T5017028

MATHEMATICS

Maximum Marks: 80
Time Allotted: Three Hours
Reading Time: Additional Fifteen minutes

Instructions to Candidates

1. You are allowed an **additional fifteen minutes** for **only** reading the paper.
2. You must **NOT** start writing during reading time.
3. The Question Paper has **11 printed pages** and **one blank page**.
4. The Question Paper is divided into **three** sections and has **22 questions** in all.
5. **Section A** is compulsory and has **fourteen** questions.
6. You are required to attempt **all** questions either from **Section B or Section C.**
7. **Section B** and **Section C** have **four** questions each.
8. Internal choices have been provided in **two** questions of **2 marks**, **two** questions of **4 marks** and **two** questions of **6 marks** in **Section A.**
9. Internal choices have been provided in **one** question of **2 marks** and **one** question of **4 marks** each in **Section B and Section C.**
10. While attempting **Multiple Choice Questions in Section A, B and C**, you are required to **write only ONE option as the answer.**
11. All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
12. Mathematical tables and graph papers are provided.
13. The intended marks for questions or parts of questions are given in the brackets [].

Instruction to Supervising Examiner

1. Kindly read aloud the instructions given above to all the candidates present in the examination hall.

SECTION A - 65 MARKS

Question 1

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) If $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, then A^{16} is: **[1]**

(a) Unit matrix

(b) Null matrix

(c) Diagonal matrix

(d) Skew matrix

(ii) Which of the following is a homogenous differential equation? **[1]**

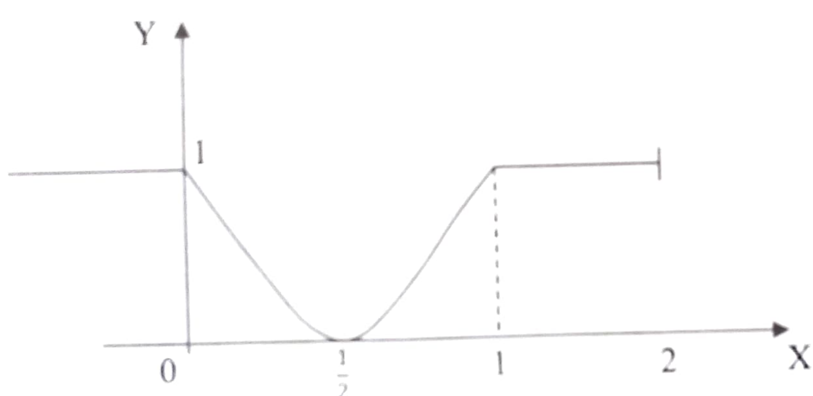
(a) $(4x^2 + 6y + 5)\,dy - (3y^2 + 2x + 4)\,dx = 0$

(b) $(xy)\,dx - (x^3 + y^3)\,dy = 0$

(c) $(x^3 + 2y^2)\,dx + 2xy\,dy = 0$

(d) $y^2\,dx + (x^2 - xy - y^2)\,dy = 0$

(iii) Consider the graph of the function $f(x)$ shown below: **[1]**



Statement 1: The function $f(x)$ is increasing in $\left(\frac{1}{2}, 2\right)$.

Statement 2: The function $f(x)$ is strictly increasing in $\left(\frac{1}{2}, 1\right)$.

Which of the following is correct with respect to the above statements?

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(iv) $\int_0^1 \frac{x^4 - 1}{x^2 + 1} dx$ is equal to: [1]

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{-2}{3}$

(d) 0

(v) **Assertion:** Consider the two events A and B such that n(A) = n(B) and $P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right)$. [1]

Reason: The events A and B are mutually exclusive.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(vi) The existence of unique solution of the system of equations $x + y = \lambda$ and $5x + ky = 2$ depends on: [1]

(a) λ only

(b) $\frac{\lambda}{k} = 1$

(c) both k and λ

(d) k only

(vii) A cylindrical popcorn tub of radius 10 cm is being filled with popcorns at the rate of 314 cm^3 per minute. The level of the popcorns in the tub is increasing at the rate of: [1]

(a) 1 cm/minute

(b) 0·1 cm/minute

(c) 1·1 cm/minute

(d) 0·5 cm/minute

(viii) If $f(x) = \begin{cases} x+2 & x<0 \\ -x^2-2 & 0 \le x < 1 \\ x & x \ge 1 \end{cases}$ [1]

then the number of point(s) of discontinuity of $f(x)$, is / are:

