

**Paper Specific Instructions**

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.
2. **Section A** contains a total of 30 **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
3. **Section B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there will be **more than one** choices that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.
4. **Section C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.
5. In all sections, questions not attempted will result in zero mark. In **Section A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1-mark questions,  $\frac{1}{3}$  marks will be deducted for each wrong answer. For all 2-mark questions,  $\frac{2}{3}$  marks will be deducted for each wrong answer. In **Section B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section C** (NAT) as well.
6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.
7. A Scribble Pad will be provided for rough work.

### Special Instructions / Useful Data

$\mathbb{N}$  = The set of all natural numbers

$\mathbb{Z}$  = The set of all integers

$\mathbb{Z}_n = \{\overline{0}, \overline{1}, \dots, \overline{n-1}\}$ , the group of integers modulo  $n$ , under addition modulo  $n$ , for  $n \in \mathbb{N}$

$\mathbb{R}$  = The set of all real numbers

$\mathbb{R}^n$  = The  $n$ -dimensional Euclidean space

$\ln x$  = The natural logarithm of  $x$  (to the base  $e$ )

$S_n$  = The symmetric group of all permutations on  $\{1, 2, \dots, n\}$

$id$  = The identity element in  $S_n$

$(a_n)$  = The infinite sequence  $a_1, a_2, a_3, \dots$

$f \circ g$  = The composition of  $f$  and  $g$ , defined by  $(f \circ g)(x) = f(g(x))$

$f'(x)$  = The first derivative of  $f$  at the point  $x$

$f''(x)$  = The second derivative of  $f$  at the point  $x$

$\text{span } S$  = The linear span of the subset  $S$  of a vector space

$P_n(\mathbb{R})$  = The real vector space of real polynomials of degree less than or equal to  $n$ , together with the zero polynomial. These polynomials can be regarded as functions from  $\mathbb{R}$  to  $\mathbb{R}$

$\ker(T)$  = The kernel of the linear transformation  $T$

$M = (m_{ij})$  = Matrix of appropriate order with the entry/element in the  $i^{\text{th}}$  row and  $j^{\text{th}}$  column denoted by  $m_{ij}$ ,  $m_{ij} \in \mathbb{R}$

$\text{gcd}(m, n)$  = The greatest common divisor of the natural numbers  $m$  and  $n$

$\det(M)$  = The determinant of the matrix  $M$

$\frac{\partial f}{\partial x}$  = The partial derivative of  $f$  with respect to  $x$

$\frac{\partial f}{\partial y}$  = The partial derivative of  $f$  with respect to  $y$

Section A: Q.1 – Q.10 Carry ONE mark each.

Q.1 The sum of the infinite series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\pi^{2n+1}}{2^{2n+1}(2n)!}$$

is equal to

(A)  $-\pi$

(B)  $\frac{\pi}{4}$

(C)  $\frac{\pi}{2}$

(D)  $-\frac{\pi}{4}$

Q.2 For which one of the following choices of  $N(x, y)$ , is the equation  $(e^x \sin y - 2y \sin x) dx + N(x, y) dy = 0$  an exact differential equation?

(A)  $N(x, y) = e^x \sin y + 2 \cos x$

(B)  $N(x, y) = e^x \cos y + 2 \cos x$

(C)  $N(x, y) = e^x \cos y + 2 \sin x$

(D)  $N(x, y) = e^x \sin y + 2 \sin x$

Q.3 Let  $f, g: \mathbb{R} \rightarrow \mathbb{R}$  be two functions defined by

$$f(x) = \begin{cases} x|x| \left| \sin \frac{1}{x} \right| & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} x^2 \sin \frac{1}{x} + x \cos \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Then, which one of the following is TRUE?

- (A)  $f$  is differentiable at  $x = 0$ , and  $g$  is NOT differentiable at  $x = 0$
- (B)  $f$  is NOT differentiable at  $x = 0$ , and  $g$  is differentiable at  $x = 0$
- (C)  $f$  is differentiable at  $x = 0$ , and  $g$  is differentiable at  $x = 0$
- (D)  $f$  is NOT differentiable at  $x = 0$ , and  $g$  is NOT differentiable at  $x = 0$

Q.4 Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be two functions defined by

$$f(x) = \begin{cases} |x|^{1/8} \left| \sin \frac{1}{x} \right| \cos x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and

$$g(x) = \begin{cases} e^x \cos \frac{1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}.$$

Then, which one of the following is TRUE?

