Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series **Y**

PAPER–II **Question Booklet No.**

(Identical with OMR Answer Sheet Number)

Subject Code : 15

MATHEMATICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D) , where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

PAPER II

(MATHEMATICAL SCIENCES)

1. Which of the following series is not uniformly convergent on [0, 1]?

(A) $\sum_{n=0}^{\infty}\left[\frac{x^{2n+1}}{(2n+1)} - \frac{x^{n+1}}{2n+2}\right]$

(B) $\sum_{n=0}^{\infty} x^n(1-x)$

(C) $\sum_{n=0}^{\infty} (-1)^n x^n(1-x)$

(D) $\sum_{n=0}^{\infty} x^n$

2. Let $u_1, u_2, \ldots u_n$ be n linearly independent solutions of the linear differential equation of order n

$P_0(x)y^{(n)} + P_1(x)y^{(n-1)} + \ldots + P_n(x)y = 0,\ P_0(x) \neq 0.$

Let W be the Wronskian of the solution $u_1, u_2, \ldots u_n$ and W_0 be the value of W at $x = x_0$. Then which of the following is true?

(A) $W = W_0 \exp\left\{\int_{x_0}^{x} \frac{P_1(x)}{P_0(x)} dx\right\}$

(B) $W = W_0 \exp\left\{\int_{x_0}^{x} \frac{P_n(x)}{P_0(x)} dx\right\}$

(C) $W = W_0 \exp\left\{-\int_{x_0}^{x} \frac{P_1(x)}{P_0(x)} dx\right\}$

(D) $W = W_0 \exp\left\{-\int_{x_0}^{x} \frac{P_n(x)}{P_0(x)} dx\right\}$

3. Let X_1, X_2, X_3 be independent and identically distributed $N(0, 1)$ random variables and $Y = \frac{2X_1^2}{X_2^2 + X_3^2}$, then the distribution of Y is

(A) $N(0, 2)$

(B) Chi-square with 3 degrees of freedom

(C) $F(1, 2)$

(D) $F(2, 1)$

4. A polynomial $p(x)$ satisfies the following:

$p(-1) = p(0) = p(1) = p(3) = p(4) = 0,\ p(2) \neq 0.$

The minimum degree of such a polynomial is

(A) 1

(B) 5

(C) 4

(D) 6

5. If X and Y are independent Cauchy (0, 1) random variables, then cov$\{I(X > 0), I(X + Y > 0)\}$ equals

(A) 0

(B) 1

(C) 1/8

(D) 3/8

6. Which of the following is true?

(A) Intervals in real line are not the only connected sets.

(B) Adjoining a limit point with a connected set would not make it a connected set.

(C) Real line is not union of two totally disconnected sets.

(D) Cantor set is a totally disconnected set.

7. If (X, T) be a topological space defined as follows : X consists of the integer 0, 1, 2 where A consists of 0, B consists of 0, 1 and T consists of void set ϕ, A, B and X and f is a continuous function of X into itself such that $f(1) = 0$ and $f(2) = 1$ then

(A) $f(0) = 1$

(B) $f(0) = 2$

(C) $f(0) = 0$

(D) $f(0)$ does not exist

8. The Average Sample Number (ASN) in a single sampling inspection plan for attribute is

(A) $n. L(p) + N. [1 - L(p)]$

(B) $n. L(p)$

(C) n

(D) $N. [1 - L(p)]$

9. Fourier transform of $\text{sgn}(t) = \begin{cases} +1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$ is

(A) $\frac{1}{i\pi K}$

(B) $-\frac{1}{i\pi K}$

(C) $\frac{i}{\pi K}$

(D) $-\frac{i}{\pi K}$

(Where K is the transform variable)

10. Let X_i, $i = 1, 2, 3$ be *iid* observations from a continuous population $F(x)$ and $Y_i, i = 1, 2, 3, 4$ be those from $F(x - \theta)$ and the two samples are independent.

Suppose the observed value of $\sum_{i=1}^{4} R_i$ is 23, where R_i is the rank of Y_i in the combined sample. If the right tailed Wilcoxon rank sum test is used for testing $H_0 : \theta = 0$ against $H_1 : \theta > 0$, then

(A) H_0 is rejected at any level of significance.

(B) H_0 must be accepted at any level of significance.

(C) P value of the test is atmost $\frac{16}{23}$.

(D) the calculation lacks consistency.

11. The values of a, b and c for which the state of stress at a given point in a continuum can be represented by the matrix

$$\begin{bmatrix} a+b & a+b-c & c-a \\ 2b & b-a & 2a-2b+c \\ c+b & a+b+c & c \end{bmatrix} \text{ are}$$

(A) 1, –1 and 2 respectively.