(a) 1

(b) 3

(c) 2

(d) 0

(ix) **Assertion:** If Set A has m elements, Set B has n elements and $n < m$, then the number of one-one function(s) from A → B is zero. [1]

Reason: A function f: A → B is defined only if all elements in Set A have an image in Set B.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(x) Let X be a discrete random variable. The probability distribution of X is given below: [1]

X	30	10	−10
P(X)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{2}$

Then E(X) will be:

(a) 1

(b) 4

(c) 2

(d) 30

(xi) **Statement 1:** If 'A' is an invertible matrix, then $(A^2)^{-1} = (A^{-1})^2$ [1]

Statement 2: If 'A' is an invertible matrix, then $|A^{-1}| = |A|^{-1}$

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(xii) Write the smallest equivalence relation from the set A to A, where A = {1, 2, 3}. [1]

(xiii) For what value of x, is $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ a skew symmetric matrix? [1]

(xiv) Three critics review a book. Odds in favour of the book are 5:2, 4:3 and 3:4 respectively for the three critics. Find the probability that all critics are in favour of the book. [1]

(xv) Evaluate: $\int \frac{5}{\sqrt{2x+7}}\, dx$ [1]

Question 2 [2]

Find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to the x-axis.

Question 3 [2]

Find the value of $\tan^{-1} x - \cot^{-1} x$, if $(tan^{-1}x)^2 - (cot^{-1}x)^2 = \frac{5\pi}{8}$

Question 4 [2]

(i) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{logx}{(1+logx)^2}$

OR

(ii) If $f(x) = \log(1 + x) + \frac{1}{1+x}$, show that $f(x)$ attains its minimum value at $x = 0$.

Question 5 [2]

Three shopkeepers Gaurav, Rizwan and Jacob use carry bags made of polythene, handmade paper and newspaper. The number of polythene bags, handmade bags and newspaper bags used by Gaurav, Rizwan and Jacob are (20, 30, 40), (30, 40, 20) and (40, 20, 30) respectively. One polythene bag costs ₹ 1, one handmade bag is for ₹ 5 and one newspaper bag costs ₹ 2. Gaurav, Rizwan and Jacob spend ₹ A, ₹ B and ₹ C respectively on these carry bags.

Using the concepts of matrices and determinants, answer the following questions:

(i) Represent the above information in Matrix form.

(ii) Find the values of ₹ A, ₹ B and ₹ C.

Question 6 [2]

(i) Differentiate $\sin^{-1}\left(\frac{2^{x+1}\,3^x}{1 + (36)^x}\right)$ with respect to x.

OR

(ii) Show that $\tan^{-1} x + \tan^{-1} y = C$ is the general solution of the differential equation $(1 + x^2)\, dy + (1 + y^2)\, dx = 0$

Question 7 [4]

If $x + y + z = 0$ then show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$, using properties of determinant.

Question 8 [4]

(i) Evaluate: $\int \frac{cosx}{3cosx - 5} dx$

OR

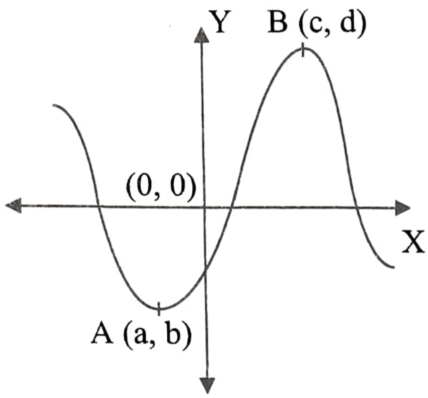
(ii) Evaluate: $\int (log\ x)^2\ dx$

Question 9

(i) If $x = \tan\left(\frac{1}{a}\ logy\right)$ then show that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$ [4]

OR

(ii) The graph of $f(x) = -x^3 + 27x - 2$ is given below:



(a) Find the slope of the above graph. [1]

(b) Find the co-ordinates of turning points, A and B. [2]

(c) Evaluate $f''(-2)$, $f(0)$ and $f'(3)$ and arrange them in ascending order. [1]

Question 10

Pia, Sia and Dia displayed their paintings in an art exhibition. The three artists displayed 15, 5 and 10 of their paintings respectively. A person bought three paintings from the exhibition.

