero.

Question 4: If the probability of an event is P, then the probability of its completmentry event will be

(A) P–1 (B) P (C) 1–P (D) $1 - \frac{1}{P}$

Solution: (C) Since, probability of an event + probability of its complementary event =1

So, probability of its complementary event =1– Probabilityofanevent = 1–P

Question 5: The probability expressed as a percentage of a particular occurrence can never be

(A) lessthan100 (B)lessthan 0

(C) greaterthan 1 (D) any thing but a whole number

Solution: (B) We know that, the probability expressed as a percentage always lie between 0 and 100. So, it cannot be less than 0.

Question 6: If P(A) denotes the probability of an event A, then

(A) $P(A) < 0$ (B) $P(A) > 1$ (C) $0 \le P(A) \le 1$ (D) $-1 \le P(A) \le 1$

Solution: (C) Since, probability of an event always lies between 0 and 1.

Question 7: If a card is selected from a deck of 52 cards, then the probability of its being a red face card is

(A) 3/26 (B) 3/13 (C) 2/13 (D) 1/2

Solution: (C) In a deck of 52 cards, there are12 face cards i.e., 6 red and 6 black cards.

So, probability of getting a red face card $= \frac{6}{52} = \frac{3}{26}$

Question 8: The probability that a non-leap year selected at random will contains 53 Sunday is

(A) 1/7 (B) 2/7 (C) 3/7 (D) 5/7

Solution: (A) Anon-leap year has 365 days and there fore 52 weeks and 1 day. This 1day may be Sunday or Monday or Tuesday or Wednesday or Thursday or Friday or Saturday.

Thus, out of 7 possibilities,1 favourable event is the event that the one day is Sunday.

∴Required probability = 1/7.

Question 9: When a die is thrown, the probability of getting an odd number less than 3 is,

(A) 1/6 (B) 1/3 (C) 1/2 (D) 0

Solution: (A) When a die-is thrown, then total number of outcomes = 6

Odd number less than 3 is1only.

Number of possible outcomes = 1

Required probability = 1/6

Question 10: A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is

(A) 4 (B)13 (C) 48 (D) 51

Solution: (D) In a deck of 52 cards, there are 13 cards of heart and 1 is ace of heart.

Hence, the number of outcomes favourable to E=51

Question 11: The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is

(A) 7 (B) 14 (C) 21 (D) 28

Solution: (B) Here, total number of eggs = 400

Probability of getting a bad egg = 0.035

$$\Rightarrow \frac{\text{Number of bad eggs}}{\text{Total number of eggs}} = 0.035$$

$$\Rightarrow \frac{\text{Number of bad eggs}}{400} = 0.035$$

$$\therefore \text{Number of bad eggs} = 400 \times 0.035 = 14$$

Question 12: A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?

(A) 40 (B) 240 (C) 480 (D) 750

Solution: (C) Given, total number of sold tickets = 6000

Let she bought x tickets. Then, the probability of her winning the first prize $= \frac{x}{6000} = 0.08$

$$\Rightarrow x = 0.08 \times 6000 = 480$$

Hence, she bought 480 tickets.

Question 13: One ticket is drawn at random from a bag containing tickets numbered 1 to 40.The probability that the selected ticket has a number which is a multiple of 5 is

(A) 1/5 (B) 3/5 (C) 4/5 (D) 1/3

Solution: (A) Number of total outcomes = 40

Multiples of 5 between 1 to 40 = 5, 10, 15, 20, 25, 30, 35, 40

$\therefore$ Total number of possible outcomes is $= 8$

$\therefore$ Required probability $= \frac{8}{40} = \frac{1}{5}$

Question 14: Some one is asked to take a number from 1 to100. The probability that it is a prime, is

(A) 1/5 (B) 6/25 (C) 1/4 (D) 13/50

Solution: (C) Total numbers of outcomes = 100

So, the prime numbers between 1 to100 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 56, 61, 67, 71, 73, 79, 83, 89 and 97.

$\therefore$ Total number of possible outcomes is $= 25$

$\therefore$ Required probability $= \frac{25}{100} = \frac{1}{4}$

Question 15: A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is

(A) 4/23 (B) 6/23 (C) 8/23 (D) 17/23

Solution: (B) Total number of students = 23

Number of students in house A, B and C = 4 + 8 + 5 = 17

$\therefore$ Remaining students $= 23 - 17 = 6$

So, probability that the selected student is not from A, B and C $= \frac{6}{23}$.

