Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series **X**

PAPER–II **Question Booklet No.**

Subject Code : 15

MATHEMATICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One Hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

Paper II
(Mathematical Sciences)

1. Let A be the set of all irrational numbers in the open interval (–1, 1). Then

(A) A has a maximum but not a minimum.
(B) A has both maximum and minimum.
(C) infimum of A = minimum of A.
(D) both Sup A and Inf A exist.

2. Let $W(t)$ be the Wronskian of two linearly independent solutions of the ordinary differential equation

$$2y'' + y' + t^2 y = 0,\ t \in \mathbb{R}.$$

Then, for all t, there exists a constant $C \in \mathbb{R}$ such that $W(t)$ is equal to

(A) Ce^{-t}
(B) $Ce^{-\frac{t}{2}}$
(C) Ce^{2t}
(D) Ce^{-2t}

3. The Fourier cosine transform of the function $F(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ is

(A) $\sqrt{\frac{2}{\pi}}\,\frac{\sin as}{s}$
(B) $\sqrt{\frac{1}{\pi}}\,\frac{\cos as}{s}$
(C) $\sqrt{\frac{1}{\pi}}\,\frac{\sin as}{s}$
(D) $\sqrt{\frac{1}{2\pi}}\,\frac{\sin as}{s}$

4. Under the transformation $w = f(z) = \left(\frac{z+i}{z-i} + 1\right)^2$ the lower half of the z-plane is mapped onto

(A) the upper half of the w-plane.
(B) a circular region in the w-plane.
(C) an elliptical region in the w-plane.
(D) a region of Cardioid shape in the w-plane.

5. Let $X = \left\{x = (x_j)_{j=1}^{\infty} \,\middle|\, x_j \in \mathbb{C}\ \forall j \text{ and there exists } N \text{ in } \mathbb{N} \text{ such that } x_j = 0\ \forall j \geq N\right\}$ be a normed linear space with the norm given by $\|x\|_\infty = \operatorname{Sup}_j |x_j|$.

Let $T : (X, \|\cdot\|_\infty) \to \mathbb{C}$ be given by $T(x) = \sum_{k=1}^{\infty} \frac{x_k}{k^2},\ x = (x_j)_{j=1}^{\infty}$.

Choose the correct option:

(A) $(X, \|\cdot\|_\infty)$ is a Banach space.
(B) T is a bounded linear functional on $(X, \|\cdot\|_\infty)$.
(C) There exists unique $y = (y_1, y_2, \ldots)$ in X such that
$$T(x) = \sum_{i=1}^{\infty} x_i \overline{y}_i,\ \forall x = (x_1, x_2, \ldots) \text{ in } X.$$
(D) There exists a y in X such that T is not continuous at y.

6. If k and τ are curvature and torsion of a geodesic and k_a and k_b are principal curvatures, then τ^2 is equal to

(A) $(k_a - k)(k_b + k)$
(B) $(k_a - k_b)\,k$
(C) $(k - k_a)(k - k_b)$
(D) $(k + k_a)(k + k_b)$

7. The spatial description of the velocity field that corresponds to the strain rate tensor

$$\begin{bmatrix} te^{tx} & 0 & 0 \\ 0 & 0 & te^{t}+1 \\ 0 & te^{y}+1 & 0 \end{bmatrix}$$ is given by

(A) $\begin{bmatrix} e^{tx}-t+1 \\ 2 \\ 2te^{y}-t \end{bmatrix}$

(B) $\begin{bmatrix} e^{tx}+t-1 \\ 2z \\ 2te^{y}-t \end{bmatrix}$

(C) $\begin{bmatrix} e^{tx}-1 \\ 2 \\ te^{y}+t \end{bmatrix}$

(D) $\begin{bmatrix} t-1 \\ 2z \\ 2e^{y}-t \end{bmatrix}$

8. The sum of residues of the function $f(z)=\dfrac{\sin z}{z\cos z}$ at its poles inside the circle $|z|=2$ is

(A) 0

(B) $\dfrac{2}{\pi}$

(C) $-\dfrac{2}{\pi}$

(D) π

9. What is the number of automorphism of the group $Z_2 \times Z_4$?

(A) 1

(B) 2

(C) 4

(D) 8

10. Let $a \equiv b \pmod n$ and the integers a, b, n be all divisible by $d > 0$. Then which of the following is *true*?

(A) $\dfrac{d}{a} \equiv \dfrac{b}{d}\left(\text{mod } \dfrac{n}{d}\right)$

(B) $\dfrac{a}{d} \equiv \dfrac{b}{d}\left(\text{mod } \dfrac{n}{d}\right)$

(C) $\dfrac{a}{d} \equiv \dfrac{d}{b}\left(\text{mod } \dfrac{n}{d}\right)$

(D) $\dfrac{d}{a} \equiv \dfrac{d}{b}\left(\text{mod } \dfrac{n}{d}\right)$

11. Laplace transform exists when

(i) the function is piecewise continuous.