(i) Find the probability that he bought one painting from each of them. [2]

(ii) Find the probability that he bought all the three paintings from the same person. [2]

Question 11 [6]

(i) Prove:

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{xdx}{1 + sinx} = (\sqrt{2} - 1)\pi$$

OR

(ii) Evaluate: $\int \frac{x^2}{(x-1)^2(x^2+1)}\,dx$

Question 12 [6]

(i) Solve the differential equation:

$(x + 5y^2)\frac{dy}{dx} = y$ when $x = 2$ and $y = 1$

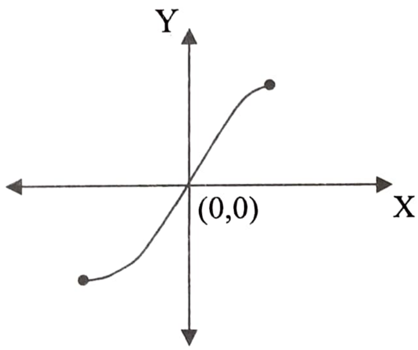
OR

(ii) Find the particular solution of the differential equation:

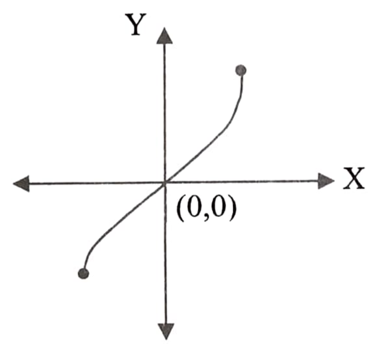
$(x^2 - 2y^2)dx + 2xydy = 0$, when $x = 1$ and $y = 1$

Question 13

Observe the two graphs, **Graph 1** and **Graph 2** given below and answer the questions that follow.



Graph 1



Graph 2

(i) Which one of the graphs represents $y = \sin^{-1} x$? [1]

(ii) Write the domain and range of $y = \sin^{-1} x$. [1]

(iii) Prove that $\sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}} = \frac{\pi}{2}$ [2]

(iv) Find the value of $\tan^{-1}\left[2sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ [2]

Question 14 [6]

An international conference takes place in a metropolitan city. International leaders, scientists and industrialists participate in it.

The organisers of the conference appoint three agencies namely X, Y and Z for the security of the participants. The track record of the success of X, Y and Z in providing security services is 99%, 98·5% and 98% respectively. The organisers assign the responsibility of ensuring the security of 1000 people to agency X, 2000 people to agency Y and 3000 people to agency Z.

At the end of the conference, one participant goes missing from the conference room.

What is the probability that the missing participant was placed under the responsibility of the security agency X?

SECTION B - 15 MARKS

Question 15

In subparts (i) and (ii) choose the correct options and in subparts (iii) and (iv), answer the questions as instructed.

(i) **Assertion:** $(\vec{a} + \vec{b})^2 + (\vec{b} - \vec{a})^2 = 2(a^2 + b^2)$ **[1]**

Reason: Dot product of any two vectors is commutative.

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(ii) The angle between the two planes $x + y + 2z = 9$ and $2x - y + z = 15$ is: **[1]**

(a) $\frac{\pi}{2}$

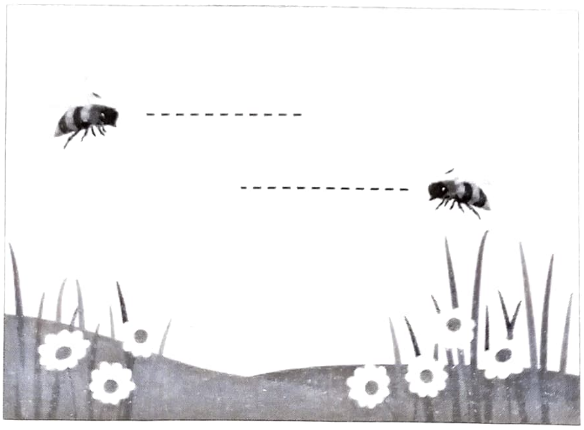
(b) $\frac{\pi}{3}$

(c) π

(d) $\frac{3\pi}{4}$

(iii) Show that points P(−2, 3, 5), Q(1, 2, 3) and R(7, 0, −1) are collinear. **[1]**

(iv) Two honeybees are flying parallel to each other in the garden to collect the nectar. The path traced by the bees is given in the form of a straight line. The equation of the path traced by one honeybee is $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$.



