Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series **X**

PAPER–II **Question Booklet No.**

(Identical with OMR Answer Sheet Number)

Subject Code : 16

PHYSICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D) , where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

Paper II

(Physical Sciences)

1. Suppose the matrix

$$M = \begin{bmatrix} x & -y \\ y & x \end{bmatrix}$$

is invertible under matrix multiplication, then its inverse is

(A) $\frac{1}{x^2+y^2}\begin{bmatrix} x & -y \\ y & x \end{bmatrix}$

(B) $\frac{1}{x^2+y^2}\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$

(C) $\frac{1}{x^2-y^2}\begin{bmatrix} -y & x \\ x & y \end{bmatrix}$

(D) $\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$

2. The value of

$$\int_0^{\infty} \delta(x+3)\,dx$$

(A) 0

(B) 1

(C) $1/2\pi$

(D) – 3

3. Identify the correct series expansion for the following periodic function of period 2π :

$$f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ 1, & 0 \le x < \pi \end{cases}$$

(A) $f(x) = 1 + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\sin nx}{n}$

(B) $f(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1,3,5...}^{\infty} \frac{\sin nx}{n}$

(C) $f(x) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} (-1)^n \frac{\sin nx}{n}$

(D) $f(x) = 1 + \frac{2}{\pi}\sum_{n=1,3,5...}^{\infty} \frac{\sin nx}{n} - \frac{2}{\pi}\sum_{n=1,3,5...}^{\infty} (-1)^{n+1} \frac{\cos nx}{n}$

4. If $A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{pmatrix}$, then which of the following statements is true about the matrix A?

(A) $A^4 = 30I$

(B) $A^4 = 36A^2$

(C) $A^4 = 20A^3$

(D) $A^4 = 30A^3$

5. The second derivative of the Dirac delta function, $\delta''(x)$, can be represented as

(A) $-\frac{1}{2\pi}\int_{-\infty}^{\infty} y^2 e^{ixy}\, dy$

(B) $\frac{i}{2\pi}\int_{-\infty}^{\infty} y e^{ixy}\, dy$

(C) $\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{ixy}}{y^2}\, dy$

(D) $\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{ixy}}{|y|}\, dy$

6. If a matrix A is both unitary and hermitian, its determinant is

(A) 1

(B) of the form $\exp(i\theta)$ for all θ

(C) 1 or – 1

(D) purely imaginary

7. Consider the complex function $f(z) = a_0 + a_1 z + a_2 z^2$, where a_0, a_1, a_2 are real constants. If $|f(z)| \le 16$ everywhere on the circle given by $|z| = 2$, the upper bounds on a_0, a_1 and a_2 are respectively

(A) 4, 4, 4

(B) 1, 4, 16

(C) 16, 8, 4

(D) 16, 4, 1

8. The vector $\vec{V} = x^2\hat{i} - y\hat{j} + xz\hat{k}$ and the vector $\vec{W} = y\hat{i} + x\hat{j} - xyz\hat{k}$, then the value of $\frac{\delta^2}{\delta x \delta y}\left(\vec{V}\times\vec{W}\right)$ at the point (1, –1, 2) is

(A) $\hat{i} - \hat{j} + 2\hat{k}$

(B) $2\hat{i} + 4\hat{k}$

(C) $-4\hat{i} + 8\hat{j}$

(D) $4\hat{i} + 4\hat{k}$

9. Let C be the unit circle, travelled counter-clockwise, evaluate $\oint_C \left[(e^{-x^2} - y^3)dx + x^3 dy\right]$.

(A) $\frac{3\pi}{2}$

(B) $\sqrt{\frac{\pi}{2}} + 2$

(C) 2π

(D) $\sqrt{\frac{\pi}{2}} - 2$

10. If M is a 3×3 matrix such that

$$M\begin{pmatrix}0\\1\\2\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} \text{ and } M\begin{pmatrix}3\\4\\5\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix},$$

then the product $M\begin{pmatrix}6\\7\\8\end{pmatrix}$ is

(A) $\begin{pmatrix}-1\\2\\0\end{pmatrix}$

(B) $\begin{pmatrix}0\\0\\1\end{pmatrix}$

(C) $\begin{pmatrix}1\\-1\\0\end{pmatrix}$

(D) $\begin{pmatrix}9\\10\\11\end{pmatrix}$

11. Which of the following functions of x could be represented by a Fourier series over the range indicated?

(A) $\tan x \qquad -\infty < x < \infty$

(B) $\sqrt{|\sin x|} \qquad -\infty < x < \infty$

(C) $\frac{\sin 2x}{\cos x} \qquad -\infty < x < \infty$

(D) $x\sin(1/x) \qquad -\infty < x < \infty$

12. $|\phi_1\rangle$ and $|\phi_2\rangle$ are normalized eigenvectors corresponding to two distinct eigenvalues of a Hermitian operator $\hat{A}$ in a two dimensional vector space and $\hat{F} = \alpha\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)$

Which of the following statements is true?

(A) $\hat{F}$ is a projection operator.