- (A)  $f$  is continuous at  $x = 0$ , and  $g$  is NOT continuous at  $x = 0$
- (B)  $f$  is NOT continuous at  $x = 0$ , and  $g$  is continuous at  $x = 0$
- (C)  $f$  is continuous at  $x = 0$ , and  $g$  is continuous at  $x = 0$
- (D)  $f$  is NOT continuous at  $x = 0$ , and  $g$  is NOT continuous at  $x = 0$

Q.5 Which one of the following is the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 2e^{4x} ?$$

- (A)  $\alpha_1 e^{4x} + \alpha_2 x e^{4x} + x^2 e^{4x}$ , where  $\alpha_1, \alpha_2 \in \mathbb{R}$
- (B)  $\alpha_1 e^{4x} + \alpha_2 x e^{4x} + 2x^2 e^{4x}$ , where  $\alpha_1, \alpha_2 \in \mathbb{R}$
- (C)  $\alpha_1 e^{-4x} + \alpha_2 e^{4x} + 2x^2 e^{4x}$ , where  $\alpha_1, \alpha_2 \in \mathbb{R}$
- (D)  $\alpha_1 x e^{-4x} + \alpha_2 x^2 e^{-4x} + x^2 e^{4x}$ , where  $\alpha_1, \alpha_2 \in \mathbb{R}$

Q.6 Define  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  by

$$T(x, y, z) = (x + z, 2x + 3y + 5z, 2y + 2z), \text{ for all } (x, y, z) \in \mathbb{R}^3.$$

Then, which one of the following is TRUE?

- (A)  $T$  is one-one and  $T$  is NOT onto
- (B)  $T$  is NOT one-one and  $T$  is onto
- (C)  $T$  is one-one and  $T$  is onto
- (D)  $T$  is NOT one-one and  $T$  is NOT onto

Q.7

Let  $M = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 1 & -4 \\ 2 & 0 & x \end{pmatrix}$  for some real number  $x$ .

If 0 is an eigenvalue of  $M$ , then  $(M^4 + M) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$  is equal to

(A)  $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(B)  $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$

(C)  $\begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix}$

(D)  $\begin{pmatrix} 17 \\ 0 \\ 17 \end{pmatrix}$

- Q.8 Let  $T : P_2(\mathbb{R}) \rightarrow P_2(\mathbb{R})$  be the linear transformation defined by  $T(p(x)) = p(x + 1)$ , for all  $p(x) \in P_2(\mathbb{R})$ . If  $M$  is the matrix representation of  $T$  with respect to the ordered basis  $\{1, x, x^2\}$  of  $P_2(\mathbb{R})$ , then which one of the following is TRUE?
- (A) The determinant of  $M$  is 2
- (B) The rank of  $M$  is 2
- (C) 1 is the only eigenvalue of  $M$
- (D) The nullity of  $M$  is 2

- Q.9 Let  $G$  be a finite abelian group of order 10. Let  $x_0$  be an element of order 2 in  $G$ . If  $X = \{x \in G : x^3 = x_0\}$ , then which one of the following is TRUE?
- (A)  $X$  has exactly one element
- (B)  $X$  has exactly two elements
- (C)  $X$  has exactly three elements
- (D)  $X$  is an empty set

Q.10 The value of

$$\int_0^1 \left( \int_{\sqrt{y}}^1 3e^{x^3} dx \right) dy$$

is equal to

(A)  $e - 1$

(B)  $\frac{e-1}{2}$

(C)  $\sqrt{e} - 1$

(D)  $\frac{\sqrt{e}-1}{2}$

**Section A: Q.11 – Q.30 Carry TWO marks each.**

Q.11 Let  $\mathcal{C}$  denote the family of curves described by  $yx^2 = \lambda$ , for  $\lambda \in (0, \infty)$  and lying in the first quadrant of the  $xy$  plane. Let  $\mathcal{O}$  denote the family of orthogonal trajectories of  $\mathcal{C}$ .