(a) Write the above-mentioned equation in cartesian form. **[1]**

(b) Find the equation of the path traced by the other honeybee passing through the point (2, 4, 5). **[1]**

Question 16

[2]

(i) Find the equation of the plane passing through the points (2, 2, −1), (3, 4, 2) and (7, 0, 6).

OR

(ii) Find the equation of the plane passing through the points (2, 3, 1), (4, −5, 3) and parallel to x-axis.

Question 17

Consider the position vectors of A, B and C as $\overrightarrow{OA} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\overrightarrow{OB} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ and $\overrightarrow{OC} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$

(i) Calculate $\overrightarrow{AB}$ and $\overrightarrow{BC}$. [1]

(ii) Find the projection of $\overrightarrow{AB}$ on $\overrightarrow{BC}$. [1]

(iii) Find the area of the triangle ABC whose sides are $\overrightarrow{AB}$ and $\overrightarrow{BC}$. [2]

Question 18

(i) The equation $y = 4 - x^2$ represents a parabola.

(a) Make a rough sketch of the graph of the given function. [1]

(b) Determine the area enclosed between the curve, the x-axis, the lines $x = 0$ and $x = 2$. [2]

(c) Hence, find the area bounded by the parabola and the x-axis. [1]

OR

(ii) A farmer has a field bounded by three lines $x + 2y = 2,\ y - x = 1,\ 2x + y = 7$. Using integration, find the area of the region bounded by these lines. [4]

SECTION C - 15 MARKS

Question 19

In subparts (i) and (ii) choose the correct options and in subpart (iii), answer the questions as instructed.

(i) The total revenue received from the sale of 'x' units of a product is $R(x) = 36x + 3x^2 + 5$. Then, the actual revenue for selling the 10^{th} item will be: [1]

(a) 27

(b) 90

(c) 93

(d) 33

(ii) Read the following statements and choose the correct option. [1]

(I) The correlation coefficient and the regression coefficients are of the same sign.

(II) The correlation coefficient is the arithmetic mean between the regression coefficients.

(III) The product of two regression coefficients is always equal to 1.

(IV) Both the regression coefficients cannot be numerically greater than unity.

(a) Only (IV) is correct.

(b) Only (I) and (II) are correct.

(c) Only (I) and (IV) are correct.

(d) Only (III) and (IV) are correct.

(iii) Consider the following data:

x	1	2	3	6
y	6	5	4	1
$x - \bar{x}$				
$y - \bar{y}$				

(a) Calculate $\bar{x}$ and $\bar{y}$ [1]

(b) Complete the table. [1]

(c) Calculate b_{xy} [1]

Question 20 [2]

(i) Find the regression line of best fit from the following data.
$\Sigma x = 24, \Sigma y = 44, \Sigma xy = 306, \Sigma x^2 = 164, \Sigma y^2 = 576, n = 4$

OR

(ii) Two lines of regression are given as $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Identify the line of regression of x on y.

Question 21

(i) A utensil manufacturer produces 'x' dinner sets per week and sells each set at ₹ p, where $x = \frac{600-p}{8}$. The cost of production of 'x' sets is ₹ $x^2 + 78x + 2000$

(a) Write the revenue function. [1]

(b) Write the profit function. [1]

(c) Calculate the number of dinner sets to be produced and sold per week to ensure maximum profit. [2]

OR

(ii) The Average Cost of producing 'x' units of commodity is given by:

$$AC = \frac{x^2}{200} - \frac{x}{50} - 30 + \frac{5000}{x}$$

(a) Find the Cost function. [1]

(b) Find the Marginal Cost function. [1]

(c) Find the Marginal Average Cost function. [1]

(d) Verify that $\frac{d}{dx}(AC) = \frac{MC-AC}{x}$ [1]

Question 22 [4]

Two different types of books have to be stacked in the shelf of a library. The first type of book weighs 1 kg and has a thickness of 6 cm. The second type of book weighs 1·5 kg and has a thickness of 4 cm. The shelf is 96 cm long and can support a maximum weight of 21 kg.

How should both the types of books be placed in the shelf to include the maximum number of books? Formulate a Linear Programming Problem and solve it graphically.