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(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series

Y

PAPER-II

Question Booklet No.

Subject Code : 16

PHYSICAL SCIENCES

Time : 2 Hours

Maximum Marks: 200

Instructions for the Candidates

- Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
- At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
 - To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
 - Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
 - Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
 - After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
- This paper consists of One Hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
- Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.
Example: (A) (B) (C) (D) , where (C) is the correct response.
- Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
- Rough work is to be done at the end of this booklet.
- If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
- Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
- You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
- Use only **Black Ball point pen**.
- Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.
- There is no negative marks for incorrect answer.

PAPER II
(PHYSICAL SCIENCES)

1. The initial and final temperatures of a water bath are $(14 \pm 0.5)^\circ\text{C}$ and $(38 \pm 0.3)^\circ\text{C}$. The rise in temperature of the bath is

- (A) $(24 \pm 0.2)^\circ\text{C}$
- (B) $(24 \pm 0.8)^\circ\text{C}$
- (C) 24°C
- (D) $(24 \pm 0.4)^\circ\text{C}$

2. The hexadecimal equivalent of $(756.603)_8$ is

- (A) $(B9F.AE)_{16}$
- (B) $(1EE.C18)_{16}$
- (C) $(2BC.D28)_{16}$
- (D) $(1DD.E42)_{16}$

3. When a substance undergoes a phase transition at constant temperature (T) and pressure (P), its chemical potential (μ)

- (A) increases.
- (B) decreases.
- (C) remains constant.
- (D) depends on the density of the substance.

4. In Young's double slit experiment, the widths of the slits are in the ratio 9 : 1, then the ratio of the intensities between central maxima and first minima in the interference pattern is

- (A) 1 : 3
- (B) 3 : 1
- (C) 1 : 4
- (D) 4 : 1

5. An optical pyrometer gives an output voltage proportional to the incident radiation. When used to measure the temperature of a blackbody at 600K, it reads 1.5V. If the temperature changes by 2K, then the change in output signal will be

- (A) 12.5 mV
- (B) 15.6 mV
- (C) 20 mV
- (D) 24.8 mV

6. The imperfect differential

$$\vec{d}q = ydx + 3x dy$$

has an integrating factor $f(x, y)$ $\vec{d}q$ where $f(x, y)$ equals to

- (A) xy
- (B) y^2
- (C) y^3
- (D) x/y

7. The density of Bose-Einstein condensate

- (A) increases with temperature as $T \rightarrow T_c$ from below.
- (B) remains constant.
- (C) may increase or decrease depending on nature of transition.
- (D) decreases with temperature as $T \rightarrow T_c$ from below.

8. A man of 90 kg weight jumps from a height of 2 m so that his center of mass moves by 1 cm. If the area of the contact of bone at each ankle is 5 cm^2 , then the force per unit area will be (Take $g = 10 \text{ m/s}^2$)

- (A) $2.4 \times 10^4 \text{ N/m}^2$
- (B) $4.8 \times 10^5 \text{ N/m}^2$
- (C) $2.4 \times 10^6 \text{ N/m}^2$
- (D) $1.8 \times 10^8 \text{ N/m}^2$

9. A particle of unit mass moves under conservative force field in such a way that $\dot{x}(t) = y(t)$ and $\dot{y}(t) = -x(t)$. The potential energy $V(x, y)$ associated with the particle is

- (A) 0
- (B) $\frac{1}{2}(x^2 - y^2)$
- (C) $\frac{1}{2}(x^2 + y^2)$
- (D) $x^2 + y^2$

10. Three matrices M_1, M_2, M_3 satisfy the relation

$$M_i M_j = -\delta_{ij} I$$

where $i, j = 1, 2, 3$ and I is the unit matrix. The eigenvalues of these matrices must be

- (A) 0, 1
- (B) 1, -1
- (C) $i, -i$
- (D) 1, i

11. Consider a paramagnetic salt containing ions with $\mu_{mj} = \pm\mu_B$. If 60% of these ions are in the lowest energy state, then the required magnetic field B to be applied at temperature T will be

- (A) $\frac{K_B T}{\mu_B} \ln\left(\frac{4}{3}\right)$
- (B) $\frac{K_B T}{2\mu_B} \ln\left(\frac{3}{2}\right)$
- (C) $\frac{K_B T}{2\mu_B} \ln\left(\frac{3}{4}\right)$
- (D) $\frac{K_B T}{\mu_B} \ln\left(\frac{6}{5}\right)$

12. The total energy stored in electromagnetic field is

- (A) $\frac{\epsilon_0}{2} \int E^2 dV$
- (B) $\frac{1}{2\mu_0} \int B^2 dV$
- (C) $\frac{1}{2} \int \left[\epsilon_0 E^2 + \left(\frac{1}{\mu_0} \right) B^2 \right] dV$
- (D) $\frac{\epsilon_0 \mu_0}{2} \int E^2 B^2 dV$

13. The Optical Theorem relates

- (A) the real and imaginary parts of the wavefunction.
- (B) the forward and backward scattering amplitudes.
- (C) the scattering cross-section to the scattering amplitude.
- (D) the scattering cross-section to the imaginary part of the forward scattering amplitude.