(B) $\langle\phi_2|\hat{F}|\phi_2\rangle = 0$.

(C) $|\phi_1\rangle$ and $|\phi_2\rangle$ are eigenvectors of $\hat{F}$.

(D) $\hat{F}$ is a unitary operator.

13. A mass m is hung from a massless spring of spring constant K. When it is displaced by an amount a from its equilibrium position, it executes simple harmonic motion and the total energy is $\frac{1}{2}m\omega^2a^2$. If the mass is doubled keeping the amplitude fixed at a, the total energy will be

(A) $m\omega^2a^2$

(B) $2m\omega^2a^2$

(C) $\frac{1}{4}m\omega^2a^2$

(D) $\frac{1}{2}m\omega^2a^2$

14. If a particle of mass m is in a potential $V(x) = ax^2 + \frac{b}{x^2}$, then the frequency of small oscillations about equilibrium is

(A) $\sqrt{\frac{2a}{m}}$

(B) $\sqrt{\frac{8a}{m}}$

(C) $\sqrt{\frac{6a}{m}}$

(D) $\sqrt{\frac{2b}{m}}$

15. If the brakes are applied suddenly to a moving car, the wheels will skid. This is because

(A) the inertia of the car will carry it forward.

(B) the momentum of the car is conserved.

(C) the impulsive retarding force exceeds the static friction.

(D) the kinetic friction is converted to static friction.

16. An elliptical orbit equation of a satellite is given by

$$\frac{8256 \text{ km.}}{r} = 1 + 0{\cdot}20 \cos\theta$$

The altitude of the apogee and perigee are

(A) 8600 km., 1720 km.

(B) 10320 km., 6880 km.

(C) 8426 km., 6380 km.

(D) 8246 km., 8360 km.

17. The escape velocity from the earth is 11·2 km/s. Consider another planet whose average density is 25% of that of the earth but whose radius is 8 times larger. The escape velocity from this planet will be

(A) 44·8 km/s

(B) 3·96 km/s

(C) 179·2 km/s

(D) 31·68 km/s

18. Consider the Sun to be a uniform solid sphere (which it is not) of radius 7×10^5 km rotating about its axis with a time period of 25 days. If it now shrinks to a sphere of radius of 7 km, but not losing any of its mass, the time period will be

(A) 21·6 s

(B) 68·305 s

(C) $2{\cdot}16 \times 10^{-4}$ s

(D) 6830·5 s

19. Hubble's law is defined as $v = H_0 r$, where v is the velocity of the galaxy, r is the distance and H_0 is the Hubble constant. Then, $\frac{H_0^2}{G}$ (G=Universal Gravitational Constant) has the dimension of

(A) mass

(B) linear momentum

(C) angular momentum

(D) density

20. Which of the following equations is Lorentz covariant?

(A) $\nabla^2\psi(\vec{r},t) + \frac{1}{c^2}\frac{\partial^2\psi(\vec{r},t)}{\partial t^2} = 0$

(B) $\nabla^2\psi(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2\psi(\vec{r},t)}{\partial t^2} = 0$

(C) $\nabla^2\psi(\vec{r},t) - i\hbar\frac{\partial^2\psi(\vec{r},t)}{\partial t^2} = 0$

(D) $\nabla^2\psi(\vec{r},t) + i\hbar\frac{\partial^2\psi(\vec{r},t)}{\partial t^2} = 0$

21. An ideal gas at a temperature T in its rest frame is viewed from a frame moving with a velocity V w.r.t. the rest frame. In this frame the temperature will appear as

(A) $T\sqrt{1-\frac{V^2}{c^2}}$

(B) $\frac{T}{\sqrt{1-\frac{V^2}{c^2}}}$

(C) $T\left(1-\frac{V}{c}\right)$

(D) $\frac{T}{1-\frac{V}{c}}$

22. We have eight diatomic non-interacting molecules in a system. The diatomic molecules are rigid. The number of degrees of the system is

(A) 4

(B) 8

(C) 40

(D) 6

23. The Lagrangian of a particle of mass m in spherical co-ordinate is given by

$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\right)$. The quantity that is conserved is

(A) $\frac{\partial L}{\partial \dot{\phi}}$

(B) $\frac{\partial L}{\partial \dot{r}}$

(C) $\frac{\partial L}{\partial \phi}$

(D) $\frac{\partial L}{\partial \dot{\phi}} + r^2\dot{\theta}$

24. Which of the following quantities remains the same at every point of the elliptic orbit of a planet around the Sun? Neglect the diurnal motion of the planet.

(A) Torque applied by the Sun.

(B) Potential energy of the planet.

(C) Kinetic energy of the planet.

(D) Angular velocity of the planet.

25. The Hamiltonian for a 1-dimensional system $H = \frac{(p + a\cos t)^2}{2} - ax\sin t$, a being a constant.

For this system.

(A) both energy and linear momentum are conserved.

(B) energy is conserved but linear momentum is not.

(C) linear momentum is conserved but energy is not.

(D) neither linear momentum nor energy is conserved.