MULTIPLE CHOICE QUESTIONS

1. **A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Manasa takes out a ball form the bag randomly, then the probability that she takes an yellow ball is**

   (A) 1 (B) 0 (C) 1/3 (D) 2/3

2. **A dice is thrown once, then the probability of getting a number greater than 4 is**

   (A) 1/3 (B) 2/3 (C) 1 (D) None

3. Which of the following is true, if $P(E)$ represents probability of an event

(A) $0 \le P(E) < 1$ (B) $0 \ge P(E) \ge 1$ (C) $0 > P(E) \le 1$ (D) $0 \le P(E) \le 1$

4. If one card is drawn from a well shuffled deck of 52 cards then the probability that the card will be an ace is

(A) 1/13 (B) 2/13 (C) 3/13 (D) None

5. Sania and Venus, play a tennis match. If the Probability of Sania winning the match is 0.71. Then the Probability of Venus winning the match

(A) 0.71 (B) 0.5 (C) 1 (D) 0.29

6. In a non-leap year, the Probability that Ramesh and Mahesh will have different birth days.

(A) 1 (B) $\frac{364}{365}$ (C) $\frac{1}{365}$ (D) None

7. A trail is made to answer a true false question. The answer is right or wrong. This event is

(A)Mutually exclusive event (B) equally likely event

(C) Exhaustive event (D) All of the above

8. Mr. Anil takes out all the hearts from the deck of playing cards. Then the Probability of picking out a diamonds

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{1}{3}$ (D) 0

9. If Rahim takes out all the spades from the deck of playing cards. The probability of picking a card that is not a spade is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{1}{3}$ (D) 0

10. A dice is thrown once then the probability of getting a number lying between 2 and 6.

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) None

11. The probability of drawing out a red king from a deck of cards is

(A) $\frac{25}{26}$ (B) $\frac{24}{26}$ (C) $\frac{1}{26}$ (D) None

12. A box contains 3 blue, 2 white and 4 red marbles. If a marble is drawn at randomly from the box. Then the Probability that it will be white marble is

(A) $\frac{2}{9}$ (B) $\frac{3}{9}$ (C) $\frac{3}{8}$ (D) $\frac{1}{8}$

13. **Arvind tosses two different coins simultaneously then the probability that he gets at least one head is**

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) 1 (D) 0

14. **A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. Then the probability that the ball drawn is not red is**

(A) $\frac{5}{8}$ (B) $\frac{4}{8}$ (C) $\frac{3}{8}$ (D) $\frac{1}{8}$

15. **One card is drawn from well shuffled deck of 52 cards then the probability of getting the queen of diamonds is**

(A) $\frac{1}{26}$ (B) $\frac{3}{13}$ (C) $\frac{3}{26}$ (D) $\frac{1}{52}$

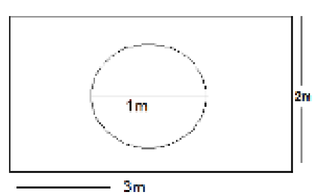
16. **12 defective pens are accidentally mixed with 132 good ones. If one pen is taken out at random from this lot. Then the probability that the pen taken out is a good one is**

(A) 1 (B) $\frac{11}{12}$ (C) $\frac{1}{12}$ (D) None

17. **A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, the probability that it is a perfect square is**

(A) $\frac{1}{10}$ (B) $\frac{11}{12}$ (C) $\frac{1}{12}$ (D) None

18. **Suppose a stone is dropped at random on the rectangular region shown in the figure, then the probability that it will land inside the circle with diameter 1m?**



(A) $\frac{11}{14}$ (B) $\frac{11}{84}$ (C) $\frac{3}{84}$ (D) None

19. **If a digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, then the probability that it is odd, is**

(A) $\frac{4}{9}$ (B) $\frac{5}{9}$ (C) $\frac{1}{9}$ (D) $\frac{2}{3}$

20. **If three coins are tossed simultaneously, then the probability of getting at least two heads, is**

(A) $\frac{1}{4}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

21. The probability of guessing the correct answer to a certain test question is $\frac{x}{12}$. If the probability of not guessing the correct answer to this question is $\frac{2}{3}$, then $x =$

(A) 2 (B) 3 (C) 4 (D) 6

22. A bag contains three green marbles, four blue marbles and two orange marbles. If a marble is picked at random, then the probability that it is not an orange marble is

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) $\frac{4}{9}$ (D) $\frac{7}{9}$

23. A number is selected at random from the numbers, 3, 5, 5, 7, 7, 7, 9, 9, 9, 9. Then the probability that the selected number is their average is