(ii) the function is of exponential order.

(iii) the function is piecewise discrete.

(iv) the function is of differential order.

Which of the following option is correct?

(A) (i) and (ii) only

(B) (iii) and (iv) only

(C) (i) and (iv) only

(D) (ii) and (iii) only

12. Let A and B be two real $n \times n$ matrices such that det $(AB) \neq 0$. Then which of the following is *true*?

(A) AB and BA have different characteristic polynomials.

(B) AB and BA have different Jordan canonical forms.

(C) AB and BA have same minimal polynomials.

(D) AB and BA have different minimal polynomials.

13. Consider a Boolean function of n variables. The order of an algorithm that determines whether the Boolean function produces an output 1 is

(A) logarithmic

(B) linear

(C) quadratic

(D) exponential

14. Let G be a group of order 175. Consider the following statements:

(I) The group G is abelian.

(II) The group G has unique Sylow p-subgroups for each prime p dividing $O(G)$.

Choose the correct option:

(A) Statement (I) is true but statement (II) is false.

(B) Statement (I) is incorrect but statement (II) is true.

(C) Both the statements are false.

(D) Both statement (I) and statement (II) are correct.

15. The general solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} + y = f(x),\ x \in (-\infty, \infty),$$ where $f(x)$ is a continuous, real valued function on $(-\infty, \infty)$ is

(A) $y(x) = \cos(x+k) + C\int_0^x f(t)\sin(x-t)dt$

(B) $y(x) = A\cos x + B\sin x + \int_0^x f(x-t)\sin t\, dt$

(C) $y(x) = A\cos x + B\sin x + \int_0^x f(x+t)\cos t\, dt$

(D) $y(x) = A\cos x + B\sin x + \int_0^x f(t)\sin(x-t)\, dt$

16. The set $\{(x_1, x_2) : x_1 \geq 1,\ x_1 \geq x_2\}$ is

(A) a convex set.

(B) not a convex set.

(C) a bounded set.

(D) an open convex set.

17. Let A and B be any two $n \times n$ matrices and $tr(A) = \sum_{i=1}^{n} a_{ii}$ and $tr(B) = \sum_{i=1}^{n} b_{ii}$. Consider the following statements:

(I) $tr(AB) = tr(BA)$

(II) $tr(A+B) = tr(A) + tr(B)$

Which of the options given below is correct?

(A) (I) only

(B) (II) only

(C) Both (I) and (II)

(D) Both (I) and (II) are incorrect

18. The initial value problem

$\frac{\partial u}{\partial t}+x\frac{\partial u}{\partial x}=x,\ 0\le x\le 1,\ t>0$ and $u(x,0)=2x$

has

(A) a unique solution $u(x, t)$ which tends to ∞ as t tends to infinity.

(B) more than one solution.

(C) a solution which remains bounded as t tends to ∞.

(D) no solution.

19. The equation

$\int_0^x \frac{\phi(y)}{(x-y)^\alpha}dy=f(x),\ (0<\alpha<1)$ is referred as

(A) Fredholm equation

(B) Able equation

(C) Maxwell equation

(D) Picard's equation

20. The plane curve of fixed perimeter and maximum enclosed area is

(A) a rectangle

(B) a circle

(C) a square

(D) an oval

21. The extremal of the functional

$I[y(x)]=\int_0^1\left(y(x)-\frac{1}{2}(y'(x))^2\right)dx$ satisfying $y(0) = 0$ and $y(1) = 1$ is

(A) $\frac{x}{2}+\frac{x^2}{2}$

(B) $\frac{3x}{2}-\frac{x^2}{2}$

(C) $\frac{x^2}{2}-\frac{x}{2}$

(D) $\frac{3}{2}x^3-\frac{x^2}{2}$

22. In Newton-Cotes formula, if $f(x)$ is interpolated at equally spaced nodes by a polynomial of degree two, then it represents

(A) Trapezoidal rule

(B) Simpson rule

(C) Three-eight rule

(D) Booles rule

23. If the set $X=\{1,2,3\}$ and $\tau=\{\phi,\{1\},\{2,3\},X\}$ be the topology on X, then which of the following is *not* τ-closed?

(A) ϕ

(B) X

(C) $\{1, 2\}$

(D) $\{2, 3\}$

24. The number of elements in the symmetric group S_5 which have their own inverse is

(A) 25

(B) 24

(C) 27

(D) 26

25. Let $(X_1,\|\cdot\|_1)$ and $(X_2,\|\cdot\|_2)$ be normed linear spaces and let $X=X_1\times X_2$ be the product vector space. Which of the following does *not* define a norm on X? Here, $x\in X$ is of the form (x_1,x_2) where $x_1\in X_1, x_2\in X_2$.