14. Total energy of the electron in the hydrogen atom in ground state is -13.6 eV. The kinetic energy of this electron is

- (A) 6.8 eV
- (B) 13.6 eV
- (C) -6.8 eV
- (D) -27.2 eV

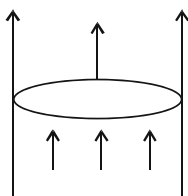
15. The probability that an energy level $3K_B T$ above the Fermi energy occupied by an electron at $T = 300$ K is (assume $e \approx 2.718$)

- (A) 3.82%
- (B) 2.86%
- (C) 4.74%
- (D) 5.64%

16. Normal Zeeman effect occurs for spectral lines from transitions between

- (A) Singlet-Singlet electronic states.
- (B) Doublet-Doublet electronic states.
- (C) Triplet-Triplet electronic states.
- (D) Singlet-Doublet electronic states.

17. A uniform current I is flowing along the surface of a hollow conducting cylinder of radius ' a ' parallel to its axis. The magnetic field inside the cylinder is



- (A) $\frac{\mu_0 I}{2\pi a}$
- (B) zero
- (C) I/a
- (D) $I/\pi a^2$

18. If the momentum of an electron moving with a velocity $0.9c$ is increased by 1%, then the increase in its energy is

- (A) 1%
- (B) 0.9%
- (C) 0.81%
- (D) 0.5%

19. A particle of mass m is moving in a potential $V(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4$, where a, b are positive constants. Frequency of small oscillations about a point of stable equilibrium is

- (A) $\sqrt{\frac{3a}{2m}}$
- (B) $\sqrt{\frac{a}{m}}$
- (C) $\sqrt{\frac{2a}{m}}$
- (D) $\sqrt{\frac{2a}{3m}}$

20. A relativistic particle moves with a constant velocity ' v ' with respect to the laboratory frame. In time τ , measured in the last frame of the particle, the distance that it travels in the laboratory frame is

- (A) $v\tau$
- (B) $v\tau\sqrt{1-v^2/c^2}$
- (C) $\frac{c\tau}{\sqrt{1-v^2/c^2}}$
- (D) $\frac{v\tau}{\sqrt{1-v^2/c^2}}$

21. Half Life of a Radioactive material is 4 days. After 20 days, the fraction remaining undecayed will be

- (A) $\frac{1}{8}$
- (B) $\frac{1}{16}$
- (C) $\frac{1}{32}$
- (D) $\frac{1}{10}$

22. The timebase of a CRO is controlled by

- (A) square waveform
- (B) sine waveform
- (C) saw-tooth waveform
- (D) staircase waveform

23. A cavity resonator can be represented by

- (A) an L-C circuit.
- (B) an L-C-R circuit.
- (C) a lossy inductor.
- (D) a lossy capacitor.

24. The dispersion relation of phonons in a solid is

$$\omega^2(k) = \omega_0^2 (3 - \cos k_x a - \cos k_y a - \cos k_z a).$$

Then, the velocity of phonons at long wavelength limit ($k \rightarrow 0$) is

- (A) $\frac{\omega_0 a}{\sqrt{3}}$
- (B) $\frac{2\omega_0 a}{\sqrt{3}}$
- (C) $\sqrt{3} \omega_0 a$
- (D) $\frac{\omega_0 a}{\sqrt{2}}$

25. The ground state of the deuteron is

- (A) pure 3S_1 state.
- (B) pure 3P_1 state.
- (C) mixture of 3S_1 and 3P_1 states.
- (D) mixture of 3S_1 and 3D_1 states.

26. The Compton Effect proves that

- (A) photons are relativistic particles.
- (B) photons carry quantized energy.
- (C) photons carry quantized linear momentum.
- (D) photons carry quantized angular momentum.