26. The value of α for which the transformation

$Q = q \cos \alpha - p \sin \alpha$

$P = q \sin \alpha + p \cos \alpha$

is not canonical is

(A) $\alpha = 0$

(B) $\alpha = \frac{\pi}{4}$

(C) $\alpha = \frac{\pi}{2}$

(D) $\alpha = \pi$

27. Two spaceships are moving towards each other with a velocity of 0·48 c each. The velocity of the second ship as seen from the first is about

(A) 0·96 c

(B) 0·39 c

(C) 0·78 c

(D) 0·65 c

28. When S^{32}_{16} nucleus is bombarded with neutrons, radioactive P^{32}_{15} and certain particles are produced. These are

(A) protons

(B) electrons

(C) neutrons

(D) α-particles

29. A Kaon K° decays to a pair of pions, i.e $K^\circ \rightarrow \pi^\circ\pi^\circ$ or $\pi^+\pi^-$. Given that K° and $\pi^\circ, \pi^\pm$ are pseudo scalar particles, the lifetime of K° will be around

(A) 10^{-10} s

(B) 10^{-16} s

(C) 10^{-23} s

(D) 10^{-45} s

30. The equation of the plane tangent to $xyz = 1$ at the point (1, 1, 1) is

(A) $x - y + z = 1$

(B) $yz + xz + xy = 3$

(C) $x^2 + y^2 + z^2 = 3$

(D) $x + y + z = 3$

31. Radii of two spherical atoms *A* and *B* are r_A and r_B respectively. The lattice constant of the alloy '*AB*', if it crystallizes in a bcc structure, will be

(A) $\frac{2(r_A + r_B)}{\sqrt{2}}$

(B) $\frac{2(r_A + r_B)}{\sqrt{3}}$

(C) $\frac{r_A + r_B}{\sqrt{3}}$

(D) $\frac{2(r_A + r_B)}{3\sqrt{3}}$

32. In a lithium atom, the outer electron is replaced by a muon (same charge, but roughly 200 times heavior). The chemical properties of the resultant atom would be

(A) very similar to He.

(B) similar to Li, but with slightly reduced excitation energy.

(C) similar to Li, but with slightly increased excitation energy.

(D) indistinguishable from normal Li.

33. Balmer series of spectral lines of hydrogen atom consists of transition of electron from a state with principal quantum number n greater than 2 to a state with $n = 2$. If fine structures are considered then each such line splits into

(A) 2 lines due to transitions between 3 distinct pairs of energy levels.

(B) 3 lines due to transitions between 3 distinct pairs of energy levels.

(C) 5 lines due to transitions between 6 distinct pairs of energy levels.

(D) 5 lines due to transitions between 7 distinct pairs of energy levels.

34. Laser cooling of atoms is produced due to

(A) absorption of photons by atoms.

(B) scattering of photons by atoms.

(C) transfer of momentum from photons to atoms.

(D) transfer of energy from photons to atoms.

35. What is the degeneracy of an Ortho-Helium atom in minimum energy state?

(A) 3

(B) 1

(C) 2

(D) 5

36. If a proton were ten times lighter, then the binding energy of the electron in a hydrogen atom would have been

(A) less

(B) more

(C) same

(D) depends on mass of electron

37. Let $|x\rangle$ denote the eigenstate of the operator $\hat{X}$ corresponding to the eigenstate x. Then the state $|\psi\rangle = \exp(iap/\hbar)|x\rangle$ is an eigenstate of $\hat{x}$ corresponding to the eigenvalue

(A) $x - a$

(B) $x + a$

(C) $a - x$

(D) $x + ia$

38. The critical coefficients of a van der Walls gas are

(A) $V_C = 5b,\ T_C = {8a}/{27bR},\ P_C = {a}/{27b^2}$

(B) $V_C = 3b,\ T_C = {8a}/{27bR},\ P_C = {a}/{27b^2}$

(C) $V_C = 3b,\ T_C = {5a}/{27bR},\ P_C = {4a}/{27b^2}$

(D) $V_C = 5b,\ T_C = {5a}/{27bR},\ P_C = {a}/{16b^2}$

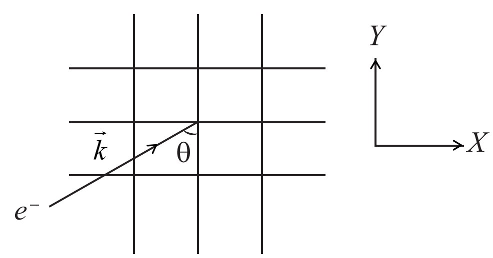
39. Consider a square lattice of lattice spacing 'a' in two dimensions, drawn in say, XY-plane. An electron of wave vector $\vec{k} = (k_x, k_y)$ is incident on a lattice-plane parallel to Y-axis, making an angle θ. The electron will be Bragg-reflected completely by all the planes parallel to both the X and Y-axes when,

(A) $k_x = 0,\ k_y = \pm \pi/2a$

(B) $k_x = 0,\ k_y = \pm \pi/2a$

(C) $k_x = \frac{\pi}{2a},\ k_y = -\frac{\pi}{2a}$

(D) $k_x = \pm\frac{\pi}{a},\ k_y = \pm\frac{\pi}{a}$



40. The fact that beta decay is due to a very weak interaction can be inferred from

(A) the lifetime of a free neutron.