(A) $\|x\|=\max\{\|x_1\|_1,\|x_2\|_2\}$

(B) $\|x\|=\{\|x_1\|_1{}^4+\|x_2\|_2{}^4\}^{1/4}$

(C) $\|x\|=\min\{\|x_1\|_1,\|x_2\|_2\}$

(D) $\|x\|=3\|x_1\|_1+5\|x_2\|_2$

26. In a fluid flow, which of the following have the same forces acting on them?

(A) Geometric similarity

(B) Conditional similarity

(C) Dynamic similarity

(D) Kinematic similarity

27. Let A, B be complex $n \times n$ matrices. Consider the following statements:

(I) If A, B and $A + B$ are invertiable, then $A^{-1} + B^{-1}$ is invertiable.

(II) If AB is nilpotent, then BA is nilpotent.

(III) $\left(A + B^{-1}\right)^2 = A^2 + \left(B^{-1}\right)^2$

Which of the following is *true*?

(A) (I) and (II)

(B) Only (I)

(C) Only (II)

(D) Only (III)

28. Which of the following sequences/series of functions are uniformly convergent on [0, 1]?

(A) $f_n(x) = \dfrac{1}{1+x^n}$

(B) $f_n(x) = (\cos \pi n! x)^n$

(C) $f(x) = \sum_{m=1}^{\infty} \dfrac{\cos(m^6 x)}{m^3}$

(D) $f_n(x) = n^2 x(1-x^2)^n$

29. The characteristic of the ideal fluid is

(A) compressible

(B) forced vortex flow

(C) fluid velocity is uniform

(D) viscous in nature

30. Let $T = \{x \in \mathbb{R} \mid x \text{ is transcendental}\}$ and let $S = \left\{ \sum_{n=1}^{\infty} \dfrac{x_n}{4^n} \mid x_n \in \{0, 2\} \right\}$. Then which of the following is *true*?

(A) Both S and T are countable sets.

(B) T is countable but S is uncountable.

(C) Both S and T are uncountable sets.

(D) T is uncountable but S is countable.

31. Consider the ordinary differential equation

$$x^2(2-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0.$$

Which of the following statements is correct?

(A) $x = 0$ is an ordinary point.

(B) $x = 2$ is the only singular point.

(C) $x = 0$ is a regular singular point and $x = 2$ is an ordinary point.

(D) $x = 0$ and $x = 2$ are regular singular points.

32. The normal curvature of the curve $u = a\sin\theta$, $v = a\cos\theta$ on the surface $r = (u, v, u^2 - v^2)$ at the origin for $\theta = \pi/6$ is

(A) 0

(B) 1

(C) 2

(D) 4

33. What is the number of non-singular 3×3 matrices over F_2, the finite field with two elements?

(A) 168

(B) 384

(C) 2^3

(D) 3^2

34. The integral, $\int\limits_{|z|=2\pi} \frac{z^2 \sin z}{(z-\pi)^3} dz$, is equal to

(A) $4\pi i$

(B) $-4\pi i$

(C) $-4\pi^2 i$

(D) $4\pi^2 i$

35. The radius of convergence of series $\sum_{n=0}^{\infty} \frac{x^n + n}{\lfloor n}$ and $\sum_{n=0}^{\infty} \lfloor n \, x^n$ respectively are

(A) ∞, 0

(B) 1, 1

(C) 1, 0

(D) 0, ∞

36. Which of the following is *not* true?

(A) $\{1, 2x, 3x^2, ...\}$ is a linearly independent set in $C[0, 1]$.

(B) Set of all Hermite polynomials forms an orthogonal set.

(C) Set of all Legendre polynomials forms an orthogonal set.

(D) Orthonormal sets in a Hilbert space are always enumerable.

37. The eigenvalues of boundary value problem

$\frac{d^2 y}{dx^2} + \lambda y = 0, x \in (0, \pi), \lambda > 0,$

$y(0) = 0, y(\pi) - y'(\pi) = 0$ are given by

(A) $\lambda = (n\pi)^2$, $n = 1, 2, 3, ...$

(B) $\lambda = n^2$, $n = 1, 2, 3, ...$

(C) $\lambda = k_n^2$, where $k_n, n = 1, 2, 3, ...$ are the roots of $k = \tan(k\pi)$.

(D) $\lambda = k_n^2$, where $k_n, n = 1, 2, 3, ...$ are the roots of $k + \tan(k\pi) = 0$.

38. What is the number of abelian groups of order 1800 up to isomorphism?

(A) 1

(B) 3

(C) 4

(D) 12

39. If an entire function $f(z)$ has a pole of order n at infinity, then

(A) $f(z)$ is a constant.

(B) $f(z)$ is a polynomial of degree n.

(C) $f(z)$ is a polynomial of degree $<n$.

(D) $f(z)$ is a transcendental entire function.

40. A general solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2} = 0$ is of the form,

(A) $z = f(y + ax) + g(y - ax)$.

(B) $z = f(y - ax) + g(x)$.

(C) $z = f(y + x) + ag(x - y)$.

(D) $z = f(y + ax) + g(y)$.