Which one of the following curves is a member of  $\mathcal{O}$ , and passes through the point  $(2, 1)$  ?

(A)  $y = \frac{x^2}{4}, x > 0, y > 0$

(B)  $x^2 - 2y^2 = 2, x > 0, y > 0$

(C)  $x - y = 1, x > 0, y > 0$

(D)  $2x - y^2 = 3, x > 0, y > 0$

Q.12 Let  $\varphi : (0, \infty) \rightarrow \mathbb{R}$  be the solution of the differential equation

$$x \frac{dy}{dx} = (\ln y - \ln x)y,$$

satisfying  $\varphi(1) = e^2$ . Then, the value of  $\varphi(2)$  is equal to

(A)  $e^2$

(B)  $2e^3$

(C)  $3e^2$

(D)  $6e^3$

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Q.13 Let  $X = \{x \in S_4 : x^3 = id\}$  and  $Y = \{x \in S_4 : x^2 \neq id\}$ .

If  $m$  and  $n$  denote the number of elements in  $X$  and  $Y$ , respectively, then which one of the following is TRUE?

(A)  $m$  is even and  $n$  is even

(B)  $m$  is odd and  $n$  is even

(C)  $m$  is even and  $n$  is odd

(D)  $m$  is odd and  $n$  is odd

Q.14 Let  $\varphi : \mathbb{R} \rightarrow \mathbb{R}$  be the solution of the differential equation

$$\frac{dy}{dx} = (y - 1)(y - 3),$$

satisfying  $\varphi(0) = 2$ . Then, which one of the following is TRUE?

(A)  $\lim_{x \rightarrow \infty} \varphi(x) = 0$

(B)  $\lim_{x \rightarrow \ln \sqrt{2}} \varphi(x) = 1$

(C)  $\lim_{x \rightarrow -\infty} \varphi(x) = 3$

(D)  $\lim_{x \rightarrow \ln \frac{1}{\sqrt{2}}} \varphi(x) = 6$

Q.15

Let  $M = \begin{pmatrix} 6 & 2 & -6 & 8 \\ 5 & 3 & -9 & 8 \\ 3 & 1 & -2 & 4 \end{pmatrix}$ . Consider the system  $S$  of linear equations given

by

$$\begin{aligned} 6x_1 + 2x_2 - 6x_3 + 8x_4 &= 8 \\ 5x_1 + 3x_2 - 9x_3 + 8x_4 &= 16 \\ 3x_1 + x_2 - 2x_3 + 4x_4 &= 32 \end{aligned}$$

where  $x_1, x_2, x_3, x_4$  are unknowns.

Then, which one of the following is TRUE?

- (A) The rank of  $M$  is 3, and the system  $S$  has a solution
- (B) The rank of  $M$  is 3, and the system  $S$  does NOT have a solution
- (C) The rank of  $M$  is 2, and the system  $S$  has a solution
- (D) The rank of  $M$  is 2, and the system  $S$  does NOT have a solution

Q.16

Let  $M = \begin{pmatrix} -2 & 0 & 0 \\ 3 & 2 & 3 \\ 4 & -1 & x \end{pmatrix}$  for some real number  $x$ . Suppose that  $-2$  and  $3$

are eigenvalues of  $M$ . If  $M^3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 125 \\ 125 \end{pmatrix}$ , then which one of the following

is TRUE?

- (A)  $x = 5$ , and the matrix  $M^2 + M$  is invertible
- (B)  $x \neq 5$ , and the matrix  $M^2 + M$  is invertible
- (C)  $x = 5$ , and the matrix  $M^2 + M$  is NOT invertible
- (D)  $x \neq 5$ , and the matrix  $M^2 + M$  is NOT invertible

Q.17 Let  $f(x) = 10x^2 + e^x - \sin(2x) - \cos x$ ,  $x \in \mathbb{R}$ . The number of points at which the function  $f$  has a local minimum is

- (A) 0
- (B) 1
- (C) 2
- (D) greater than or equal to 3

Q.18 For  $n \in \mathbb{N}$ , define  $x_n$  and  $y_n$  by

$$x_n = (-1)^n \cos \frac{1}{n} \quad \text{and} \quad y_n = \sum_{k=1}^n \frac{1}{n+k}.$$

Then, which one of the following is TRUE?