27. A rigid body is rotating about an axis fixed in space. Total degrees of freedom it has

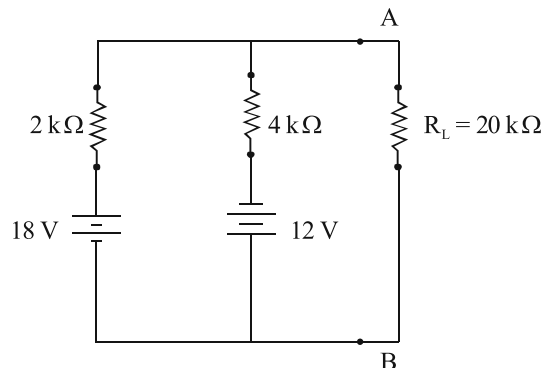
- (A) 6
- (B) 3
- (C) 2
- (D) 1

28. The density of a thin rod of length l varies with the distance x from one end as

$\rho = \rho_0 \frac{x^2}{l^2} (\rho_0 > 0)$. Then, the position of the center of mass is

- (A) $\frac{l}{2}$
- (B) $\frac{l}{3}$
- (C) $\frac{2l}{3}$
- (D) $\frac{3l}{4}$

29. The Thevenin equivalent voltage (V_{TH}) and Thevenin equivalent resistance (R_{TH}) of the following circuits are respectively



- (A) $8V, \frac{4}{3}k\Omega$
- (B) $6V, \frac{2}{3}k\Omega$
- (C) $30V, 24k\Omega$
- (D) $15V, \frac{1}{3}k\Omega$

30. According to the Shell Model, the ground state of ${}_8\text{O}^{15}$ nucleus is

- (A) $\frac{3^-}{2}$
- (B) $\frac{3^+}{2}$
- (C) $\frac{1^+}{2}$
- (D) $\frac{1^-}{2}$

31. The vibrational frequencies associated with the fundamental and first overtone transitions are centered at 1870 cm^{-1} and 3720 cm^{-1} respectively. Considering the oscillator to be anharmonic, the anharmonicity constant (x_e) is

- (A) 0.001
- (B) 0.005
- (C) 0.023
- (D) 0.421

32. The simplified Boolean expression of the following function

$f(X_1, X_2, X_3, X_4) = \sum m(0, 1, 2, 3, 4, 6, 8, 9) + d(10, 11)$ is

- (A) $\bar{X}_2 + \bar{X}_1 \bar{X}_4$
- (B) $X_1 + X_2 X_3$
- (C) $\bar{X}_1 + X_3 X_4$
- (D) $\bar{X}_2 + X_1 \bar{X}_3$

33. A grating is used to resolve sodium D lines whose wavelengths are 589 nm and 589.6 nm . The minimum number of lines, the grating must have to resolve them in the second order is approximately

- (A) 815
- (B) 492
- (C) 250
- (D) 112

34. The Lagrangian $\mathcal{L}(q, \dot{q})$ of a particle of mass m is represented as $\mathcal{L}(q, \dot{q}) = \frac{1}{2}m\dot{q}^2 - \frac{\lambda}{2}q\dot{q}^2$ where λ is a constant. The corresponding Hamiltonian of the system is

- (A) $\frac{p^2}{m + \lambda q}$
- (B) $\frac{p^2}{m - \lambda q}$
- (C) $\frac{p^2}{2(m + \lambda q)}$
- (D) $\frac{p^2}{2(m - \lambda q)}$

35. The capacitance per unit length and characteristic impedance of a lossless transmission line are C and Z_0 respectively. The velocity of a travelling wave on the transmission line is

- (A) $Z_0 C$
- (B) $\frac{1}{Z_0 C}$
- (C) $\frac{Z_0}{C}$
- (D) $\frac{C}{Z_0}$

36. Consider a spin Hamiltonian

$H = DS_z^2$ ($D > 0$) of a spin 1 system. The average energy of the system at $T \rightarrow \infty$ limit is

- (A) 0
- (B) $D/2$
- (C) $D/3$
- (D) $2D/3$

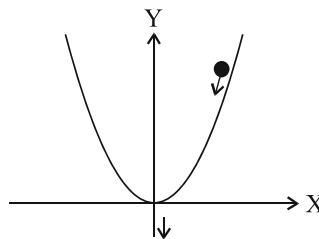
37. An electromagnetic wave at the interface between two homogeneous dielectric media of dielectric constant ϵ_1 and ϵ_2 where $\epsilon_1 < \epsilon_2$ and no charge on the surface. The electric field vector (\vec{E}) and displacement vector (\vec{D}) in the two media will satisfy the inequalities

- (A) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$
 (B) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$
 (C) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$
 (D) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

38. The group velocity of electromagnetic waves moving with phase velocity 'c' in a dispersive medium of refractive index 'n' is given by

- (A) $\frac{c}{\omega + n \frac{dn}{d\omega}}$
 (B) $\frac{c}{\omega + n \frac{d\omega}{dn}}$
 (C) $\frac{c}{n + \omega \frac{dn}{d\omega}}$
 (D) $\frac{c}{n + \omega \frac{d\omega}{dn}}$

39. A particle of mass 'm' slides under the gravity without friction along the path $y = \beta x^2$ as shown in the figure below. Here β is constant. The Lagrangian of the particle is



- (A) $\mathcal{L} = \frac{1}{2} m \dot{x}^2 - mg\beta x^2$
 (B) $\mathcal{L} = \frac{1}{2} m \dot{x}^2 (1 + 4\beta^2 x^2) + \frac{1}{2} mg\beta x^2$
 (C) $\mathcal{L} = \frac{1}{2} m \dot{x}^2 (1 + 4\beta^2 x^2) - mg\beta x^2$
 (D) $\mathcal{L} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} mg\beta x^2$

[g is the acceleration due to gravity.]