41. Which of the following statements is *true*?

(A) Let $f : (0, 1) \to (0, 1)$ be such that $|f(x) - f(y)| \le \frac{1}{2} |x - y|$ for all $x, y \in (0, 1)$, then f has a fixed point.

(B) Let $f : [-1, 1] \to [-1, 1]$ be continuous, then f has a fixed point.

(C) Let $f : [0, 1] \to [3, 4]$ be continuous, then f has a fixed point.

(D) Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and periodic with period $T > 0$, then there exists no point $x_0 \in \mathbb{R}$ *s. t.*

$f(x_0) = f\left(x_0 + \frac{T}{2}\right)$.

42. Consider a particle of unit mass falling freely from rest under gravity with velocity v. If the air resistance retards the acceleration by cv, where c is a constant, then

(A) $v = \frac{g}{c}\left[1 + e^{ct}\right]$.

(B) $v = \frac{g}{c}\left[1 + e^{-ct}\right]$.

(C) $v = \frac{g}{c}\left[1 - e^{-ct}\right]$.

(D) $v = \frac{g}{c}\left[1 - e^{ct}\right]$.

43. Which of the following real-valued function defined on open interval (0, 1) is uniformly continuous?

(A) $f(x) = \cos\left(\frac{1}{x}\right)$

(B) $f(x) = \frac{\sin x}{x}$

(C) $f(x) = \frac{1}{x}$

(D) $f(x) = \frac{1}{x}\ \sin\frac{1}{x}$

44. Let p be a prime number and n be a natural number. The order of the Galois group of the polynomial $x^{p^n} - 1$ over the finite field F_p is

(A) $p - 1$

(B) 1

(C) p

(D) n

45. Which of the following is *not* true?

(A) Products of Hausdorff spaces are Hausdorff.

(B) Products of compact spaces are compact.

(C) Products of regular spaces are regular.

(D) Products of normal spaces are normal.

46. In which of the following cases, there is no differentiable function f from the interval S onto the interval T?

(A) $S = [0, 1], T = \mathbb{R}$

(B) $S = (0, 1), T = \mathbb{R}$

(C) $S = (0, 1), T = (0, 1)$

(D) $S = \mathbb{R}, T = (0, 1]$

47. The function $\phi(x) = 1 + \lambda x$ is a solution of the integral equation $x = \int_0^x e^{x-\xi}\phi(\xi)d\xi$ if

(A) $\lambda = -1$

(B) $\lambda = 1$

(C) $\lambda = -2$

(D) $\lambda = 2$

48. Let W_s and W_q be the expected waiting time in system and queue and L_s and L_q be the expected number of customers in system and queue, then

(A) $\frac{L_s}{W_s} < \frac{L_q}{W_q}$

(B) $\frac{L_s}{W_s} > \frac{L_q}{W_q}$

(C) $\frac{L_s}{W_s} = \frac{L_q}{W_q}$

(D) $\frac{L_s}{W_s} + \frac{L_q}{W_q} = 1$

49. Let $L = \int_0^1 \frac{dx}{1+x^4}$, then

(A) $L = 1$

(B) $L = \frac{\pi}{4}$

(C) $\frac{\pi}{4} < L < 1$

(D) $0 < L < \frac{\pi}{4}$

50. The values of a, b, c such that

$$\int_0^h f(x)dx = h\left[af(0) + bf\left(\frac{h}{2}\right) + cf(h)\right]$$

is exact for polynomials $f(x)$ of degree two are

(A) $a = 0, b = \frac{1}{2}, c = \frac{1}{3}$

(B) $a = 0, b = \frac{2}{3}, c = \frac{1}{6}$

(C) $a = \frac{1}{4}, b = \frac{1}{2}, c = \frac{1}{4}$

(D) $a = 0, b = \frac{1}{2}, c = \frac{1}{2}$

51. Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of bounded faces in any embedding of G on the plane is equal to

(A) 6

(B) 5

(C) 4

(D) 3

52. For a metric space X, which of the following is correct?

(A) If X is countably compact, then X need not be compact.

(B) If X is compact, then X need not be countably compact.

(C) If X is compact, then X need not be sequentially compact.

(D) If X is compact, then X need not be connected.

53. If $f(z)$ is analytic in the region $|z| \le R$ and $|f(z)| \le M$ on $|z| = R$, then

(A) $f'(0) = 0$

(B) $|f'(0)| \le \frac{M}{R}$

(C) $|f'(0)| \le M R$

(D) $|f'(0)| \le M$

54. Which of the following statements is *false*?

(A) In a metric space (X, d) every convergent sequence is a Cauchy sequence.

(B) In a metric space (X, d) a Cauchy sequence may not be convergent.

(C) In a metric space (X, d) containing infinite number of elements, for any two distinct elements $x, y \in X$, there exists a point z in X such that $d(x,z) = d(z,y) = \frac{1}{2}d(x,y)$.

(D) For a metric space (X, d), the set X consists of one point if any bounded sequence in X is convergent.