- (A)  $\sum_{n=1}^{\infty} x_n$  converges, and  $\sum_{n=1}^{\infty} y_n$  does NOT converge
- (B)  $\sum_{n=1}^{\infty} x_n$  does NOT converge, and  $\sum_{n=1}^{\infty} y_n$  converges
- (C)  $\sum_{n=1}^{\infty} x_n$  converges, and  $\sum_{n=1}^{\infty} y_n$  converges
- (D)  $\sum_{n=1}^{\infty} x_n$  does NOT converge, and  $\sum_{n=1}^{\infty} y_n$  does NOT converge

Q.19 Let  $x_1 = \frac{5}{2}$ . For  $n \in \mathbb{N}$ , define

$$x_{n+1} = \frac{1}{5}(x_n^2 + 6).$$

Then, which one of the following is TRUE?

- (A)  $(x_n)$  is an increasing sequence, and  $(x_n)$  is NOT a bounded sequence
- (B)  $(x_n)$  is NOT an increasing sequence, and  $(x_n)$  is NOT a bounded sequence
- (C)  $(x_n)$  is NOT a decreasing sequence, and  $(x_n)$  is a bounded sequence
- (D)  $(x_n)$  is a decreasing sequence, and  $(x_n)$  is a bounded sequence

Q.20

Let  $x_1 = 2$  and  $x_{n+1} = 2 + \frac{1}{2x_n}$  for all  $n \in \mathbb{N}$ .

Then, which one of the following is TRUE?

- (A)  $x_{n+1} \geq \frac{4}{x_n}$  for all  $n \in \mathbb{N}$ , and  $(x_n)$  is a Cauchy sequence
- (B)  $x_{n+1} < \frac{4}{x_n}$  for some  $n \in \mathbb{N}$ , and  $(x_n)$  is a Cauchy sequence
- (C)  $x_{n+1} \geq \frac{4}{x_n}$  for all  $n \in \mathbb{N}$ , and  $(x_n)$  is NOT a Cauchy sequence
- (D)  $x_{n+1} < \frac{4}{x_n}$  for some  $n \in \mathbb{N}$ , and  $(x_n)$  is NOT a Cauchy sequence

Q.21 For  $n \in \mathbb{N}$ , define  $x_n$  and  $y_n$  by

$$x_n = (-1)^n \frac{3^n}{n^3} \text{ and } y_n = (4^n + (-1)^n 3^n)^{1/n}.$$

Then, which one of the following is TRUE?

- (A)  $(x_n)$  has a convergent subsequence, and NO subsequence of  $(y_n)$  is convergent
- (B) NO subsequence of  $(x_n)$  is convergent, and  $(y_n)$  has a convergent subsequence
- (C)  $(x_n)$  has a convergent subsequence, and  $(y_n)$  has a convergent subsequence
- (D) NO subsequence of  $(x_n)$  is convergent, and NO subsequence of  $(y_n)$  is convergent

Q.22

Let  $M = (m_{ij})$  be a  $3 \times 3$  real, invertible matrix and  $\sigma \in S_3$  be the permutation defined by  $\sigma(1) = 2$ ,  $\sigma(2) = 3$  and  $\sigma(3) = 1$ . The matrix  $M_\sigma = (n_{ij})$  is defined by  $n_{ij} = m_{i\sigma(j)}$  for all  $i, j \in \{1, 2, 3\}$ .

Then, which one of the following is TRUE?

- (A)  $\det(M) = \det(M_\sigma)$ , and nullity of the matrix  $M - M_\sigma$  is 0
- (B)  $\det(M) = -\det(M_\sigma)$ , and nullity of the matrix  $M - M_\sigma$  is 1
- (C)  $\det(M) = \det(M_\sigma)$ , and nullity of the matrix  $M - M_\sigma$  is 1
- (D)  $\det(M) = -\det(M_\sigma)$ , and nullity of the matrix  $M - M_\sigma$  is 0

Q.23 Let  $\mathbb{R}/\mathbb{Z}$  denote the quotient group, where  $\mathbb{Z}$  is considered as a subgroup of the additive group of real numbers  $\mathbb{R}$ .