(B) –1, 1 and 2 respectively.

(C) –1, 1 and 1 respectively.

(D) 1, 1 and 0 respectively.

12. The polynomial $x^3 - 312312x + 123123$ is irreducible in $F[x]$ if F is

(A) Z_3

(B) Z_7

(C) Z_{13}

(D) Q

13. If the variables (y, x_1, x_2, x_3) are related by $y - 2x_1 = 3(x_2 - x_3)$ then

(A) $r_{y\,.\,x_1 x_2 x_3} = 1$ and $r_{y x_1\,.\,x_2 x_3} = -1$

(B) $r_{y\,.\,x_1 x_2 x_3} = 0$ and $r_{y x_1\,.\,x_2 x_3} = -1$

(C) $r_{y\,.\,x_1 x_2 x_3} = 1$ and $r_{y x_3\,.\,x_1 x_2} = -1$

(D) $r_{y\,.\,x_1 x_2 x_3} = 0$ and $r_{y x_1\,.\,x_2 x_3} = 1$

14. Which of the following is not correct?

(A) Countable set has Lebesgue measure zero.

(B) Uncountable set has always non-zero Lebesgue measure.

(C) All subsets of real line are not Lebesgue measurable.

(D) Borel sets are also Lebesgue measurable.

15. If $\underset{\sim}{X} \sim N_P(\mu, \Sigma)$, which of the following statements is correct?

(A) For any square matrix A of order P, $X'AX$ follows chi-square distribution with 'P' degree of freedom.

(B) Two quadratic forms $X'AX$ and $X'BX$ are independent if and only if $AB = BA$.

(C) Solutions of the determinental equation $|\Sigma - \lambda I| = 0$ provides the variances of all principal components of $\underset{\sim}{X}$.

(D) Solutions of the determinental equation $|\Sigma + \lambda I| = 0$ provides the variances of all principal components of $\underset{\sim}{X}$.

16. A particle of mass m slides down without friction along a curve $z = 1 + \frac{x^2}{2}$ in the x-z plane under the action of constant gravity. Suppose the z-axis be vertically upwards. Then Lagrangian of the motion is (with $\dot{x} = dx/dt$)

(A) $\frac{1}{2}m\dot{x}^2\left(1+x^2\right) - mg\left(1+\frac{x^2}{2}\right)$

(B) $\frac{1}{2}m\dot{x}^2\left(1+x^2\right) + mg\left(1+\frac{x^2}{2}\right)$

(C) $\frac{1}{2}mx^2\dot{x}^2 - mg\left(1+\frac{x^2}{2}\right)$

(D) $\frac{1}{2}m\dot{x}^2\left(1-x^2\right) - mg\left(1+\frac{x^2}{2}\right)$

17. Let a_{-2} be the coefficient of $\frac{1}{z^2}$ in the Laurent series of $f(z) = \frac{z+1}{z^3(z^2+1)}$ around $z = 0$ in the region $0 < |z| < 1$. Then a_{-2} is equal to

(A) 1

(B) 0

(C) –1

(D) 2

18. Let $f \in C\,[a, b]$, $m(x) = \inf\,\{f(t) : t \in [a, x]\}$, $M(x) = \sup\,\{f(t) : t \in [a, x]\}$, where $a \le x \le b$. Then

(A) $m \in C\,[a, b]$, $M \notin C\,[a, b]$

(B) $m \notin C\,[a, b]$, $M \in C\,[a, b]$

(C) $m \notin C\,[a, b]$, $M \notin C\,[a, b]$

(D) $m \in C\,[a, b]$, $M \in C\,[a, b]$

19. If X is distributed as exponential with mean 5. Then which of the following is not true?

(A) $P(X \ge 6 \mid X \ge 5) = e^{-2/5}$

(B) $\exp\left(-\frac{X}{5}\right) \sim$ uniform (0, 1)

(C) $E\{(X-10) \mid X > 5\} = 0$

(D) $E\{(X-5) \mid X > 5\} > 0$

20. Let X and Y be independent random variables with distribution function $F(x)$ and $G(x)$ respectively. Then $P(Y < X)$ is

(A) $\int_{-\infty}^{\infty} F(x)\,dG(x)$

(B) $\int_{-\infty}^{\infty} F^2(x)\,dx$

(C) $\int_{-\infty}^{\infty} G^2(x)\,dx$

(D) $\int_{-\infty}^{\infty} G(x)\,dF(x)$

21. Let X and Y be independent random variables on a probability space and $Z(\omega) = c$, for every $\omega \in \Omega$. Then which of the following is not correct?