55. For $E \subset \mathbb{R}$ and $a \in \mathbb{R}$, define
$E + a = \{x + a : x \in \mathbb{R}\}$ and
$aE = \{ax : x \in \mathbb{R}\}$,
Which of the following is correct?

(A) $m(\pi(E+e)) = \pi(m(E)) + e$
(B) $m(\pi(E+e)) = m(E) + \pi e$
(C) $m(\pi(E+e)) = \pi m(E)$
(D) $m(\pi(E+e)) = m(\pi E) + e$

where m is the Lebesgue measure.

56. The function $f(z) = \left(\dfrac{e^z - 1}{2}\right)^2$, $z \in \mathbb{C}\setminus\{0\}$ has

(A) a removable singularity at $z = 0$.
(B) a simple pole at $z = 0$.
(C) a simple pole at $z = 1$.
(D) a pole of order two at $z = 2$.

57. For $n \in \mathbb{N}$, define

$$f_n(x) = \begin{cases} \dfrac{nx}{1+n^2x^2}, & \text{if } x \text{ is irrational} \\ n, & \text{if } x \text{ is rational} \end{cases}$$ and let

$\alpha = \lim_{n\to\infty} \int_0^1 f_n(x)dx$. Then

(A) each f_n is Lebesgue integrable and $\alpha > 0$.
(B) not all f_n's are Lebesgue integrable and $\alpha = 0$.
(C) each f_n is Lebesgue integrable and $\alpha = 0$.
(D) not all f_n's are Lebesgue integrable and $\alpha > 0$.

58. Let A be a connected open subset of $\mathbb{R}^2$. Then the number of continuous surjective functions from closure of A in $\mathbb{R}^2$ to the set of rationals Q is

(A) 1
(B) 0
(C) 2
(D) infinite

59. What is the total variation of $f(x) = \sin 2x$ on $[0, 2\pi]$?

(A) 1
(B) 2
(C) 4
(D) 8

60. Let A be a 3 × 3 real matrix having eigenvalues 1, 0 and 1. Then the determinant of the matrix $I + A^{2022}$ is

(A) 4
(B) 5
(C) 0
(D) 2022

61. Let $\mathbb{C}$ be the field of complex numbers. Then the vectors $\alpha = (a_1, a_2)$, $\beta = (b_1, b_2)$ in $V_2(\mathbb{C})$, the two-dimensional vector space over $\mathbb{C}$, are linearly dependent if

(A) $a_1 b_1 = a_2 b_2$
(B) $a_1 a_2 = b_1 b_2$
(C) $a_1 b_2 + a_2 b_1 = 0$
(D) $a_1 b_2 - a_2 b_1 = 0$

62. The series $\dfrac{x}{1+x} - \dfrac{x^2}{1+x^2} + \dfrac{x^3}{1+x^3} - \dfrac{x^4}{1+x^4} + \cdots$ is

(A) divergent for all $x > 1$.
(B) uniformly convergent for $0 \le x < 1$.
(C) not uniformly convergent for $0 \le x < 1$.
(D) divergent for $0 \le x < 1$.

63. A particle of mass m is moving under an inverse square central force $\frac{\mu}{r^2}$ (μ, a constant). Then the Lagrangian and Hamiltonian of the particle are

(A) $L=\frac{m}{2}(\dot{r}^2+r^2\dot{\theta}^2)+\frac{\mu}{r},\ H=\frac{1}{2m}\left[p_r^2+\frac{p_\theta^2}{r^2}\right]-\frac{\mu}{r}$

(B) $L=\frac{m}{2}(\dot{r}^2+r^2\dot{\theta}^2)-\frac{\mu}{r},\ H=\frac{1}{2m}\left[p_r^2+\frac{p_\theta^2}{r^2}\right]-\frac{\mu}{r}$

(C) $L=\frac{m}{2}(\dot{r}^2+r^2\dot{\theta}^2)+\frac{\mu}{r},\ H=\frac{1}{2m}\left[p_r^2+\frac{p_\theta^2}{r^2}\right]+\frac{\mu}{r}$

(D) $L=\frac{m}{2}(\dot{r}^2+r^2\dot{\theta}^2)-\frac{\mu}{r},\ H=\frac{1}{2m}\left[p_r^2+\frac{p_\theta^2}{r^2}\right]+\frac{\mu}{r}$

64. The dual of the Linear Programming problem Minimize $c^T x$ subject to $Ax \geq b$ and $x \geq 0$ is

(A) Maximize $b^T w$ subject to $A^T w \geq c$ and $w \geq 0$.

(B) Maximize $b^T w$ subject to $A^T w \leq c$ and $w \geq 0$.

(C) Maximize $b^T w$ subject to $A^T w \leq c$ and w is unrestricted.

(D) Maximize $b^T w$ subject to $A^T w \geq c$ and w is unrestricted.

65. Which of the following permutations is an even permutation?

(A) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 4 & 5 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 4 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & 2 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \end{pmatrix}$

66. Suppose that we have a data set consisting of 25 observations, where each value is either 0 or 1. Consider the following statements:

(i) The mean of the data cannot be larger than the variance.