40. If $y_1(x)$ and $y_2(x)$ are two solutions of the differential equation

$$A(x) \frac{d^2 y}{dx^2} + B(x) \frac{dy}{dx} + C(x)y = D(x)$$

then the condition for $y_1(x)$ and $y_2(x)$ to be linearly independent is

- (A) $y_1(x) + y_2(x) \neq 0$ for at least one value of x .
 (B) $y_1(x) y'_2(x) - y_2(x) y'_1(x) \neq 0$ for all values of x .
 (C) $y_1(x) y'_2(x) - y_2(x) y'_1(x) \neq 0$ for at least one value of x .
 (D) $y_1(x) y'_1(x) - y_2(x) y'_2(x) \neq 0$ for all values of x .

41. If M is the matrix

$$M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

and θ is a real parameter, then $e^{\theta M}$ equals to

- (A) $\frac{1}{2}e^{2\theta}M$
- (B) $\frac{1}{2}e^{\theta}M$
- (C) $e^{\theta}e^M$
- (D) $e^{2\theta}e^M$

42. A parallel plate capacitor is connected across a potential difference of 40 V and holds 36 C of charge. The capacitor has a separation of 3 mm. The area of the plate is approximately (Take the value of ϵ_0 approximated to $1 \times 10^{-4} \text{ F/m}$)

- (A) $1 \times 10^8 \text{ m}^2$
- (B) $2 \times 10^8 \text{ m}^2$
- (C) $3 \times 10^8 \text{ m}^2$
- (D) $5 \times 10^8 \text{ m}^2$

43. A box contains 8 black balls and 4 white balls. If two balls are drawn with replacement, what is the probability of getting two black balls?

- (A) $\frac{1}{3}$
- (B) $\frac{1}{4}$
- (C) $\frac{1}{6}$
- (D) $\frac{4}{9}$

44. In a 4-bit DAC, for a digital input of 0100, an output current of 10 mA is produced. What will be the output for a digital input of 1011?

- (A) 2.5 mA
- (B) 7.5 mA
- (C) 11 mA
- (D) 27.5 mA

45. Guide wavelength (λ_g), cut-off frequency (λ_c) and free space wavelength (λ_0) of a waveguide are related as

- (A) $\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$
- (B) $\frac{1}{\lambda_0^2} = \frac{1}{\lambda_g^2} - \frac{1}{\lambda_c^2}$
- (C) $\frac{1}{\lambda_c^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_g^2}$
- (D) $\frac{1}{\lambda_g^2} = \frac{1}{\lambda_0^2} - \frac{1}{\lambda_c^2}$

46. A hunter has a rifle that can fire 0.06 kg bullets with a muzzle velocity of 500 m/second. A 40 kg leopard springs at him at a speed of 9 m/second. The number of bullets the hunter must fire on the leopard to stop it in its track is

- (A) 8
- (B) 10
- (C) 12
- (D) 40

[Neglect air resistance for the leopard as well as the bullets.]

47. The magnetic field associated with the electric field vector $\vec{E} = E_0 \sin(kz - \omega t) \hat{j}$ is given by

- (A) $\vec{B} = \frac{E_0}{c} \sin(kz - \omega t) \hat{j}$
- (B) $\vec{B} = \frac{E_0}{c} \sin(kz - \omega t) \hat{i}$
- (C) $\vec{B} = -\frac{E_0}{c} \sin(kz - \omega t) \hat{i}$
- (D) $\vec{B} = \frac{E_0}{c} \sin(kz - \omega t) \hat{k}$

48. The determinant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

evaluates to

- (A) $(a - b)(b - c)(c - a)$
- (B) $(ab - bc)(bc - ca)(ca - ab)/abc$
- (C) $3abc - a^3 - b^3 - c^3$
- (D) 0

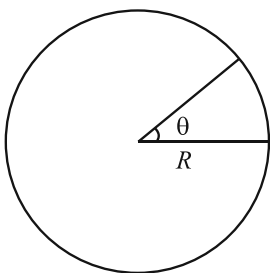
49. A harmonic oscillator with Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

is subjected to a perturbation λx^3 where λ is a small real positive constant. It is observed that

- (A) the energy levels remain almost unchanged.
- (B) the oscillatory motion gradually stops.
- (C) the particle suddenly expels from the well.
- (D) the motion becomes anharmonic and chaotic.