(A) X^2 and Y^2 are independent.

(B) X and Z are independent.

(C) X^2 and Z are independent.

(D) X^2 and Z^2 are not independent.

22. There are different columns of life tables like l_x, q_x, L_x and T_x. e°_{15} (expectation of life at age 15) is computed as

(A) $\frac{L_{15}}{l_{15}}$

(B) $\frac{T_{15}}{l_{15}}$

(C) $\frac{T_{15}}{L_{15}}$

(D) $\frac{L_{15}}{T_{15}}$

23. The plane curve passing through two points, not in the same vertical line, which when rotated about the x-axis gives a minimum surface area, is a

(A) Straight line

(B) Catenary

(C) Parabola

(D) Cycloid

[Please Turn Over]

24. Suppose the minimal polynomial of a 5×5 matrix A over $\mathbb{R}$ is $x(x-1)(x-2)^2$. Then

(A) A is a singular matrix and is not diagonalisable.

(B) A is a singular matrix and is diagonalisable.

(C) A is a nonsingular matrix and is diagonalisable.

(D) A is a nonsingular matrix and is not diagonalisable.

25. If ϕ is the solution of

$$\phi(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x-t) - 4(x-t)^2]\, \phi(t)dt,$$

then $\phi(\log(2))$ is equal to

(A) 10

(B) 4

(C) 2

(D) 8

26. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{x}$, for all $x \in [a,b]$ with $0 < a < b$. Then by Cauchy's mean value theorem, there exists $c \in (a,b)$ such that $\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}$, where c is

(A) arithmetic mean of a and b.

(B) geometric mean of a and b.

(C) harmonic mean of a and b.

(D) $\frac{1}{a} + \frac{1}{b}$.

27. For what values of m and n the transformation $(q, p) \to (Q, P)$ given by

$Q = q^m \cos(np)$

$P = q^m \sin(np)$

represents a canonical transformation?

(A) $m = \frac{1}{2}, n = 1$

(B) $m = 1, n = 2$

(C) $m = \frac{1}{2}, n = 2$

(D) $m = 1, n = 1$

28. Let $\underset{\sim}{X} \sim N_P(\mu, \Sigma)$, where Σ is a known matrix. To test the hypothesis $H_0 : \mu = \mu_0$, a given vector in $\mathbb{R}^P$, which of the following is used?

(A) F distribution

(B) Chi-square distribution

(C) Hotelling's T^2

(D) Mahalanobis D^2

29. The partial differential equation,

$U_{xx} + 2U_{xy} + U_{yy} = x$ has

(A) only one particular integral.

(B) a particular integral which is linear in x and y.

(C) a particular integral which is quadratic polynomial in x and y.

(D) more than one particular integral.

30. If $A = \begin{pmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-i\sqrt{3}}{2} \end{pmatrix}$ then the trace of A^{102} is

(A) 0

(B) 1

(C) 2

(D) 3

31. State which of the following is not correct:

(A) Singleton sets in a T_1-space are closed.

(B) Every separable metric space is second countable.

(C) The boundary of a closed set is nowhere dense.

(D) Product of two normal spaces is necessarily a normal space.

32. Suppose X is a single observation from an exponential distribution with mean θ, consider testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$. Then the type-I and type-II error probabilities of most powerful test satisfy

(A) $x + y = 1$

(B) $x + y = 1/2$

(C) $y + \sqrt{x} = 1$

(D) $x + \sqrt{y} = 1$

[$x(y)$ = type I(II) error probabilities]

33. Let π be the probability that a die shows an even number. The die is tossed twice and $H_0 : \pi = \frac{1}{2}$ is accepted if an even sum is observed. Then type-I error probability is

(A) 1/4

(B) 1/8

(C) 3/4

(D) 1/2

34. Let $A : H \to H$ be any bounded linear operator on a complex Hilbert space H such that $\| Ax\| = \| A^{*}x\|$ for all x in H where A^{*} is the adjoint of A. If there is a non-zero x in H such that $A^{*}(x) = (2 + 3i)x$, then A is

(A) an unitary operator on H.

(B) a self-adjoint operator on H but not unitary.

(C) a self-adjoint operator on H but not normal.

(D) a normal operator.