(ii) The mean of the data cannot be smaller than the variance.

(iii) The mean being same as the variance implies that the mean is zero.

(iv) The variance will be 0 if and only if the mean is either 1 or 0.

Which of the above statements is/are *true*?

(A) (i) only

(B) (i) and (iii) only

(C) (i), (iii) and (iv) only

(D) (ii), (iii) and (iv) only

67. In observing height (x) and weight (y) of some individuals, one gets measurements on them along with some error, i.e., $x' = x + \varepsilon_x$ and $y' = y + \varepsilon_y$ are observed, where the two error terms are uncorrelated and also ε_x is uncorrelated with x and ε_y is uncorrelated with y. Then the correlation coefficient between x' and y' is numerically

(A) smaller than that between x and y.

(B) greater than that between x and y.

(C) equal to that between x and y.

(D) smaller in some situations and greater in other situations than that between x and y.

68. A box contains np white and nq black balls, where $p + q = 1$. It is required to obtain the probability that exactly $(x + k)$ draws will be required to produce 'k' white balls. Which of the following would be applicable here?

(A) Binomial distribution

(B) Geometric distribution

(C) Multinomial distribution

(D) Negative Binomial distribution

69. Suppose $X_1, X_2, \ldots, X_n$ are *i.i.d.* observations from U (0, 1) distribution and let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the corresponding order statistics. Then for $r < s$, the correlation coefficient between $X_{(r)}$ and $X_{(s)}$ is

(A) $\left(\frac{n-s}{n-r}\right)^{\frac{1}{2}}$

(B) $\left(\frac{n-s+1}{n-r+1}\right)^{\frac{1}{2}}$

(C) $\left\{\frac{r(n-s+1)}{s(n-r+1)}\right\}^{\frac{1}{2}}$

(D) $\left\{\frac{r(n-s)}{s(n-r)}\right\}^{\frac{1}{2}}$

70. The points of discontinuity of the distribution function $F(\cdot)$

(A) do not always exist.
(B) form an uncountable set.
(C) form atmost a countable set.
(D) cannot be determined.

71. Suppose X and Y are random variables with $E(X) = E(Y) = 0$, $Var(X) = Var(Y) = 1$ and $Cov(X,Y) = 0{\cdot}25$. Then which of the following is/are always *true*?

(a) $P\{|X+2Y| \geq 4\} \geq \frac{4}{16}$

(b) $P\{|X+2Y| \geq 4\} \geq \frac{5}{16}$

(c) $P\{|X+2Y| \geq 4\} \leq \frac{6}{16}$

(d) $P\{|X+2Y| \geq 4\} = \frac{7}{16}$

(A) (a) only
(B) (b) only
(C) (b), (c) and (d) only
(D) (c) only

72. Let X_1, X_2, X_3 and X_4 be i.i.d. random variables having a continuous distribution.

Then $P\{X_3 < X_2 < max(X_1, X_4)\}$ equals

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

73. A and B have, respectively, $(n+1)$ and n coins. They toss all their coins simultaneously. Then the probability that A will have more heads then B is

(A) $\frac{n+1}{2n+1}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{n}$

(Assume all the coins are fair.)

74. Which of the following statements is correct?

(A) An extremely small p-value indicates that the actual data differ markedly from that expected, if the null hypothesis were true.
(B) The p-value measures the probability that the null hypothesis is true.
(C) The p-value measures the probability of making a type II error.
(D) The larger the p-value, the stronger the evidence against the null hypothesis.

75. A decision rule δ is said to be admissible if

(A) δ is as good as any other rule.

(B) δ is better than some other rule.

(C) there exists no other rule which is as good as δ.

(D) there exists no other rule which is better than δ.

76. For a sequential probability ratio test of strength (α, β), for testing $H_0 : \theta = \theta_0$, against $H_1 : \theta = \theta_1$, if $Z = ln \dfrac{f(X, \theta_1)}{f(X, \theta_0)}$ and $L(\theta)$ denotes the OC function, where $f(X, \theta)$ is the density of X under θ, the ASN function is given by

(A) $$\frac{[1 - L(\theta)]\, ln \dfrac{1-\beta}{\alpha} + L(\theta)\, ln \dfrac{\beta}{1-\alpha}}{E_\theta(Z)}$$

(B) $$\frac{[1 - L(\theta)]\, ln \dfrac{\beta}{1-\alpha} + L(\theta)\, ln \dfrac{1-\beta}{\alpha}}{E_\theta(Z)}$$

(C) $$\frac{L(\theta)\, ln \dfrac{1-\alpha}{\beta} + [1 - L(\theta)]\, ln \dfrac{\alpha}{1-\beta}}{E_\theta(Z)}$$

(D) $$\frac{L(\theta)\, ln \dfrac{\alpha}{1-\beta} + [1 - L(\theta)]\, ln \dfrac{1-\alpha}{\beta}}{E_\theta(Z)}$$

77. Suppose $X_1, X_2, ..., X_n$ are *i.i.d.* $N(\mu, \sigma^2)$, where both μ and σ^2 are unknown. Consider the following statements:

(a) Uniformly most accurate confidence interval for σ^2 with confidence coefficient $(1-\alpha)$ exists.