50. A particle is constrained to move in a circle of radius R as shown below:



A possible wavefunction $\psi(\theta)$ of this particle is

- (A) $\psi(\theta) = A \sin \theta$
- (B) $\psi(\theta) = A \sin \frac{\theta}{2\pi}$
- (C) $\psi(\theta) = A \sin \frac{\theta}{\pi}$
- (D) $\psi(\theta) = A \sin \frac{2\theta}{\pi}$

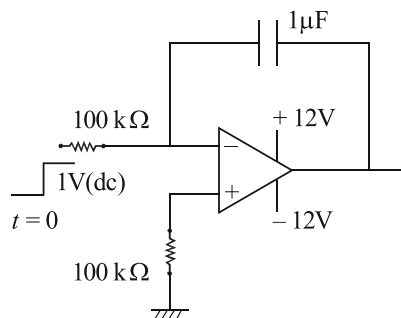
51. A diffusion pump works on the principle of

- (A) Kinetics of gases
- (B) Vapour flight
- (C) Electron trapping
- (D) Electron flight

52. The strangeness Quantum number is conserved in

- (A) strong and weak interactions.
- (B) strong and electromagnetic interactions.
- (C) strong, weak and electromagnetic interactions.
- (D) weak and electromagnetic interactions.

53.



With respect to the figure above, the output voltage after 1 sec of the application of input voltage is

- (A) - 1V
- (B) - 4V
- (C) - 10V
- (D) - 12V

54. The wavefunction

$$\psi(x) = Ae^{-\frac{1}{2}bx^2}$$

where A and b are real constants, is a normalized eigenfunction of the Schrödinger Equation for a particle of mass m and energy E in a one-dimensional potential such that $V(x) = 0$ at $x = 0$. Which of the following is correct?

(A) $V(x) = \frac{\hbar^2 b^4}{m} x^2$

(B) $V(x) = \frac{\hbar^2 b^4 x^2}{2m}$

(C) $E = \frac{\hbar^2 b^2}{4m}$

(D) $E = \frac{\hbar^2 b^2}{m}$

55. For an atom in the state of $^2D_{5/2}$, the Lande-g factor should be

(A) 2

(B) 1.5

(C) 1.4

(D) 1.2

56. According to the Liquid drop model, when a nucleus is bombarded by neutrons, the compound nucleus attains the given shapes in the sequence

(A) Ellipsoidal, Spherical, Dumb-bell

(B) Spherical, Dumb-bell, Ellipsoidal

(C) Spherical, Ellipsoidal, Dumb-bell

(D) Dumb-bell, Ellipsoidal, Spherical

57. The free energy of a system follows the relation

$$f(t) = 0.3t^{1/2} + 0.6t^{1/4}, \text{ where } t = \frac{T - T_c}{T_c}$$

and T_c being the critical temperature. The critical exponent is

(A) 0.3

(B) 0.6

(C) 0.5

(D) 0.25

58. Graphene is a planar monatomic layer of carbon atoms. The dispersion relation for the electrons in this material is $E(k) = \hbar v_F |\vec{k}|$. The Fermi energy E_F of this material depends on the no. of atoms per unit area (f) as

(A) $E_F \propto \sqrt{f}$

(B) $E_F \propto f^{2/3}$

(C) $E_F \propto f^2$

(D) $E_F \propto f^{2/5}$

59. A particle is moving along the curve $r = ae^{\theta}$ with constant angular velocity ω and 'a' is a constant. The radial acceleration of the particle is

(A) $r\dot{\theta}$

(B) 0

(C) $r^2\dot{\theta}$

(D) $2\omega^2 r$

60. Electrons in a given system of hydrogen atoms are described by the wavefunction

$$\psi(\vec{x}) = 0.8 \psi_{1s}(\vec{x}) + 0.6 e^{i\pi/3} \psi_{3p}(\vec{x})$$

where ψ_{1s} and ψ_{3p} are the 1s and 3p states respectively. The expectation value of the total angular momentum L^2 will be

(A) \hbar^2

(B) $0.72\hbar^2$

(C) $0.36\hbar^2$

(D) $0.6\hbar^2$

61. Neutrons can be diffracted from a crystalline solid since

(A) Neutrons are chargeless.

(B) Neutrons have very high energy.

(C) Neutrons can have wavelength of the order of the interplanar spacing of solids.

(D) Neutrons have high penetration power.

62. A blue light emits light of mean wavelength of 4500\AA . The lamp is rated at 100 Watt and 10% of the energy appears as emitted light. Number of Photons emitted by the lamp per second is

- (A) 4.32×10^{12}
- (B) 8.45×10^{14}
- (C) 3.62×10^{16}
- (D) 2.26×10^{18}

63. A Geiger counter measures 1250 events near a radioactive source during 10 second and 350 events during 10 second when the source is removed. What is the uncertainty of the rate of events due to the source?