(B) the decay rate of a nuclide undergoing beta decay.

(C) the fraction of beta versus alpha particles omitted in radioactive decay.

(D) the penetrating power of beta rays.

41. Which of the following phenomena is NOT evidence for an isospin symmetry between proton and neutron?

(A) Charge symmetry of the strong interaction

(B) Charge independence of the strong interaction

(C) Anomalous magnetic moment of the neutron

(D) Mass degeneracy between proton and neutron

42. Which one of the following decay is permissible?

(A) $n \to p + \beta^- + \nu$

(B) $n \to p + \beta^- + \overline{\nu}$

(C) $p \to n + \beta^- + \nu$

(D) $p \to n + \beta^+ + \overline{\nu}$

43. If $\vec{p}$ is the linear momentum and $\vec{J}$ the angular momentum, the Poisson bracket $\{\vec{p}, \vec{J}.\hat{n}\}$ equals (here $\hat{n}$ is an arbitrary constant vector)

(A) $\vec{p} \times \hat{n}$

(B) $\hat{n} \times \vec{p}$

(C) $2\vec{p} \times \hat{r}$

(D) $\hat{p} \times \hat{r}$

44. The neutral pion π° is produced in strong interactions, but it decays electromagnetically $\pi^\circ \to \gamma\gamma$. However the π° is not considered to be a strange particle because

(A) it is not pair-produced in strong interactions.

(B) it carries no isospin.

(C) it is the lightest hadron.

(D) it forms an iso-triplet with $\pi^\pm$, which are not strange particles.

45. A certain OPAMP has an open-loop gain of 10^5 and a common mode gain of 0·2. The CMRR is

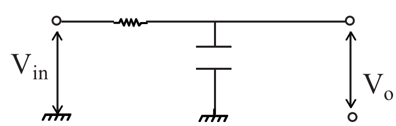
(A) 500000 dB

(B) 114 dB

(C) 11·4 dB

(D) 1140 dB

46. A sinusoidal voltage $V_{in}(t) = V\sin\omega t$ is applied to the input of the following circuit with RC = 0·1 sec. The output voltage V_o will



(A) lead the input voltage by an angle $\varphi = \tan^{-1}\left(\frac{\omega}{10}\right)$.

(B) lead the input voltage by an angle $\varphi = \tan^{-1}(10\omega)$.

(C) lag behind the input voltage by an angle $\varphi = \tan^{-1}\left(\frac{\omega}{10}\right)$.

(D) lag behind the input voltage by an angle $\varphi = \frac{\pi}{2}$ radian.

47. A signal of frequency 20 kHz is being digitalized by an A/D converter. A possible sampling time which can be used is

(A) 25 μs

(B) 100 μs

(C) 40 μs

(D) 60 μs

48. The decimal equivalent of hexadecimal B2F8 is

(A) 40968

(B) 45808

(C) 46816

(D) 45816

49. The following Boolean expression

$$\overline{\bar{a}_1 + a_4} \cdot \overline{a_2 + \bar{a}_3} \cdot \overline{\bar{a}_3 + a_4}$$

simplifies to

(A) $a_1 \cdot \bar{a}_2 \cdot a_3 \cdot \bar{a}_4$

(B) $a_1 \cdot \bar{a}_2 + a_3 \cdot \bar{a}_4$

(C) $a_1 \cdot a_2 \cdot a_3 \cdot \bar{a}_4$

(D) $a_1 \cdot \bar{a}_3 \cdot a_2 \cdot \bar{a}_4$

50. To obtain a first order diffraction peak for a crystalline solid with interplane distance equal to wavelength of incident X-ray radiation, the angle of incidence with the normal should be

(A) 90°

(B) 0°

(C) 30°

(D) 60°

51. A bridge rectifier is preferred over an ordinary full wave rectifier, because

(A) it uses four diodes.

(B) its transformer does not require a centre trap.

(C) it needs much smaller transformer for the same output.

(D) it has higher efficiency.

52. In penning gauge, the pressure P is related to the discharge current I via the relation

(A) $I = k P^2$; k is a constant

(B) $I = k\sqrt{P}$; k is a constant

(C) $I = k P^n$; k is a constant and n has a value ranging between 1·1 — 1·2

(D) $I = k P^n$; k is a constant and n has a value ranging between 2·2 — 2·3

53. Which of the following is an absolute gauge?

(A) Knudsen gauge

(B) Pirani gauge

(C) Ionisation gauge

(D) Mcleod gauge

54. Which of the following functions may be fitted with experimental data using the technique of linear least square fit?