Let  $m$  denote the number of injective (one-one) group homomorphisms from  $\mathbb{Z}_3$  to  $\mathbb{R}/\mathbb{Z}$  and  $n$  denote the number of group homomorphisms from  $\mathbb{R}/\mathbb{Z}$  to  $\mathbb{Z}_3$ .

Then, which one of the following is TRUE?

(A)  $m = 2$  and  $n = 1$

(B)  $m = 3$  and  $n = 3$

(C)  $m = 2$  and  $n = 3$

(D)  $m = 1$  and  $n = 1$

Q.24 Let  $f_1, f_2, f_3$  be nonzero linear transformations from  $\mathbb{R}^4$  to  $\mathbb{R}$  and

$$\ker(f_1) \subseteq \ker(f_2) \cap \ker(f_3).$$

Let  $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear transformation defined by

$$T(v) = (f_1(v), f_2(v), f_3(v)), \quad \text{for all } v \in \mathbb{R}^4.$$

Then, the nullity of  $T$  is equal to

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Q.25 Let  $x_1 = 1$ . For  $n \in \mathbb{N}$ , define

$$x_{n+1} = \left( \frac{1}{2} + \frac{\sin^2 n}{n} \right) x_n.$$

Then, which one of the following is TRUE?

- (A)  $\sum_{n=1}^{\infty} x_n$  converges
- (B)  $\sum_{n=1}^{\infty} x_n$  does NOT converge
- (C)  $\sum_{n=1}^{\infty} x_n^2$  does NOT converge
- (D)  $\sum_{n=1}^{\infty} x_n x_{n+1}$  does NOT converge

Q.26 Let  $x_1 > 0$ . For  $n \in \mathbb{N}$ , define  $x_{n+1} = x_n + 4$ . If

$$\lim_{n \rightarrow \infty} \left( \frac{1}{x_2 x_3} + \frac{1}{x_3 x_4} + \dots + \frac{1}{x_{n+1} x_{n+2}} \right) = \frac{1}{24},$$

then the value of  $x_1$  is equal to

- (A) 1
- (B) 2
- (C) 3
- (D) 8

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Q.27 Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$f(x, y) = e^y(x^2 + y^2) \quad \text{for all } (x, y) \in \mathbb{R}^2.$$

Then, which one of the following is TRUE?

- (A) The number of points at which  $f$  has a local minimum is 2
- (B) The number of points at which  $f$  has a local maximum is 2
- (C) The number of points at which  $f$  has a local minimum is 1
- (D) The number of points at which  $f$  has a local maximum is 1

Q.28 Let  $\Omega$  be the bounded region in  $\mathbb{R}^3$  lying in the first octant ( $x \geq 0$ ,  $y \geq 0$ ,  $z \geq 0$ ), and bounded by the surfaces  $z = x^2 + y^2$ ,  $z = 4$ ,  $x = 0$  and  $y = 0$ .

Then, the volume of  $\Omega$  is equal to

- (A)  $\pi$
- (B)  $2\pi$
- (C)  $3\pi$
- (D)  $4\pi$

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Q.29 Let  $\varphi : [0, \infty) \rightarrow \mathbb{R}$  be the continuous function satisfying

$$\varphi(x) = \left( \int_0^x \varphi(t) dt \right) + \sin x, \text{ for all } x \in [0, \infty).$$

Then, the value of  $\lim_{x \rightarrow \pi/2} (2\varphi(x) - e^x)$  is equal to

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Q.30 The number of elements in the set

$$\{x \in \mathbb{R} : 8x^2 + x^4 + x^8 = \cos x\}$$

is equal to

(A) 0

(B) 1

(C) 2

(D) greater than or equal to 3

**Section B: Q.31 – Q.40 Carry TWO marks each.**

Q.31 Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$f(x, y) = \begin{cases} \frac{xy^2 + y^5}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Then, which of the following is/are TRUE?