- (A) 3 Hz
- (B) 4 Hz
- (C) 40 Hz
- (D) 36.8 Hz

64. At $z = 0$, the function $f(z) = z^{1/2}$ has

- (A) a simple pole.
- (B) a branch point.
- (C) a removable singularity.
- (D) a point of accumulation.

65. If a system is submitted to a rotation by an angle θ about the z -axis, the angular momentum operators $\hat{L}_x, \hat{L}_y, \hat{L}_z$ will change to

- (A) L_x, L_y
- (B) $\hat{L}_x \cos \theta - \hat{L}_y \sin \theta, \hat{L}_x \sin \theta + \hat{L}_y \cos \theta$
- (C) $\hat{L}_x \sin \theta - \hat{L}_y \cos \theta, \hat{L}_x \cos \theta + \hat{L}_y \sin \theta$
- (D) $L_x + iL_y, L_x - iL_y$

66. The band structure of a crystal is given by $E = -\alpha k^2$. The velocity of an electron with wave vector $\vec{k} = 10^{10} m^{-1} \hat{k}_x$ is $-1.9 \times 10^5 \hat{k}_x$ m/s. It is removed to higher band. Then, the value of α (in Jm^2) will be

- (A) 3.5×10^{-35}
- (B) 0.95×10^{-35}
- (C) 6×10^{-39}
- (D) 2×10^{-59}

67. If the angle between the direction of incident X-ray and diffracted one is 40° , then the angle of incidence will be

- (A) 40°
- (B) 20°
- (C) 60°
- (D) 70°

68. The integral

$$I = \int_0^\infty dx e^{ix} \delta(\sin x)$$

where $\delta(x)$ is the Dirac delta function evaluates to

- (A) $\frac{1}{2}$
- (B) $\frac{1}{i\pi}$
- (C) 0
- (D) ∞

69. The vectors $|\varphi_1\rangle$, $|\varphi_2\rangle$ and $|\varphi_3\rangle$ form an orthonormal basis in a linear vector space. A linear operator \hat{A} has the following action:

$$\hat{A}|\varphi_1\rangle = |\varphi_2\rangle$$

$$\hat{A}|\varphi_2\rangle = |\varphi_3\rangle$$

$$\hat{A}|\varphi_3\rangle = |\varphi_1\rangle$$

The matrix representation of \hat{A} in this basis is

(A)
$$\begin{pmatrix} 0 & i & -i \\ -i & 0 & i \\ 0 & i & 0 \end{pmatrix}$$

(B)
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(C)
$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

(D)
$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

70. Indicate which of the following is largest as $x \rightarrow \infty$.

- (A) x
- (B) $x^2/\ln x$
- (C) $(\ln x)^2$
- (D) $\ln x^2$

71. The dynamics of a system is described by a Schrödinger Equation with a potential

$$V(x) = V_1 + iV_2$$

where V_1 and V_2 are both real. From this we can conclude that the system

- (A) has imaginary energy.
- (B) cannot be in an eigenstate of the Hamiltonian.
- (C) cannot have bound or metastable states.
- (D) is in a state which decays with time.

72. A system of N localized particles, each of which can exist in levels of energy $0, \epsilon, 2\epsilon$ and 3ϵ having degeneracies 1, 3, 3 and 1 respectively. The free energy F for the system at a given temperature T is

- (A) $-3NK_B T \ln(1 + e^{-\epsilon/K_B T})$
- (B) $-4NK_B T \ln(1 + e^{-2\epsilon/K_B T})$
- (C) $-3NK_B T \ln(3 + e^{-\epsilon/K_B T})$
- (D) $-3NK_B T \ln(2 + 3e^{-\epsilon/K_B T})$

73. If the binding energy (E_B) of deuteron is very small compared to depth of the barrier (V_0), the radius of the deuteron becomes

- (A) $\frac{2r_0}{\pi} \sqrt{\frac{E_B}{V_0}}$
- (B) $\frac{2r_0}{\pi} \sqrt{\frac{E_B}{E_B - V_0}}$
- (C) $\frac{2r_0}{\pi} \sqrt{\frac{V_0}{E_B}}$
- (D) $\frac{2r_0}{\pi} \sqrt{E_B + V_0}$

[Where r_0 is the prefactor in the relation between radius (r) and atomic mass number (A) of a nucleus ($r = r_0 A^{1/3}$)]

74. If the velocity of light, acceleration due to gravity and normal pressure are taken as the fundamental units, then the unit of time will be

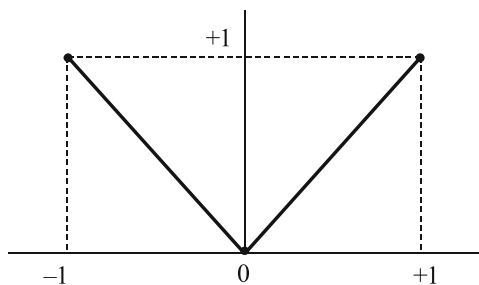
- (A) 3×10^6 s
- (B) 3×10^4 s
- (C) 3×10^8 s
- (D) 3×10^7 s