(b) Shortest length confidence interval for μ with confidence coefficient $(1-\alpha)$ exists.

(c) Fixed-length confidence interval for μ with confidence coefficient $(1-\alpha)$ exists.

(d) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$ is minimal sufficient for (μ, σ^2).

Which of the following statements is/are *true*?

(A) (a) and (c) only

(B) (d) only

(C) (b) and (d) only

(D) (b), (c) and (d) only

78. The probability mass functions $f_0(x)$ and $f_1(x)$ of a discrete random variable X under H_0 and H_1, respectively, are given in the following table:

x	1	2	3	4	5	6
$f_0(x)$	0·01	0·01	0·01	0·01	0·01	0·95
$f_1(x)$	0·05	0·04	0·03	0·02	0·01	0·85

The best test of size 0·03 for testing $H_0 : X \sim f_0$ against $H : X \sim f_1$ has power

(A) 0·09

(B) 0·12

(C) 0·14

(D) 0·86

79. Let $X_1, X_2, ..., X_n$ be i.i.d. random variables having $N(\mu, \sigma^2)$ distribution; $\mu \in R, \sigma^2 > 0$. Define

$$W = \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(X_i - X_j)^2 .$$

Then W, as an estimator of σ^2, is

(A) biased and consistent.

(B) unbiased and consistent.

(C) biased and inconsistent.

(D) unbiased and inconsistent.

80. Suppose $X_1, X_2, ...$ is a sequence of i.i.d. random variables with common variance $\sigma^2 > 0$. Let $Y_n = \frac{1}{n}\sum_{i=1}^{n} X_{2i-1}$ and $Z_n = \frac{1}{n}\sum_{i=1}^{n} X_{2i}$.

Then the asymptotic distribution (as $n \to \infty$) of $\sqrt{n}\left(Y_n - Z_n\right)$ is

(A) $N(0, 1)$

(B) $N(0, \sigma^2)$

(C) $N(0, 2\sigma^2)$

(D) degenerate at 0.

81. $\{X_n\}$ is a sequence of uniformly bounded and independent random variables. Define

$s_n^2 = \sum_{i=1}^{n} Var(X_i)$.

Then

(A) CLT holds if and only if $s_n^2 \to \infty$ as $n \to \infty$.

(B) CLT holds if $s_n^2 \to \infty$ as $n \to \infty$.

(C) CLT holds if and only if s_n^2 remains bounded as $n \to \infty$.

(D) CLT holds only if s_n^2 remains bounded as $n \to \infty$.

82. In ANOVA, if observed F-statistic's value is less than unity for different levels of any effect, which of the following is *true*?

(A) The null hypothesis is always accepted.

(B) The null hypothesis is always rejected.

(C) Decision on acceptance/rejection of null hypothesis depends on the tabulated F-value.

(D) No conclusion can be made.

83. For a BIBD (10, 3; 6, 5; 2), the number of plots in the design is

(A) 15

(B) 6

(C) 10

(D) 30

84. Suppose that we are carrying out a 2^5 factorial experiment. Each replication is divided into 2^3 blocks. It is known that AD, BE and ABC are confounded along with the generalised interactions. The treatments in the control block/principal block are

(A) (1), acd, bce, abde

(B) (1), ad, be, abc

(C) (1), abde, bcd, ace

(D) (1), bcd, ace, cde

85. In a Randomised Block Design with 'p' levels of treatment, 'q' blocks and 'm' observations per cell, the d.f. (degrees of freedom) due to error is

(A) $(p - 1)(q - 1)(m - 1)$

(B) $pq(m - 1)$

(C) $(p - 1)(q - 1)m$

(D) $pqm - 1$

86. Consider a stratified random sampling scheme with cost function

$$C = a + \sum_{i=1}^{k} c_i n_i$$

(Symbols have their usual meaning)

Neyman's allocation holds good if the following is satisfied:

(A) $S_1 = S_2 = S_3 = \dots = S_k$

(B) $n_1 = n_2 = \dots = n_k$

(C) $N_1S_1 = N_2S_2 = \dots = N_kS_k$

(D) $C_1 = C_2 = \dots = C_k$

87. In survey sampling, the regression estimator and the ratio estimator are equally good and are in fact identical if

(A) the regression of y on x is a straight line through (0, 0).

(B) the regression of y on x is a straight line with a non-zero intercept.

(C) the regression co-efficient is 0.

(D) the regression co-efficient is 1.