35. For the function $f(z) = \frac{1-e^{-z}}{z}$, the point $z = 0$ is a(an)

(A) essential singularity

(B) pole of order zero

(C) pole of order one

(D) removable singularity

36. Residue of $f(z) = \frac{\log(1+z)}{(1+z^2)^2}$ at $z = i$ is given by

(A) $\frac{1}{8} - \frac{\pi}{16} + i\left[-\frac{1}{8} + \frac{1}{4}\log\sqrt{2}\right]$

(B) $-\frac{1}{8} + \frac{\pi}{16} + i\left[\frac{1}{8} - \frac{1}{4}\log\sqrt{2}\right]$

(C) $-\frac{1}{8} + \frac{\pi}{16} + i\left[-\frac{1}{8} + \frac{1}{4}\log\sqrt{2}\right]$

(D) $\frac{1}{8} - \frac{\pi}{16} + i\left[\frac{1}{8} - \frac{1}{4}\log\sqrt{2}\right]$

37. State which of the following is not correct:

(A) For a closed system, isotropy of space implies conservation of angular momentum.

(B) For a closed system, homogeneity of space implies conservation of linear momentum.

(C) For a closed system, homogeneity in time implies conservation of energy.

(D) For a closed system, homogeneity of space implies conservation of energy.

38. If X is a single observation from $U(\theta, \theta + 1)$, then the coverage probability of the set $C = [X - 1, X]$ for θ equals

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

39. If X_i, $i = 1(1)n$ are *iid* observations from the distribution $f(x, \theta) = \frac{1}{2}e^{-|x-\theta|}$, then which of the following is not true?

(A) MLE of θ is sample median.

(B) The family has MLR in sample median.

(C) No single sufficient statistic exists for θ.

(D) $E(\bar{X}) = \theta$.

40. The mode of the geometric distribution with probability mass function

$$P_k = \left(\frac{1}{2}\right)^k, \; k = 1, 2, 3, \; .... \text{ is}$$

(A) 0.5

(B) 0

(C) 1

(D) does not exist

41. Consider a design with 4 treatments 1, 2, 3 and 4 and 5 blocks (1, 2), (1, 3), (1, 2, 3), (1, 2, 4) and (1, 2, 3, 4). Then

(A) the design is connected but non-orthogonal.

(B) the design is connected and orthogonal.

(C) all treatment contrasts are estimable.

(D) only some elementary treatment contrasts are estimable.

42. Let $\varepsilon_t, \varepsilon_{t+1}, ...$ and ζ be independent variables with zero mean and unit variance. Suppose $U_t = a.\zeta + \varepsilon_t, |a| < 1, -\infty < t < \infty$. The process is

(A) stationary

(B) non-stationary

(C) oscillatory

(D) evolutionary

43. Suppose y_1 and y_2 are independent observations with $E(y_1) = E(y_2) = \beta$, $\text{var}(y_1) = 2\sigma^2$, $\text{var}(y_2) = 3\sigma^2$. Which of the following is BLUE of β?

(A) $\frac{y_1 + y_2}{2}$

(B) $\left(y_1/5 + y_2/2\right) \Big/ \left(\frac{1}{5} + \frac{1}{2}\right)$

(C) $\frac{2y_1 + 3y_2}{5}$

(D) $\frac{3y_1 + 2y_2}{5}$

44. A TV mechanic finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day, the expected idle time of each day is equal to

(A) 2 hours

(B) 3 hours

(C) 4 hours

(D) 1 hour

45. Suppose Q_1 is the optimal lot size when inventories and shortages are equally costly and Q_2 is the optimal lot size when shortages are not allowed. Then,

(A) $Q_1 = \frac{1}{2} Q_2$

(B) $Q_1 = \sqrt{2}\, Q_2$

(C) $\sqrt{2}\, Q_1 = Q_2$

(D) $\frac{1}{2} Q_1 = Q_2$

46. Given any m positive integers $a_1, a_2, ... a_m$, a non-empty subset of these integers can always be selected whose sum is a multiple of

(A) m

(B) $2m$

(C) $m + 1$

(D) $m + 2$

47. Given that

x	1	2	3	4	5
u	2	5	10	20	30

and Δ, ∇ and E denote the forward difference, backward difference and shift operator respectively. Which of the following is correct?

(A) $\Delta \equiv \nabla E^{-1}$

(B) $\Delta \equiv E + 1$

(C) $\nabla^2 u_4 = u_4 + 2u_3 - u_2$

(D) $\nabla^2 u_4 = 5$

48. A function $u(x, t)$ satisfies the wave equation $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$.

If $u\left(\frac{1}{2}, 0\right) = \frac{1}{4}$, $u\left(1, \frac{1}{2}\right) = 1$ and $u\left(0, \frac{1}{2}\right) = \frac{1}{2}$

then $u\left(\frac{1}{2}, 1\right)$ is

(A) $\frac{7}{4}$

(B) $\frac{5}{4}$

(C) $\frac{4}{5}$

(D) $\frac{4}{7}$

49. For the differential equation

$$x(x-1)\frac{d^2y}{dx^2} + \cot(\pi x)\frac{dy}{dx} + \operatorname{cosec}^2(\pi x)y = 0$$

which of the following statements is true?