(Take $c = 3 \times 10^{10}$ cm/s; $g = 1000$ cm/s²; $p = 10^6$ dyne/cm²)

75. The temperature at which the rates of spontaneous and stimulated emission for wavelength λ become equal is

- (A) $\frac{hc}{\lambda K_B \ln 2}$
- (B) $\frac{hc \ln 2}{\lambda K_B}$
- (C) $\frac{\lambda K_B \ln 2}{hc}$
- (D) $\frac{\lambda K_B}{hc \ln 2}$

76. A Fourier series is used to approximate the function $f(x)$ shown in the graph below, in the domain $-1 \leq x \leq +1$.



The principal Fourier mode will be proportional to

- (A) $\sin x$
- (B) $\cos \pi x$
- (C) $\sin \frac{\pi}{2} x$
- (D) $\cos 2\pi x$

77. Given a function $f(x)$ which is continuous in the closed interval $a \leq x \leq b$ and differentiable in the open interval $a < x < b$, we can always find a point c ($a < c < b$) such that the derivative

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

This result is known as

- (A) Rolle's Theorem
- (B) Newton-Leibnitz Theorem
- (C) Lagrange's Mean Value Theorem
- (D) Cauchy's Mean Value Theorem

78. Classify the unit cell of the given lattice parameters into proper crystal system: $a = 1.1$ nm, $b = 0.5$ nm, $c = 0.3$ nm; and $\alpha = \beta = \gamma = 90^\circ$.

- (A) Triclinic
- (B) Rhombohedral
- (C) Orthorhombic
- (D) Tetragonal

79. A hydrogen atom in its ground state is collided with an electron of energy 13.377 eV. The radius of the atom can increase by a maximum of

- (A) 7 times
- (B) 8 times
- (C) 49 times
- (D) 64 times

80. What does the equiprobability postulate in statistical mechanics state?

- (A) All microstates of a system are equally likely to occur.
- (B) The macrostate with higher entropy is the most probable.
- (C) The number of particles remains constant.
- (D) The system will reach its thermal equilibrium.

81. A thermal neutron having speed v impinges on a ^{235}U nucleus. The reaction cross-section is proportional to

- (A) v
- (B) $1/v$
- (C) $v^{1/2}$
- (D) $v^{-1/2}$

82. The Poynting theorem is the mathematical statement of the conservation of

- (A) Momentum
- (B) Charge
- (C) Electromagnetic energy
- (D) Angular momentum

83. The resistivity of a normal metal at low temperature varies with temperature T as $\rho(T) = \rho_0 + \alpha T^5$. The origin of T^5 term is due to

- (A) phonon scattering.
- (B) impurity and defect scattering.
- (C) electron-electron scattering.
- (D) impurity and electron-electron scattering.

84. For a dielectric material, the Clausius-Mossotti relation is written as

- (A) $\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$
- (B) $\frac{\epsilon_r + 1}{\epsilon_r + 2} = \frac{N\alpha^2}{3\epsilon_0}$
- (C) $\frac{\epsilon_r + 1}{\epsilon_r - 2} = \frac{N^2\alpha}{3\epsilon_0}$
- (D) $\frac{\epsilon_r - 1}{\epsilon_r - 2} = \frac{N\alpha}{3\epsilon_0}$

(where ϵ_r is dielectric constant, α is polarizability, N is the number of molecules/atoms and ϵ_0 is permittivity of free space)

85. The pressure at $T = 0$ of an ultra-relativistic free Fermi gas in three dimensions depends on the density n as

- (A) $n^{5/3}$
- (B) $n^{1/3}$
- (C) $n^{2/3}$
- (D) $n^{4/3}$

86. An electric dipole having dipole moment of magnitude p is rotating about an axis passing through its mid-point and perpendicular to \vec{p} . The power radiated by the dipole is

- (A) $\frac{p^2\omega^2}{3\pi\epsilon_0 c^3}$
- (B) $\frac{p^2\omega^4}{6\pi\epsilon_0 c^3}$
- (C) $\frac{p^2\omega^4}{8\pi\epsilon_0 c^3}$
- (D) $\frac{p^2\omega^4}{12\pi\epsilon_0 c^3}$

87. Consider the operator

$$\hat{A} = \vec{\alpha} \cdot \vec{\sigma}$$

where $\vec{\alpha}$ is a vector and $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the three Pauli matrices. The eigenvalues of \hat{A} will be

- (A) $\alpha_x + \alpha_y, \alpha_z$
- (B) $\alpha_x + \alpha_z \pm i\alpha_y$
- (C) $\pm (\alpha_x + \alpha_y + \alpha_z)$
- (D) $\pm |\vec{\alpha}|$