(A) $f_1(x) = a_1 e^{\lambda_1 x} + b_2 e^{\lambda_2 x}$

(B) $f_2(x) = c_1 \sin(k_1 x)$

(C) $f_3(x) = d_1 x^m$

(D) $f_4(x) = k_1 \tan(q x)$

55. The wavefunction of a state of the one-dimensional harmonic oscillator with angular frequency ω is given by $\psi = \psi_0 + 2\psi_1$, where ψ_0 and ψ_1 are the ground state and the first excited state respectively. The expectation value of the energy in the state ψ is

(A) $\frac{13}{10}\hbar\omega$

(B) $\frac{13}{2}\hbar\omega$

(C) $\frac{7}{2}\hbar\omega$

(D) $\frac{6}{5}\hbar\omega$

56. In estimating a physical quantity $I = \frac{m}{P} x^4 d^{3/2}$ one measures the variables m, P, x and d. In a given observation relative errors in these quantities are 0·010, 0·030, 0·025 and 0·020 respectively. What will be the maximum proportional error in measurement of I?

(A) 17%
(B) 5%
(C) 11%
(D) 8·5%

57. A mineral consists of a cubic close packed structure formed by O^{2-} ions where half of the octahedral voids are occupied by Al^{3+} and one-eighth of the tetrahedral voids are occupied by Mn^{2+}. The chemical formula of the mineral is

(A) $Mn_3Al_2O_6$
(B) $MnAl_2O_4$
(C) $MnAl_4O_7$
(D) $Mn_2Al_2O_5$

58. A solid contains N spin-half magnetic atoms. At sufficiently high temperatures, the atoms are randomly oriented, while at sufficiently low temperatures, they are perfectly aligned. The heat capacity is given by

$$C(T) = \begin{cases} C_0\left(\frac{T}{T_0} - 1\right), & T_0 \le T \le 3T_0 \\ 0, & \text{otherwise} \end{cases}$$

where C_0 and T_0 are constants. Determine the maximum value of C_0.

(A) $\frac{Nk_B \ln 2}{2 - \ln 3}$

(B) $\frac{Nk_B \ln 2}{\ln 3}$

(C) $\frac{Nk_B \ln 3}{2}$

(D) $\frac{2Nk_B \ln 2}{2 + \ln 3}$

59. A system uses a 12bit word to represent the input signal. If the maximum peak to peak voltage at the output is set for 4V, then the typical value for the step size will be

(A) 48μV
(B) 976μV
(C) 4096μV
(D) 720μV

60. The short wavelength cut-off of the continuous X-ray spectrum from a nickel target is 0·0825nm. The voltage required to be applied on an X-ray tube is

(A) 0·15 kV
(B) 1·5 kV
(C) 15 kV
(D) 150 kV

61. A thermal neutron having speed v impinges on a ^{235}U nucleus. The reaction cross-section is proportional to

(A) v^{-1}
(B) v
(C) $\sqrt{v}$
(D) $v^{-1/2}$

62. The approximate force exerted on a perfectly reflecting mirror by an incident laser beam of power 10mW at normal incidence is

(A) 10^{-13} N
(B) 10^{-11} N
(C) 10^{-9} N
(D) 10^{-15} N

63. A circular loop of radius R, carries a uniform line charge density λ. The electric field calculated at a distance z directly above the centre of the loop, is maximum if z is equal to

(A) $\frac{R}{\sqrt{3}}$

(B) $\frac{R}{\sqrt{2}}$

(C) $\frac{R}{2}$

(D) $\frac{R}{3}$

64. A charge $+q$ is kept at a distance of $2R$ from the centre of a grounded conducting sphere of radius R. The image charge and its distance from the centre respectively, are

(A) $+\frac{q}{2}$ and $\frac{R}{4}$

(B) $-q$ and $\frac{R}{4}$

(C) $-\frac{q}{2}$ and $\frac{R}{4}$

(D) $-\frac{q}{2}$ and $\frac{R}{2}$

65. The capacitance of two concentric spherical metal shells, with radii a and b is

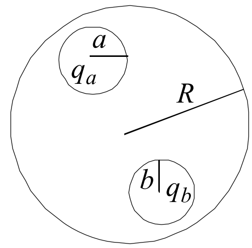
(A) $\frac{1}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$

(B) $4\pi\epsilon_0 \frac{ab}{b-a}$

(C) $4\pi\epsilon_0 \left[\ln b - \ln a\right]$

(D) $\frac{1}{4\pi\epsilon_0}\left[\ln b - \ln a\right]$

66. Two spherical cavities, of radii a and b, are hollowed out from the interior of a (neutral) conducting sphere of radius R. At the centre of each cavity a point charge is placed—called these charges $+q_a$ and $+q_b$. Find the surface charge density σ_a, σ_b and σ_R.