(A) Both 0 and 1 are irregular singular points.

(B) 0 is an irregular and 1 is a regular singular point.

(C) Both 0 and 1 are regular singular points.

(D) 0 is a regular and 1 is an irregular singular point.

50. The transformation $\omega = e^{i\theta}\left(\dfrac{z-p}{\bar{p}z-1}\right)$, where p is a constant, maps $|z| < 1$ onto

(A) $|\omega| = 1$ if $|p| = 1$

(B) $|\omega| < 1$ if $|p| < 1$

(C) $|\omega| > 1$ if $|p| > 1$

(D) $|\omega| = 3$ if $p = 0$

51. Which of the following mappings is conformal everywhere?

(A) $\omega = z^2$

(B) $\omega = \frac{1}{2}\left(z + z^{-1}\right)$

(C) $\omega = e^z$

(D) $\omega = c \sin z$, c being a constant

52. Let ω be a complex number such that $\omega^3 = 1$ and $\omega \neq 1$. Suppose L is the field $Q\left(\sqrt[3]{2}, \omega\right)$ generated by $\sqrt[3]{2}$ and ω over the field Q of rational numbers. Then the number of sub fields K of L such that $Q \subset K \subset L$ is

(A) 2

(B) 3

(C) 4

(D) 5

53. Which one of the following systems of linear congruences is not solvable?

(A) $x \equiv 2 \pmod 5$
$x \equiv 4 \pmod{21}$
$x \equiv 3 \pmod{14}$

(B) $x \equiv 5 \pmod 8$
$x \equiv 9 \pmod{12}$
$x \equiv 3 \pmod{18}$

(C) $x \equiv 1 \pmod 4$
$x \equiv 2 \pmod 3$
$x \equiv 4 \pmod 7$

(D) $x \equiv 7 \pmod{11}$
$x \equiv 3 \pmod{13}$
$x \equiv 1 \pmod 4$

54. Let $X = \{1, 2, 3\}$, $S = \{\{1\}, \{2, 3\}, X, \phi\}$ and $T = \{\{1, 2\}, \{3\}, X, \phi\}$. Then which one of the following is correct?

(A) S is a σ-algebra but T is not.

(B) T is a σ-algebra but S is not.

(C) Both S and T are σ-algebra.

(D) Neither S nor T is a σ-algebra.

55. Given the trend equation $Y = 118.5 + 2.2X + 1.4X^2$ with origin 2010, then trend equation with origin 2011 is

(A) $Y = 117.7 - 0.6X + 1.4X^2$

(B) $Y = 108.5 + 1.2X + 1.4X^2$

(C) $Y = 122.1 + 5X + 1.4X^2$

(D) $Y = 117.7 + 5X + 1.4X^2$

56. A two component series system consisting of independent and identical components following exponential distribution with mean 2. The failure rate of the system is

(A) 4

(B) 1

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

57. Suppose y_1, y_2 and y_3 are independent observations with

$E(y_1) = \beta_1 + \beta_2$
$E(y_2) = \beta_1 + \beta_3$
$E(y_3) = \beta_1 + \beta_2$

having common unknown variance σ^2. Which of the following is true?

(A) $\beta_1 - \beta_2$ is estimable.

(B) Dimension of the error space is 2.

(C) $SSE = \frac{1}{2}(y_1 - y_2)^2$.

(D) Dimension of estimation space is unity.

58. Consider a sampling design with 5 population units $p(\{1, 2\}) = \frac{1}{2}$,

$p(\{3, 4\}) = p(\{3, 5\}) = p(\{4, 5\}) = \frac{1}{6}$ then

(A) $\pi_{12} - \pi_1 \pi_2 = 1/16$

(B) $\pi_{12} - \pi_1 \pi_2 = 0$

(C) $\pi_{12} - \pi_1 \pi_2 = -1/8$

(D) $\pi_{12} - \pi_1 \pi_2 = -1/16$

[π_i and π_{ij} are inclusion probabilities of first and second orders.]

59. Let $X_n, n \geq 1$ are uncorrelated standard normal random variables. Then which of the following is always true with regard to the stochastic process $\{X_n, n \geq 1\}$?

(A) It is a covariance stationary process which is not strictly stationary.

(B) It is a strictly stationary process which is not covariance stationary.

(C) It is covariance stationary and strictly stationary.

(D) It is an evolutionary process.