- (A) The iterated limits  $\lim_{x \rightarrow 0} \left( \lim_{y \rightarrow 0} f(x, y) \right)$  and  $\lim_{y \rightarrow 0} \left( \lim_{x \rightarrow 0} f(x, y) \right)$  exist
- (B) Exactly one of the partial derivatives  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  exists at  $(0, 0)$
- (C) Both the partial derivatives  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  exist at  $(0, 0)$
- (D)  $f$  is NOT differentiable at  $(0, 0)$

Q.32 If  $M, N, \mu, w: \mathbb{R}^2 \rightarrow \mathbb{R}$  are differentiable functions with continuous partial derivatives, satisfying

$$\mu(x, y)M(x, y)dx + \mu(x, y)N(x, y)dy = dw$$

then which of the following is/are TRUE?

- (A)  $\mu w$  is an integrating factor for  $M(x, y)dx + N(x, y)dy = 0$
- (B)  $\mu w^2$  is an integrating factor for  $M(x, y)dx + N(x, y)dy = 0$
- (C)  $w(x, y) = w(0, 0) + \int_0^x (\mu M)(s, 0)ds + \int_0^y (\mu N)(x, t)dt$ , for all  $(x, y) \in \mathbb{R}^2$
- (D)  $w(x, y) = w(0, 0) + \int_0^x (\mu M)(s, y)ds + \int_0^y (\mu N)(0, t)dt$ , for all  $(x, y) \in \mathbb{R}^2$

Q.33 Let  $\varphi : (-1, \infty) \rightarrow (0, \infty)$  be the solution of the differential equation

$$\frac{dy}{dx} - 2ye^x = 2e^x\sqrt{y},$$

satisfying  $\varphi(0) = 1$ .

Then, which of the following is/are TRUE?

(A)  $\varphi$  is an unbounded function

(B)  $\lim_{x \rightarrow \ln 2} \varphi(x) = (2e - 1)^2$

(C)  $\lim_{x \rightarrow \ln 2} \varphi(x) = \sqrt{2e - 1}$

(D)  $\varphi$  is a strictly increasing function on the interval  $(0, \infty)$

Q.34 Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$f(x, y) = \begin{cases} \frac{(x^2 + \sin x)y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Then, which of the following is/are TRUE?

- (A)  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exists and  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 1$
- (B)  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exists and  $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$
- (C)  $f$  is differentiable at  $(0, 0)$
- (D)  $f$  is NOT differentiable at  $(0, 0)$

Q.35

Let

$$u_1 = (1,0,0,-1), u_2 = (0,2,0,-1), u_3 = (0,0,1,-1) \text{ and } u_4 = (0,0,0,1)$$

be elements in the real vector space  $\mathbb{R}^4$ .

Then, which of the following is/are TRUE?

- (A)  $\{u_1, u_2, u_3, u_4\}$  is a linearly independent set in  $\mathbb{R}^4$
- (B)  $\{u_1 - u_2, u_2 - u_3, u_3 - u_4, u_4 - u_1\}$  is NOT a linearly independent set in  $\mathbb{R}^4$
- (C)  $\{u_1, -u_2, u_3, -u_4\}$  is NOT a linearly independent set in  $\mathbb{R}^4$
- (D)  $\{u_1 + u_2, u_2 + u_3, u_3 + u_4, u_4 + u_1\}$  is a linearly independent set in  $\mathbb{R}^4$

Q.36 For  $n \in \mathbb{N}$ , let

$$x_n = \sum_{k=1}^n \frac{k}{n^2 + k}.$$

Then, which of the following is/are TRUE?

- (A) The sequence  $(x_n)$  converges
- (B) The series  $\sum_{n=1}^{\infty} x_n$  converges
- (C) The series  $\sum_{n=1}^{\infty} x_n$  does NOT converge
- (D) The series  $\sum_{n=1}^{\infty} x_n^n$  converges

Q.37 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a twice differentiable function such that  
 $f(0) = 0, f'(0) = 2$  and  $f(1) = -3$ .

Then, which of the following is/are TRUE?