(A) $\sigma_a = -\frac{q_a}{4\pi\epsilon_0 a^2}$; $\sigma_b = -\frac{q_b}{4\pi\epsilon_0 b^2}$;
$\sigma_R = \frac{q_a + q_b}{4\pi\epsilon_0 R^2}$

(B) $\sigma_a = +\frac{q_a}{4\pi\epsilon_0 a^2}$; $\sigma_b = +\frac{q_b}{4\pi\epsilon_0 b^2}$;
$\sigma_R = -\frac{q_a + q_b}{4\pi\epsilon_0 R^2}$

(C) $\sigma_a = -\frac{q_a}{4\pi\epsilon_0 a^2}$; $\sigma_b = -\frac{q_b}{4\pi\epsilon_0 b^2}$;
$\sigma_R = -\frac{q_a + q_b}{4\pi\epsilon_0 R^2}$

(D) $\sigma_a = \frac{q_a}{4\pi\epsilon_0 a^2}$; $\sigma_b = \frac{q_b}{4\pi\epsilon_0 b^2}$;
$\sigma_R = \frac{q_a + q_b}{4\pi\epsilon_0 R^2}$

67. A long cylindrical conductor of radius R and $\sigma = \infty$ carries a current $I = I_0 \sin\omega t$. The conduction current density $J(r)\left[\text{for } r > R\right]$ is

(A) $\sigma\vec{E}$

(B) $\frac{I_0}{2\pi r}$

(C) $\frac{I_0 r}{\pi R^2}$

(D) zero

68. A diffraction grating used at normal incidence gives a green line (540nm) at a certain order superimposed on a violet line (405nm) of the next higher order. If the angle of diffraction be 30°, the number of lines per cm on the grating is

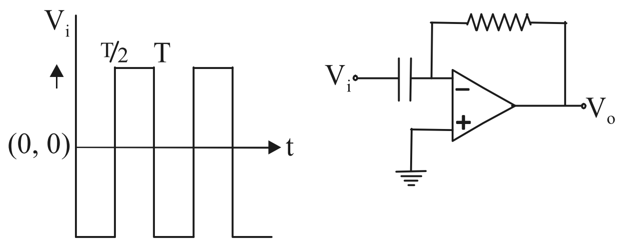
(A) 4065

(B) 4051

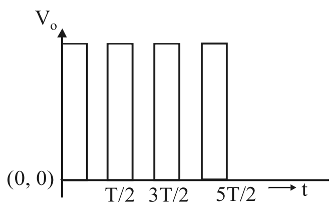
(C) 4025

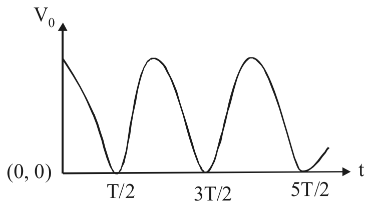
(D) 3086

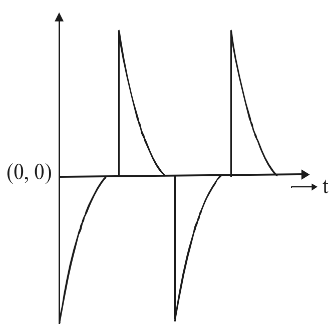
69. The input V_i to the following circuit is a square wave as shown in the following figure.

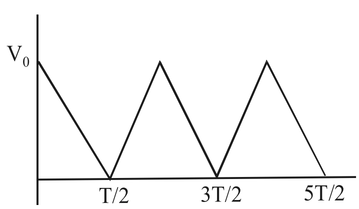


Which of the waveforms is the best suitable output?

(A) 

(B) 

(C) 

(D) 

70. A long straight wire carrying a uniform line charge density λ, is surrounded by rubber insulation out to a radius *R*. The electric displacement at a distance *r* (*r* > 0) from the axis of the wire is

(A) $\frac{\lambda}{2\pi r}\hat{r}$

(B) $2\pi r\lambda\,\hat{r}$

(C) $\frac{2\pi r}{\lambda}\hat{r}$

(D) zero

71. An observer in an inertial frame finds that at a point *P*, the electric field vanishes but the magnetic field does not. This implies that in any other inertial frame, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ satisfy

(A) $|\vec{E}|^2 = |\vec{B}|^2$

(B) $\vec{E}.\,\vec{B} = 0$

(C) $\vec{E} \times \vec{B} = 0$

(D) $\vec{E} = 0$

72. A large spherical soap film of thickness *d* has a refractive index $\frac{4}{3}$. A narrow beam of yellow light $(\lambda \approx 6400\text{Å})$ is incident on the film at an angle of 30°. What is the value of *d* for which a constructive second order interference would occur for the reflected wave?

(A) 3900Å

(B) 5200Å

(C) 4800Å

(D) 5500Å

73. x-z plane is the interface between two dielectric media. An electromagnetic wave with electric field vector $\vec{E} = E_0 \sin(ky - \omega t)\hat{k}$ is incident on this interface. Then which of the following statements is true?

(A) The electric fields in the incident and transmitted waves are in the opposite phase.

(B) The magnetic field in the transmitted wave is along $\hat{k}$ direction.

(C) The magnetic field in the incident and the reflected waves are in the same phase.

(D) The electric field in the reflected wave is along $\hat{i}$ direction.