88. In estimating population mean, the ratio estimator will be more efficient than the ordinary estimator $\overline{y}$ under SRSWOR if

(A) $\rho < \frac{1}{2}\frac{C_x}{C_y}$

(B) $\rho = \frac{1}{2}\frac{C_x}{C_y}$

(C) $\rho > \frac{1}{2}\frac{C_x}{C_y}$

(D) $\frac{S_x}{S_y} > \frac{1}{2}\frac{C_x}{C_y}$

89. Two judges rank four competitors as follows:

Judge	Competitor			
	1	2	3	4
X	3	4	2	1
Y	3	1	4	2

For testing independence of rankings of the two judges, Kendall's t coefficient has the value

(A) – 0·33

(B) – 0·10

(C) – 0·23

(D) 0·33

90. Suppose $A_1 \sim W_p(n_1, \Sigma), A_2 \sim W_p(n_2, \Sigma)$ and A_1 and A_2 are independent. Also suppose $\lambda_1(A)$ and $\lambda_p(A)$ denote, respectively, the largest and the smallest characteristic root of the matrix A. Then Wilk's Λ statistic is defined as

(A) $\Lambda = |A_1|(|A_1+A_2|)^{-1}$

(B) $\Lambda = |A_1|(|A_1|+|A_2|)^{-1}$

(C) $\Lambda = \lambda_p(A_1(A_1+A_2)^{-1})$

(D) $\Lambda = \lambda_1(A_1(A_1+A_2)^{-1})$

91. $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ are two p-component random vectors such that $\underset{\sim}{X} \sim N_p(0, \Sigma)$, $\underset{\sim}{Y} \sim N_p(\underset{\sim}{0}, \Sigma^{-1})$ and $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ are not necessarily independent. Then $var(\underset{\sim}{X}'\underset{\sim}{Y})$ equals

(A) $tr(\Sigma)$

(B) $tr(\Sigma^{-1})$

(C) p

(D) $2p$

92. Consider the linear model

$y_1 = \theta_1 + 2\theta_2 - 2\theta_3 + \varepsilon_1$

$y_2 = \theta_1 + 3\theta_2 - \theta_3 + \varepsilon_2$

$y_3 = \theta_2 + \theta_3 + \varepsilon_3$

where y_i are observations, θ_i are parameters and ε_i are uncorrelated random variables with mean zero and constant variance for $i = 1,2,3$. Then, which of the following is *true*?

(A) $2y_1 - y_2 - y_3$ is an unbiased estimator of $\theta_1 - 4\theta_3$.

(B) $2y_1 - y_2 + y_3$ is the BLUE of $\theta_1 - 4\theta_3$.

(C) $y_2 - 3y_3$ is the BLUE of $\theta_1 - 4\theta_3$.

(D) $y_1 - 4y_3$ is an unbiased estimator of $\theta_1 - 4\theta_3$.

93. Consider a linear model $E(Y) = X\beta$, $Var\ (Y) = \sigma^2 \cdot I$, where X is the design matrix of order $n \times p$ $(n > p)$ having rank $r \le p$. Then, which of the following statements is necessarily *true*?

(A) The set of estimable linear functions form a vector space of dimension r.

(B) If $E(c'Y) = 0$ for some non-zero vector c, then there exists a function $l'\beta$ which is not estimable.

(C) If all linear functions $l'\beta$ are estimable, then $r \neq p$.

(D) The set of functions $c'Y$ with $E(c'Y) = 0$ form a vector space with dimension r.

94. Which one of the following is *true*?

(The functions are defined in the context of a life-table)

(A) $T_x = \frac{1}{2} l_x + \sum_{t=0}^{\infty} l_{x+t}$

(B) $T_x = \sum_{t=0}^{\infty} l_{x+t}$

(C) $T_x = \frac{1}{2} l_x + \sum_{t=1}^{\infty} l_{x+t}$

(D) $T_x = \sum_{t=1}^{\infty} l_{x+t}$

95. Crude Birth Rate (CBR) is to be considered as

(A) ratio

(B) probability

(C) rate

(D) All of the above

96. A three-unit parallel system has independent components with reliabilities 0·2, 0·3 and 0·4, respectively. The reliability of the system is

(A) 0·886

(B) 0·664

(C) 0·336

(D) 0·024

97. In a 3σ-control chart, if the quality characteristic follows the normal distribution, then the probability that the subgroup quality characteristic will be between two limits under control situation is

(A) 0·9793

(B) 0·9379

(C) 0·9739

(D) 0·9973

98. Let $\varepsilon_t, \varepsilon_{t+1}, \ldots$ and ξ be independent variables with zero mean and unit variance. Suppose $U_t = a\,\xi + \varepsilon_t, -\infty < t < \infty$. The process is

(A) stationary

(B) non-stationary

(C) oscillatory

(D) evolutionary

99. Trend value of a time series of each timepoint is not available in use of the method of

(A) Graphical Approach

(B) Semi-Averages

(C) Least Squares

(D) Moving Averages

100. For a transition probability matrix P, sum of the entries of each row of P^4 is

(A) 0

(B) 1

(C) 2

(D) 4

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work