- (A)  $|f'(x)| \leq 2$  for all  $x \in [0, 1]$
- (B)  $|f'(x_1)| > 2$  for some  $x_1 \in [0, 1]$
- (C)  $|f''(x)| < 10$  for all  $x \in [0, 1]$
- (D)  $|f''(x_2)| \geq 10$  for some  $x_2 \in [0, 1]$

Q.38 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a twice differentiable function such that

$$f(0) = 4, f(1) = -2, f(2) = 8 \text{ and } f(3) = 2.$$

Then, which of the following is/are TRUE?

- (A)  $|f'(x)| < 5$  for all  $x \in [0, 1]$
- (B)  $|f'(x_1)| \geq 5$  for some  $x_1 \in [0, 1]$
- (C)  $f'(x_2) = 0$  for some  $x_2 \in [0, 3]$
- (D)  $f''(x_3) = 0$  for some  $x_3 \in [0, 3]$

Q.39 For  $n \in \mathbb{N}$ , consider the set  $U(n) = \{\bar{x} \in \mathbb{Z}_n : \gcd(x, n) = 1\}$  as a group under multiplication modulo  $n$ .

Then, which of the following is/are TRUE?

- (A)  $U(8)$  is a cyclic group
- (B)  $U(5)$  is a cyclic group
- (C)  $U(12)$  is a cyclic group
- (D)  $U(9)$  is a cyclic group

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Q.40 Consider the following subspaces of the real vector space  $\mathbb{R}^3$  :

$$V_1 = \text{span} \{(1, 2, 3), (1, 1, 0)\},$$

$$V_2 = \text{span} \{(1, -1, 0)\},$$

$$V_3 = \text{span} \{(1, 1, 1)\},$$

$$V_4 = \text{span} \{(1, 3, 6)\} \text{ and}$$

$$V_5 = \text{span} \{(1, 0, -3)\}.$$

Then, which of the following is/are TRUE?

(A)  $V_1 \cup V_2$  is a subspace of  $\mathbb{R}^3$

(B)  $V_1 \cup V_3$  is a subspace of  $\mathbb{R}^3$

(C)  $V_1 \cup V_4$  is a subspace of  $\mathbb{R}^3$

(D)  $V_1 \cup V_5$  is a subspace of  $\mathbb{R}^3$

Section C: Q.41 – Q.50 Carry ONE mark each.

Q.41 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{\left(x + \frac{1}{4}\right)^n}{(-2)^n n^2}$$

about  $x = -\frac{1}{4}$ , is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.42 The value of

$$\lim_{n \rightarrow \infty} 8n \left( e^{\left(\frac{1}{2n}\right)} - 1 \right) \left( \sin \frac{1}{2n} + \left| \cos \frac{1}{2n} \right| \right)$$

is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.43 Let  $\alpha$  be the real number such that

$$\lim_{x \rightarrow 0} \frac{(1 - \cos x)(2^{2+x} - 4)}{x^3} = \alpha \ln 2 .$$

Then, the value of  $\alpha$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.44 Let  $\varphi: \mathbb{R} \rightarrow \mathbb{R}$  be the solution of the differential equation

$$4 \frac{d^2 y}{dx^2} + 16 \frac{dy}{dx} + 25y = 0$$

satisfying  $\varphi(0) = 1$  and  $\varphi'(0) = -\frac{1}{2}$ .

Then, the value of  $\lim_{x \rightarrow \pi/6} e^{2x} \varphi(x)$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.45 Let  $S$  be the surface area of the portion of the plane  $z = x + y + 3$ , which lies inside the cylinder  $x^2 + y^2 = 1$ .

Then, the value of  $\left(\frac{S}{\pi}\right)^2$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.46 Consider the following subspaces of  $\mathbb{R}^4$ :

$$V_1 = \{(x, y, z, w) \in \mathbb{R}^4 : x + y + 2w = 0\},$$

$$V_2 = \{(x, y, z, w) \in \mathbb{R}^4 : 2y + z + w = 0\},$$

$$V_3 = \{(x, y, z, w) \in \mathbb{R}^4 : x + 3y + z + 3w = 0\}.$$

Then, the dimension of the subspace  $V_1 \cap V_2 \cap V_3$  is equal to \_\_\_\_\_.