74. Suppose a 4 dimensional isotropic harmonic oscillator is in third excited state. The degeneracy of the state is

(A) 20

(B) 36

(C) 24

(D) 10

75. The electric field of an electromagnetic wave is given by $\vec{E} = 3\sin(kz - \omega t)\,\hat{i} + 4\cos(kz - \omega t)\,\hat{j}$. The wave is

(A) linearly polarised at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ with the X-axis.

(B) linearly polarised at an angle $\tan^{-1}\left(\frac{3}{4}\right)$ with the X-axis.

(C) elliptically polarised in clockwise direction and it is travelling towards the observer.

(D) elliptically polarised in counter clockwise direction and it is travelling towards the observer.

76. Two homonuclear diatomic molecules produce identical rotational spectra, even though the atoms are known to have different chemical properties. From this one can conclude that

(A) the atoms are isotopes with same atomic number.

(B) the atoms are isobars with same atomic weight.

(C) the rotational spectra depend only on the electronic configuration.

(D) the inter-atomic separation is smaller for the heavier molecule.

77. A particle of mass m has a wavefunction satisfying the Schrödinger equation with a potential $V(\vec{r}) = V_1(\vec{r}) + i\,V_2(\vec{r})$, where $V_1(\vec{r})$ and $V_2(\vec{r})$ are real functions. From this one can conclude that

(A) the particle is in a bound state.

(B) the particle cannot be in an eigenstate of the Hamiltonian.

(C) the particle is in a state which decays with time.

(D) the particle cannot have stationary states.

78. In computing the splitting of spectral lines in a weak magnetic field the Landé g-factor arises because of

(A) Pauli exclusion principle

(B) Spin-orbit coupling

(C) Larmor precession

(D) Relativistic corrections

79. A quantum particle is moving in one dimension under the influence of a symmetric potential $V(x) = V(-x)$. If $V(x)$ is finite everywhere then which of the following functions may be a valid wavefunction of the ground state of this particle?

(A) $N\exp(-\gamma x)$

(B) $N\exp(-\gamma x^2)$

(C) $N\exp(-\gamma|x|)$

(D) $Nx\exp(-\gamma x^2)$

80. Consider a one dimensional free particle, for which the Hamiltonian commutes with the momentum operator. Which of the following statements is false?

(A) All energy eigenfunctions are eigenfunctions of momentum too.

(B) All momentum eigenfunctions are eigenfunctions of Hamiltonian too.

(C) The position operator does not commute with the Hamiltonian.

(D) The position uncertainty of a momentum eigenfunction is infinite.

81. A particle is confined in a one dimensional box between $x = -a$ to $x = +a$ and is in its ground state. Suddenly the walls of the box are shifted to $x = \pm 2a$. The probability that the particle is now at the first excited state of the new potential is

(A) $\frac{1}{\pi}$

(B) 0·5

(C) 0·2

(D) zero

82. A quantum mechanical Harmonic oscillator is in the state $|\psi\rangle$ with angular frequency (ω). For $|\psi\rangle = \frac{\sqrt{2}}{4}|0\rangle + i\,\frac{2}{4}|1\rangle - i\,\frac{1}{4}|2\rangle + \frac{3}{4}\,e^{i\frac{\pi}{3}}\,|3\rangle$ the average energy of the system is

(A) 2·03 $\hbar\omega$

(B) 0·53 $\hbar\omega$

(C) 2·56 $\hbar\omega$

(D) None of the above

83. The Hamiltonian for an N-state system is given by $H = E_0 \sum_{k,j=1}^{N} |k\rangle\langle j|$. $|k\rangle$ and $|j\rangle$ are orthonormal basis vectors. The energy eigenvalues E_j are given by

(A) $E_N = NE_0$ and $E_j = 0 \;\; j = 1,\ldots,N-1$

(B) $E_j = jE_0 \;\; j = 1,\ldots,N$

(C) $E_0 = 0$ and $E_j = jE_0 \;\; j = 1,\ldots,N-1$

(D) $E_j = E_0 \;\; j = 1,\ldots,N$

84. A particle of mass m is in the ground state of a 1d-harmonic oscillator potential of angular frequency ω. The entire system is placed in the centre of a 1d-box of length $\sqrt{\frac{\hbar}{m\omega}}$. The ground state energy

(A) decreases.

(B) increases.

(C) remains unchanged.

(D) splits into two energy levels.

85. If V_s is the stopping potential of the photocurrent I, generated by shining a light beam of frequency ν to a metal surface then which of the following statements is true?

(A) V_s increases by increasing I.

(B) V_s decreases by increasing I.

(C) V_s increases by increasing ν.

(D) V_s is independent of ν.