88. Consider the state

$$\begin{pmatrix} 1/2 \\ 1/2 \\ 1/\sqrt{2} \end{pmatrix}$$

corresponding to the angular momentum $l = 1$, in the L_z basis of states $m = 1, 0, -1$. If L_z^2 is measured on this state with a result $+1$, the state after measurement can be

(A) $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

(B) $\begin{pmatrix} 1/\sqrt{3} \\ 0 \\ \sqrt{2/3} \end{pmatrix}$

(C) $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{pmatrix}$

89. The average time associated with an atomic transition is about 10 picoseconds. This gives rise to a yellow line in the optical spectrum of these atoms. The width $\Delta\lambda$ of this spectral line can be estimated as around

- (A) 0.12 nanometres
- (B) 0.0012 nanometres
- (C) 0.0144 nanometres
- (D) 14.4 nanometres

90. A classical system has three particles, each with two accessible energy level $\epsilon = 0$ and $\epsilon = 1$. What is the total number of microstates (Ω) in the system?

- (A) 4
- (B) 6
- (C) 9
- (D) 8

91. How many gate inputs are required to realize the Boolean expression

$$F = ABC + \bar{A}\bar{B}CD + E\bar{F} + AD?$$

- (A) 12
- (B) 11
- (C) 14
- (D) 15

92. A closed container initially contains only water vapour at 50°C and 2 atm pressure. If the container volume is decreased, what will happen to the temperature and pressure?

- (A) Temperature and pressure will decrease.
- (B) Temperature will increase and pressure will decrease.
- (C) Temperature will decrease and pressure will increase.
- (D) Temperature and pressure will increase.

93. The specific heat of a two-dimensional material possesses both electronic and phononic contributions with corresponding coefficients α and β respectively. Then, the specific heat at constant volume at $t = -173^\circ\text{C}$ is given by

- (A) $C_V = -173\alpha + (173)^2\beta$
- (B) $C_V = 100\alpha + (100)^3\beta$
- (C) $C_V = (100)^3\alpha + 100\beta$
- (D) $C_V = 100\alpha + (100)^2\beta$

94. The state of polarization of the emergent ray whose x and y components of electric fields are $E_x = E_0 \cos(\omega t + kz)$ and

$$E_y = \frac{E_0}{\sqrt{2}} \cos(\omega t + kz + \pi) \text{ respectively is}$$

- (A) linearly polarized making an angle $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ with the x -axis.
- (B) linearly polarized making an angle $\tan^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ with the x -axis.
- (C) elliptically polarized in the clockwise direction and propagating towards the observer.
- (D) elliptically polarized in the anticlockwise direction and is propagating towards the observer.

95. A surface is defined in three dimensions by the function

$$f(x, y, z) = xy + yz + zx.$$

The normal to this surface at a point $(x, y, 0)$ is

$$\hat{n} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k}).$$

The value of the function at this point $(x, y, 0)$ will be

- (A) 0
- (B) 1
- (C) $\sqrt{6}$
- (D) $\frac{2}{\sqrt{3}}$

96. Two reversible engines are connected in series. The first one receives heat at T_0K and rejects heat at T_1K . The second one receives the heat rejected by the first one and then rejects heat at T_2K . Find the temperature T_0K , if efficiencies of the two engines are the same.

- (A) $\frac{T_1}{T_2}$
- (B) $\frac{T_1^2}{T_2}K$
- (C) $\frac{T_2^2}{T_1}K$
- (D) $\frac{T_2}{T_1}$

97. A tunnel diode can work at very high frequencies of the order of 10 GHz or so because

- (A) it has very high capacitance and inductance.
- (B) it has negative as well as positive resistance.
- (C) the tunneling takes place very slow.
- (D) the tunneling takes place very fast.

98. An amplitude modulated wave is represented by the expression $E_m = 10(1 + 0.6 \cos 6280t) \sin 2\pi \times 10^4 t$ volts(V). The minimum and maximum amplitudes of the AM wave are respectively

- (A) 12V and 8V
- (B) 16V and 4V
- (C) 20V and 16V
- (D) 24V and 16V

99. If (p, q) and (P, Q) represent two pairs of canonical variables, then the transformation $Q = q^\alpha \cos(\beta p), P = q^\alpha \sin(\beta p)$ is canonical for

- (A) $\alpha = 2, \beta = \frac{1}{2}$
- (B) $\alpha = \frac{1}{2}, \beta = 2$
- (C) $\alpha = 1, \beta = 1$
- (D) $\alpha = 2, \beta = 2$

100. The Strange baryon Σ^+ has the quark structure

- (A) uds
- (B) uud
- (C) uus
- (D) $u\bar{s}$

Space for Rough Work

Space for Rough Work

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