- Q.47 Consider the real vector space  $\mathbb{R}^3$ . Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}$  be a linear transformation such that

$$T(1, 1, 1) = 0, \quad T(1, -1, 1) = 0 \text{ and } T(0, 0, 1) = 16.$$

Then, the value of  $T\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right)$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

- Q.48 Let  $T$  denote the triangle in the  $xy$  plane bounded by the  $x$  axis and the lines  $y = x$  and  $x = 1$ . The value of the double integral (over  $T$ )

$$\iint_T (5 - y) dx dy$$

is equal to \_\_\_\_\_ (rounded off to two decimal places).

- Q.49 Let  $T, S : P_4(\mathbb{R}) \rightarrow P_4(\mathbb{R})$  be the linear transformations defined by

$$T(p(x)) = xp'(x) \text{ and } S(p(x)) = (x + 1)p'(x)$$

for all  $p(x) \in P_4(\mathbb{R})$ .

Then, the nullity of the composition  $S \circ T$  is \_\_\_\_\_.

- Q.50 Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by

$$f(x, y) = \begin{cases} \frac{(x^2 - y^2)xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Then, the value of  $\frac{\partial f}{\partial y}(1, 0) - \frac{\partial f}{\partial x}(0, 2)$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

**Section C: Q.51 – Q.60 Carry TWO marks each.**

Q.51 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function satisfying

$$\int_0^{\pi/4} \left( \sin(x) f(x) + \cos(x) \int_0^x f(t) dt \right) dx = \sqrt{2}.$$

Then, the value of  $\int_0^{\pi/4} f(x) dx$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.52 Let  $\sigma \in S_4$  be the permutation defined by  $\sigma(1) = 2$ ,  $\sigma(2) = 3$ ,  $\sigma(3) = 1$  and  $\sigma(4) = 4$ . The number of elements in the set

$$\{\tau \in S_4 : \tau\sigma\tau^{-1} = \sigma\}$$

is equal to \_\_\_\_\_.

Q.53 Let  $f(x) = 2x - \sin x$ , for all  $x \in \mathbb{R}$ . Let  $k \in \mathbb{N}$  be such that

$$\lim_{x \rightarrow 0} \left( \frac{1}{x} \sum_{i=1}^k i^2 f\left(\frac{x}{i}\right) \right) = 45.$$

Then, the value of  $k$  is equal to \_\_\_\_\_.

Q.54 The value of the infinite series

$$\sum_{n=1}^{\infty} n \left(\frac{3}{4}\right)^{2(n-1)}$$

is equal to \_\_\_\_\_ (rounded off to two decimal places)

Q.55 Let  $\varphi: (0, \infty) \rightarrow \mathbb{R}$  be the solution of the differential equation

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 6x \ln x$$

satisfying  $\varphi(1) = -3$  and  $\varphi(e) = 0$ .

Then, the value of  $|\varphi'(1)|$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.56 Let  $\varphi: \mathbb{R} \rightarrow \mathbb{R}$  be the solution of the differential equation

$$\frac{dy}{dx} + 2xy = 2 + 4x^2$$

satisfying  $\varphi(0) = 0$ .

Then, the value of  $\varphi(2)$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.57 Let  $\Omega$  be the solid bounded by the planes  $z = 0$ ,  $y = 0$ ,  $x = \frac{1}{2}$ ,  $2y = x$  and  $2x + y + z = 4$ .

If  $V$  is the volume of  $\Omega$ , then the value of  $64V$  is equal to \_\_\_\_\_ (rounded off to two decimal places).

Q.58 Let the subspace  $H$  of  $P_3(\mathbb{R})$  be defined as

$$H = \{p(x) \in P_3(\mathbb{R}) : xp'(x) = 3p(x)\}.$$

Then, the dimension of  $H$  is equal to \_\_\_\_\_.

Q.59 Let  $G$  be an abelian group of order 35. Let  $m$  denote the number of elements of order 5 in  $G$ , and let  $n$  denote the number of elements of order 7 in  $G$ .

Then, the value of  $m + n$  is equal to \_\_\_\_\_.

Q.60 The number of surjective (onto) group homomorphisms from  $S_4$  to  $\mathbb{Z}_6$  is equal to \_\_\_\_\_.

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