60. Which of the following differential equations is a quasi linear partial differential equation?

(A) $2xu\frac{\partial u}{\partial x} + 2yu\frac{\partial u}{\partial y} = x^2y^2u^2 + 2xu$

(B) $2x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = x^2y^2u^2 + 2xu$

(C) $2x\frac{\partial u}{\partial x} + 2y\frac{\partial u}{\partial y} = 2xu$

(D) $2xu\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} = x^2y^2u^2 + 2xu$

61. If α, β are the roots of the equation $ax^2 + bx + c = 0$, then $\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is

(A) 0

(B) $\frac{a}{2}(\alpha - \beta)^2$

(C) $-\frac{a^2}{2}(\alpha - \beta)^2$

(D) $\frac{a^2}{2}(\alpha - \beta)^2$

62. If $\underset{\sim}{X} \sim N_P(\mu, \Sigma)$ which of the following is not a correct statement?

(A) $\varphi_{\underset{\sim}{X}}(t) = e^{it'\mu - \frac{1}{2}t\Sigma t'}$

(B) For any $\underset{\sim}{l} \in \mathbb{R}^P$, $l'\underset{\sim}{X}$ is univariate normal.

(C) If X_i and X_j are the i^{th} and j^{th} components of $\underset{\sim}{X}$, then $X_i - X_j$ is univariate normal.

(D) $\varphi_{\underset{\sim}{X}}(t) = e^{-it'\mu + \frac{1}{2}t\Sigma t'}$.

63. For a BIBD (b, v, r, k, λ), which of the following conditions is sufficient?

(A) $bk = vr$

(B) $\lambda(v - 1) = r(k - 1)$

(C) $(r - \lambda)$ is a perfect square for even v.

(D) Information is insufficient.

64. I is an approximate value of the integral $\Delta = \int_{-2}^{2} \big|\,|x+1| - |x-1|\,\big|\,dx$, obtained through Trapezoidal rule with five points. Then which one of the following is correct?

(A) $\Delta = 0$

(B) $\Delta = 6$

(C) $I \neq \Delta/(0.9)$

(D) $I = 5$

65. Let $X = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$ and $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ be a linear mapping defined by $T(A) = XA - AX$ for all $A \in M_2(\mathbb{R})$. Then the dimension of $Ker(T)$ is

(A) 0

(B) 1

(C) 2

(D) 4

66. A matrix M has eigenvalues 1 and 4 with corresponding eigenvactors $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ respectively. Then M is

(A) $\begin{bmatrix} -4 & -8 \\ 5 & 9 \end{bmatrix}$

(B) $\begin{bmatrix} 9 & -8 \\ 5 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

67. In a queuing system arrival process is Poisson. Then the inter-arrival process is

(A) Poisson

(B) Normal

(C) Binomial

(D) Exponential

68. For a plane curve one has

(A) $\kappa = 0, \vec{b} = \vec{0}$

(B) $\tau = 0, \vec{b} \neq \vec{0}$

(C) $\tau \neq 0, \vec{b} = \vec{0}$

(D) $\tau = 0, \vec{b} = \vec{0}$

(where symbols have their usual meaning)

69. In measurement of population growth, Vital Index is defined as

(A) $\dfrac{CBR}{CDR} \times 100$

(B) $\dfrac{CDR}{CBR} \times 100$

(C) $CBR \times CDR \times 100$

(D) $\dfrac{CBR \times CDR}{100}$

70. Suppose $\{X_n, n \geq 1\}$ is a sequence of random variables on a probability space. Which of the following statements is always correct?

(A) If $X_n \xrightarrow{P} X$, then $X_n \xrightarrow{a.s.} X$.

(B) If $X_n \xrightarrow{P} X$ and $\{X_n\}$ is a monotone sequence, then $X_n \xrightarrow{a.s.} X$.

(C) If $X_n \xrightarrow{L} X$, then $X_n \xrightarrow{P} X$.

(D) If $X_n \xrightarrow{L} X$, then there exists a subsequence $\{X_{n_k}\}$ of $\{X_n\}$ such that $X_{n_k} \xrightarrow{a.s.} X$.

71. If X_i, $i = 1(1)n$ are *iid* $N(\theta, 1)$ observations and $\hat{\theta}$ is the maximum likelihood estimator, then

(A) $\lim P\left(\hat{\theta} - \theta \le \frac{1}{n}\right) = 1$

(B) $\lim P\left(\hat{\theta} - \theta \le \frac{1}{n}\right) = 0$

(C) $\lim P\left(\hat{\theta} - \theta \le \frac{1}{n}\right) = 0.5$

(D) $\lim P\left(\hat{\theta} - \theta \le \frac{1}{n}\right)$ does not exist

72. The function $f(z) = |z|^2$, $z \in \mathbb{C}$, is

(A) continuous nowhere.