86. Five non-interacting identical spin $\frac{3}{2}$ particles are in the one-dimensional harmonic oscillator potential $V(x) = \frac{1}{2}m\omega^2 x^2$. The ground state energy of the system is

(A) $\frac{7}{2}\hbar\omega$

(B) $\frac{9}{2}\hbar\omega$

(C) $\frac{11}{2}\hbar\omega$

(D) $\frac{13}{2}\hbar\omega$

87. Two angular momenta $\bar{L}_1$ and $\bar{L}_2$ are added to form $\vec{J} = \bar{L}_1 + \bar{L}_2$. If the eigenstates of $\bar{L}_{1,2}$ are denoted by $|l_{1,2} m_{1,2}\rangle$ and those of $\vec{J}$ as $|j, m_j\rangle$, then $|j, m_j\rangle$ cannot be written as a direct product $|l_1, m_1\rangle \otimes |l_2, m_2\rangle$ because

(A) J_z does not commute with L_{1z} and L_{2z}.

(B) J^2 does not commute with J_z.

(C) J^2 does not commute with L_{1z} and L_{2z}.

(D) J_z does not commute with L_1^2 and L_2^2.

88. A particle of mass m is trapped in an infinite, quantum well of width L. The expectation value of its momentum in the third excited state is

(A) Infinity

(B) $\frac{\pi}{L}$

(C) Zero

(D) $\sqrt{\frac{\pi}{L}}$

89. Consider the free energy as a function of order parameter M :

$$F(M,T) = F_0 - \frac{\alpha(T - T_c)}{2}M^2 + \frac{b}{4}M^4$$

The discontinuity of specific heat at T_c is

(A) $-\frac{\alpha^2}{2b}$

(B) $-\frac{\alpha^2}{4b}$

(C) $\frac{\alpha^2}{6b}$

(D) $\frac{2\alpha^2}{3b}$

90. Suppose an ideal gas is having a transition following VP^3 = const. equation. Initial temperature and volume of the gas are T and V respectively. If the gas expands to $27V$, then the temperature will become

(A) $9T$

(B) $27T$

(C) $\frac{T}{9}$

(D) T

91. A rubber band is stretched up to 1·1 times of its original length isothermally, its entropy will be

(A) increased by 1·1 times.

(B) increased by 0·1 times.

(C) same.

(D) decreased by 0·1 times.

92. An ensemble of N three level systems with energies $E = -\varepsilon_0, 0$ and $+\varepsilon_0$ is in thermal equilibrium at temperature T. At high temperature, the free energy is approximately $(x = \beta\varepsilon_0)$

(A) $-NK_BT\ x^2$

(B) $-NK_BT\left[ln 2 + \frac{x^2}{2}\right]$

(C) $-NK_BT\left[ln 3 + \frac{x^2}{3}\right]$

(D) $-NK_BT\ ln 3$

93. N identical fermions occupy $N + 2$ states in 28 different ways. The value of N is

(A) 12

(B) 10

(C) 8

(D) 6

94. The thermodynamic relation expressing $T\,ds$ equation is

(A) $T\,ds = C_V dT - T\left(\frac{\partial P}{\partial T}\right)_V dV$

(B) $T\,ds = C_V dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$

(C) $T\,ds = C_V dT - T\left(\frac{\partial S}{\partial V}\right)_T dV$

(D) $T\,ds = C_V dT + T\left(\frac{\partial S}{\partial V}\right)_T dV$

95. 540 gm of ice at 0°C is mixed with 540 gm of water at 80°C. The resultant temperature of the mixture would be

(A) 0°C

(B) 30°C

(C) 80°C

(D) 40°C

96. What is the minimum attainable pressure of an ideal gas in the process given by $T = a + bV^2$?

(A) $\sqrt{ab}$

(B) $R\sqrt{ab}$

(C) $2R\sqrt{ab}$

(D) $R\sqrt{\frac{a}{b}}$

97. According to Debye theory, the phonon mode of density for two-dimensional solid will vary with frequency(ω) as

(A) ω

(B) ω^2

(C) ω^3

(D) ω^{-1}

98. In case of Bose-Einstein condensation

(A) number of particles increase in lower energy levels at low temperature and high pressures.

(B) number of particles decreases in lower energy levels at low temperatures and high pressures.

(C) number of particles increase in lower energy levels at high temperatures and low pressures.

(D) number of particles decreases in lower energy levels at high temperatures and low pressures.

99. The density of a quantum system with two microstates is given by $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. Choose the correct option.

(A) $\langle \sigma_z \rangle = -\frac{1}{3}, \langle \sigma_x \rangle = \frac{2}{3}$

(B) $\langle \sigma_z \rangle = \frac{1}{3}, \langle \sigma_y \rangle = \frac{2}{3}$

(C) $\langle \sigma_z \rangle = \frac{1}{3}, \langle \sigma_x \rangle = \frac{2}{3}$

(D) $\langle \sigma_z \rangle = \frac{1}{3}, \langle \sigma_x \rangle = -\frac{2}{3}$

100. Consider a system of N weakly interacting particles at sufficiently high enough temperature T. Suppose, the restoring force acting is proportional to x^5, then the heat capacity at temperature T will be

(A) $\frac{3}{2}NK_B$

(B) $\frac{3}{4}NK_B$

(C) $\frac{1}{3}NK_B$

(D) $\frac{2}{3}NK_B$

Space for Rough Work

Space for Rough Work

Space for Rough Work