(A) $\frac{1}{10}$ (B) $\frac{3}{10}$ (C) $\frac{7}{10}$ (D) $\frac{9}{10}$

24. Which of the following cannot be the probability of an event?

(A) $\frac{2}{3}$ (B) -1.5 (C) 15% (D) 0.7

25. If $P(E) = 0.05$, then $P(not\ E) =$

(A) -0.05 (B) 0.5 (C) 0.9 (D) 0.95

26. The probability of a certain event is

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) 2

27. The probability that a non-leap year has 53 Sundays is

(A) $\frac{2}{7}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) $\frac{1}{7}$

28. The probability that a non- leap year has 53 Mondays is

(A) $\frac{2}{7}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) $\frac{1}{7}$

29. If a two digit number is chosen at random, then the probability that the number chosen is a multiple of 3 is

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{3}{4}$ (D) None

30. A month is selected at random in a year. The probability that it is March or October is

(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{3}{4}$ (D) None

31. The probability of getting king or queen card from the play cards (1 deck)

(A) 1 (B) $\frac{3}{52}$ (C) $\frac{1}{13}$ (D) $\frac{2}{13}$

32. Among the numbers 1, 2, 3….15 the probability of choosing a number which is a multiple of 4 is

(A) 1 (B) $\frac{1}{5}$ (C) $\frac{1}{15}$ (D) $\frac{4}{15}$

33. Gita said that the probability of impossible events is 1, Pravallika said that probability of sure events is 0 and Gowthami said that the probability of any event lies in between 0 & 1. In above with whom you will agree

(A) Gita (B) Pravallika (C) Gowthami (D) All the above

34. The probability of a sure event is

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) -1

35. If a die is rolled then the probability of getting an even number is

(A) 1 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

36. If $P(E)=0.5,$ then $P(not\ E)=$

(A) -0.05 (B) 0.5 (C) 0.9 (D) 0.95

37. No of playing cards in a deck of cards is

(A) 13 (B) 53 (C) 52 (D) 26

38. In a single throw of two dice the probability of getting distinct number is

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

39. A card is pulled from a desk of 52 cards, the probability of obtaining a club is

(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

40. If two dice are rolled at a time then the probability that the two faces show same number is

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$ (C) $\frac{5}{36}$ (D) $\frac{25}{36}$

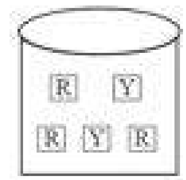
41. If three coins are tossed simultaneously then the probability of getting at least two heads is

(A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{2}{3}$ (D) $\frac{5}{8}$

42. A number is selected from numbers 1 to 25. The probability that it is prime is

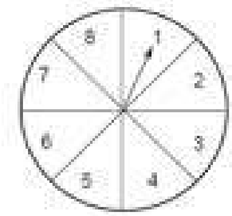
(A) $\frac{1}{25}$ (B) $\frac{9}{25}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

43. R = Red, Y = yellow, from the figure, the probability to get yellow colour ball is



(A) $\frac{4}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $\frac{1}{5}$

44. A game of chance consists of spinning an arrow which comes to rest at one of the number 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes the possibilities that the arrow will point at a number greater than 2 is



(A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$

45. From the above problem, the probability that the arrow will point at a prime number is

(A) $\frac{3}{8}$ (B) $\frac{5}{8}$ (C) $\frac{3}{4}$ (D) $\frac{1}{2}$

46. Getting "7" when a single die is throw is an example of

(A) certain event (B) uncertain event (C) complementary event (D) None

47. The probability of an event lies between and

(A) $0,1$ (B) $-1,1$ (C) $1,\infty$ (D) $-\infty,\infty$

48. When a die is thrown once, the possible number of outcomes is

(A) 1 (B) 2 (C) 6 (D) 36

49. If two events have same chances to happen then they are called

(A) unequally likely events (B) equally likely events

(C) complementary events (D) None

50. For any event E, $P(E)+P(\overline{E})$ is [where $\overline{E}$ stands for "not E"]

(A) 0 (B) 2 (C) 1 (D) -1

ANSWERS

1	C	2	A	3	D	4	A	5	D	6	C	7	B	8	C	9	B	10	c
11	C	12	A	13	A	14	D	15	D	16	B	17	A	18	C	19	B	20	C
21	C	22	D	23	B	24	B	25	D	26	B	27	D	28	D	29	C	30	B
31	D	32	B	33	C	34	A	35	C	36	B	37	C	38	B	39	A	40	A
41	B	42	B	43	C	44	C	45	D	46	A	47	A	48	C	49	B	50	C