(B) continuous everywhere but nowhere differentiable.

(C) continuous everywhere but nowhere differentiable except at the origin.

(D) continuous at the origin only.

73. Solution of the differential equation $y'' - x(y')^2 = 0$, $y(0) = 0$, $y'(0) = -1$ is

(A) $y = \sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

(B) $y = -\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

(C) $y = \frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}x\right)$

(D) $y = -\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}x\right)$

74. The value of the integral $\int_0^\infty e^{-x^2}\cos(\alpha x)dx$ is

(A) $\frac{1}{2}\sqrt{\pi}\,e^{-\alpha^2/4}$

(B) $\frac{1}{2}\sqrt{\pi}\,e^{-\alpha/2}$

(C) $\frac{\alpha^2}{4}$

(D) $\frac{\alpha}{2}$

75. The class equation of a group of order 10 is

(A) $1 + 2 + 3 + 4 = 10$

(B) $1 + 1 + 3 + 5 = 10$

(C) $1 + 2 + 2 + 5 = 10$

(D) $2 + 3 + 5 = 10$

76. Suppose X_1, X_2 and X_3 are *iid* $N(0, 1)$ variables. If $Q = P\left(X_1^2 + X_2^2 + X_3^2 > 1\right)$ then

(A) $Q = 1.0$

(B) $Q = 0.5$

(C) $Q < 0.5$

(D) $Q > 0.5$

77. Let $\{A_n\}$ be a sequence of events on a probability space and $\sum_{n=1}^{\infty} P(A_n) < \infty$, then $P\left(\overline{\lim}\,A_n\right)$ is

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) $\frac{1}{4}$

78. Let $\{X_n\}$ be a Markov chain with transition probability matrix P, which is doubly stochastic, then the stationary distribution of this Markov chain is

(A) (1, 0, 0)

(B) (0, 1, 0)

(C) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(D) (0, 0, 1)

79. An RBD is conducted in 2 blocks with 6 treatments. If the average yield for the blocks are x and y then SS(Blocks) can be expressed as

(A) $\frac{1}{2}(x - y)^2$

(B) $3(x - y)^2$

(C) $2(x - y)^2$

(D) $6(x - y)^2$

80. The resolvent kernel of the integral equation

$$u(x) = x^2 + \int_{\log 2}^{x} e^{t-x} u(t)\, dt \text{ is}$$

(A) $\cos(x-t)$

(B) 1

(C) e^{t-x}

(D) $\sin(x-t)$

81. Which of the following statements is not correct?

(A) Centre of a complete graph G is G itself.

(B) Complement of complete bipartite graph is disconnected.

(C) Spanning subgraph of the complete bipartite graph $K_{3,6}$ has a Hamilton cycle.

(D) If for a graph G a closed walk is a Hamilton cycle, then G is itself a cycle.

82. Consider an SRSWOR(n) sample S from a population of N units. Then for any two distinct units i and j, which one is true?

(A) $P(i \in S,\ j \notin S) = \left(\frac{n}{N}\right)^2$

(B) $P(i \in S,\ j \notin S) = \frac{n(n-1)}{N(N-1)}$

(C) $P(i \in S,\ j \notin S) = \frac{n(N-n)}{N(N-1)}$

(D) $P(i \in S,\ j \notin S) = \frac{n}{N}$

83. A survey is conducted by interviewers A and B. The cost of interviewing m persons by A is $4m^2$ while that through B is $6m^2$. If the survey is to be conducted on N individuals, how to distribute these N between A and B to minimize cost?

(A) In the 4 : 9 proportion

(B) In the 2 : 3 proportion

(C) In the 3 : 2 proportion

(D) In the 1 : 1 proportion

84. Consider the L.P.P. :

Maximize $z = x_1 + 3x_2$

subject to $3x_1 + 6x_2 \le 8$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \ge 0$

The problem

(A) has no feasible solution.

(B) has infinitely many optimal solutions.

(C) has a unique solution.

(D) has an unbounded solution.

85. Consider the nonlinear optimization problem: Minimize $Z = \sin(x_1 x_2) - \cos(x_1 - x_2)$.

$Z_{\min}$ occurs at

(A) $x_1 \ne x_2$

(B) $x_1 = x_2 = \frac{\pi}{2}$

(C) $x_1 = x_2 = \sqrt{\frac{3\pi}{2}}$

(D) $x_1 = 0,\ x_2 = \pi$

86. Let $f(t)$, for $t \ge 0$, be a continuous function and is of exponential order in t. Let $F(s)$ be the Laplace transform of $f(t)$ and $F(s)$ satisfies $s^2F''(s) - 6F(s) = 0$. If $f(2) = 8$, then $f(t)$ is equal to

(A) $4t$

(B) $2t$

(C) $t + 1$

(D) $2t + 4$

87. For $E \subset \mathbb{R}$, $E \ne \phi$, which of the following is correct?

(A) If E is closed then $m(A) > 0$

(B) If E is open then $m(A) > 0$

(C) If E is dense then $m(A) > 0$

(D) If E is perfect then $m(A) > 0$

where $m(A)$ denotes the Lebesgue measure of the set A.

88. Consider the differential equation $a\frac{dy}{dx}+by=ce^{-\lambda x}$ where a, b, c are positive constants and λ is a non-negative constant. Every solution of the differential equation approaches to $\frac{c}{b}$ as $x \to \infty$ when

(A) $\lambda > 0$

(B) $\lambda = 0$

(C) $\lambda = \frac{b}{a}$

(D) $\lambda = \frac{a}{b}$

89. There is a source of strength m at the origin of coordinates. Combine its flow pattern with that of uniform flow at velocity u parallel to the line $\theta = 0$. The fluid is assumed to extend to infinity in all directions. The stream function for the resulting flow is

(A) $-u-\frac{m\theta}{2\pi}$

(B) $-u\,r\sin\theta+\frac{m}{2\pi}\cos\theta$

(C) $u\,r\cos\theta-m\theta$

(D) $-u\,r\sin\theta-\frac{m\theta}{2\pi}$

90. Let T be a self-adjoint operator on a Hilbert space H, then which of the following statements is not correct?

(A) $\langle x, Tx\rangle$ is real for all x in H

(B) $\|T\| = \sup\limits_{\substack{x\in H \\ \|x\|=1}} |\langle x, Tx\rangle|$

(C) Eigenvalues of T (if exists) are real.

(D) T is always invertible.

91. Let γ be any positively oriented circle whose centre is at origin. Then the value of the integral $\int_\gamma \frac{\cos z}{z^2}\,dz$ is

(A) $2\pi i$

(B) $-2\pi i$

(C) 0

(D) πi

92. The value of $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{\sin^2 x}}$ is

(A) $1/e$

(B) e

(C) $1/\sqrt{e}$

(D) $\sqrt{e}$

93. For the Cantor middle one third set $C \subset [0, 1]$,

(A) $C + C = C$

(B) $C + C = 2C$

(C) $C + C = [0, 2]$

(D) $C + C = [0, 1]$

where $A+B=\{x+y : x\in A, y\in B\}$, $A, B\subset \mathbb{R}$ and $\lambda A = \{\lambda x : x\in A\}$, $\lambda\in\mathbb{R}$, $A\subset\mathbb{R}$.

94. The function $f(x) = x^{1/3}$, $x\in\mathbb{R}$ (set of all reals) is

(A) not continuous.

(B) not differentiable.

(C) of bounded variation on every finite interval in $\mathbb{R}$.

(D) not of bounded variation on every finite interval in $\mathbb{R}$.

95. In $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, \infty\}$, every sequence

(A) has a convergent subsequence.

(B) is convergent.

(C) is monotone.

(D) is unbounded.

96. The degree of $\sqrt{3}+\sqrt{5}$ over field of rationals Q is

(A) 2

(B) 3

(C) 5

(D) 4

97. The number of elements in the quotient ring $z\,[i]\,|\,\langle 3+i \rangle$ is

(A) 3

(B) 10

(C) 9

(D) infinite

98. Let p, q be distinct primes and Z be the set of integers. Then

(A) $Z/[p^2qZ]$ has exactly three distinct ideals.

(B) $Z/[p^2qZ]$ has exactly three distinct prime ideals.

(C) $Z/[p^2qZ]$ has exactly two distinct ideals.

(D) $Z/[p^2qZ]$ has a unique maximal ideal.

99. The row space of a matrix A of order 20×50 has dimension 13. Then the dimension of the space of solutions of $AX = O$ is

(A) 7

(B) 13

(C) 33

(D) 37

100. Let $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ and let α_n and β_n be two eigenvalues of A^n such that $|\alpha_n| \geq |\beta_n|$. Then

(A) $\alpha_n \to 0$ and $\beta_n \to \infty$ as $n \to \infty$

(B) $\alpha_n \to \infty$ and $\beta_n \to 0$ as $n \to \infty$

(C) $\alpha_n \to 0$ and $\beta_n \to 0$ as $n \to \infty$

(D) $\alpha_n \to \infty$ and $\beta_n \to \infty$ as $n \to \infty$

Space for Rough Work

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