Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series **Y**

PAPER–II **Question Booklet No.**

Subject Code : 15

MATHEMATICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the Question Booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet Number is identical with OMR Sheet Number; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One Hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D), where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

The Question Booklet is encrypted with QR code for security purpose.

MATHEMATICAL SCIENCES

1. In a 7-node directed cyclic graph, the number of Hamiltonian cycles is

(A) 720

(B) 540

(C) 360

(D) 180

2. If eigen functions corresponding to distinct eigenvalues of the boundary value problem $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = \lambda y$, $0 < x < \pi$, $y(0) = y(\pi) = 0$ are orthogonal with respect to the weight function $w(x)$, then $w(x)$ equals

(A) e^{x}

(B) e^{-x}

(C) e^{3x}

(D) e^{-3x}

3. Let $x_1 = -2, x_2 = 1, x_3 = 3$ and $x_4 = -4$ be the observed values of a random sample from the distribution having p.d.f.

$$f(x|\theta) = \frac{e^{-x}}{e^{\theta} - e^{-\theta}}, \quad -\theta \le x \le \theta, \ \theta > 0$$

Then the MLE of θ is

(A) 3

(B) 0·5

(C) 4

(D) any value between 1 and 2

4. For any integer $n \geq 3$, the value of $\sum_{k=1}^{n} \mu(\lfloor k)$, where μ is the Mobius function, is

(A) 0

(B) 3

(C) 1

(D) 2

5. How many non-isomorphic groups of order 30 are there?

(A) 16

(B) 15

(C) 8

(D) 4

6. The relationship between L_s, the expected number of customers in the system, L_q, the expected number of customers in queue, arrival rate of customers at system λ and service rate of customers in system μ is

(A) $L_s - L_q = \frac{\lambda}{\mu}$

(B) $L_s + L_q = \frac{\lambda}{\mu}$

(C) $L_s / L_q = \frac{\lambda}{\mu}$

(D) $L_s + L_q = \frac{\lambda}{2\mu}$

7. Let q_α, p_α $(\alpha = 1, 2, ..., n)$ be the generalized coordinates and the generalized moments, respectively. If H denotes the Hamiltonian and q_α (for some $\alpha = \alpha_0$) is an ignorable coordinate, then

(A) $\dot{p}_{\alpha_0} = -\frac{\partial H}{\partial q_{\alpha_0}}, \ \dot{q}_{\alpha_0} = 0$

(B) $\dot{p}_{\alpha_0} = 0, \ \dot{q}_{\alpha_0} = \frac{\partial H}{\partial p_{\alpha_0}}$

(C) $\dot{p}_{\alpha_0} = \frac{\partial H}{\partial q_{\alpha_0}}, \ \dot{q}_{\alpha_0} = 0$

(D) $\dot{p}_{\alpha_0} = 0, \ \dot{q}_{\alpha_0} = -\frac{\partial H}{\partial p_{\alpha_0}}$

8. The partial differential equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+u$ can be transformed to $\frac{\partial v}{\partial t}=\frac{\partial^2 v}{\partial x^2}$ with the transformation

(A) $v=e^{-t}u$

(B) $v=e^{t}u$

(C) $v = tu$

(D) $v = -tu$

9. Let $I=\int_c z^4 e^{1/z}dz$, where c is given by $|z|=4$. Then

(A) $I=0$

(B) $I=\frac{\pi}{60}$

(C) $I=\frac{\pi i}{6}$

(D) $I=\frac{\pi i}{60}$

10. Consider the following linear model:

$$y_1=2\theta+\beta+e_1$$
$$y_2=\beta+2\gamma+e_2$$
$$y_3=\theta+\beta+\gamma+e_3,$$

where θ, β, γ are unknown parameters and e_1, e_2, e_3 are uncorrelated random errors with mean zero and constant variance (> 0). Then, which of the following statements is true?

(A) θ, β and γ are estimable.

(B) θ – γ is estimable.

(C) 4θ – 2β is estimable.

(D) θ + γ is estimable.

11. The domain of convergence of $\sum_{n=0}^{\infty} n^2\left(\frac{z^2+1}{1+i}\right)^n$ is

(A) $|z+1|<2$

(B) $|z^2+1|<\sqrt{2}$

(C) $|z^2+1|<2$

(D) $|z+1|<\sqrt{2}$

12. If V_{ran} is the variance of sample mean under simple random sampling and V_{prop} is the variance of the estimator (with the same sample size) under stratified random sampling with proportional allocation, then the two variances will be equal iff

(A) the stratum means are all equal

(B) the stratum sizes (N_i) are all equal

(C) the stratum mean square (S_i^2) are all equal

(D) the cost per unit (C_i) are all equal

13. Let G(x, y) be the Green's function of the boundary value problem $[(1+x)\,u']'+(\sin x)u=0$, $x \in [0, 1]$, $u(0) = u(1) = 0$. Then the function $g(x)$ defined by

$$g(x)=G\left(x,\frac{1}{2}\right),\quad x\in[0,1]$$

(A) is continuous

(B) is discontinuous at $x=\frac{1}{2}$

(C) is differentiable

(D) does not have left derivative at $x=\frac{1}{2}$

14. Let B be a finite Boolean algebra. Then

(A) B may be a lattice without universal lower bound.

(B) B may be a lattice with universal lower bounds but not necessarily complemented.

(C) B may have 100 elements.

(D) B may have 1024 elements.

15. Let $\tau=\{\phi, X, \{p\}, \{p,q\}, \{q,p,t\}, \{p,r,s\}, \{p,q,r,s\}\}$ be the topology on $X=\{p,q,r,s,t\}$. Then the set of limit points of the set $\{r, s, t\}$ is

(A) $\{r, s, t\}$

(B) $\{r, s\}$

(C) $\{q, p, t\}$

(D) $\{p, q, r, s\}$

16. Let X_1, X_2, ..., X_n be iid r.v,s with mean zero and unit variance. Consider their sum $X = X_1 + X_2 + ... + X_n$. Using Chebyshev's inequality, which of the following would hold?

(A) $P\{|X| \geq \sqrt{2n}\} \leq \frac{1}{2}$

(B) $P\{|X| \geq n\} \leq \frac{1}{2n}$

(C) $P\{|X| \geq 2n\} \leq \frac{1}{3n}$

(D) $P\{|X| \geq 3n\} \leq \frac{1}{4n}$

17. The standard error of sample mean under SRSWOR is given by

(A) $\frac{\sigma}{\sqrt{n}}$

(B) $\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$

(C) $\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{Nn}}$

(D) $\frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-1}{Nn}}$

18. The Galois group of the polynomial $x^3 - 3x - 3$ over the field Q of rationals is

(A) $\mathbb{Z}_3$

(B) S_3

(C) $\mathbb{Z}_6$

(D) A_3

19. Let $X_1, X_2, ...$ be a sequence of i.i.d. random variables with mean 2 and variance 4. Denote by $\bar{X}_n$ the mean of the first n observations. Then which of the following statements is/are *true*?

(a) $\bar{X}_n \xrightarrow{P} 2$ as $n \to \infty$

(b) $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \xrightarrow{P} 4$ as $n \to \infty$

(c) $\frac{\sqrt{n}\left(\bar{X}_n - 2\right)}{2} \xrightarrow{L} N(0,1)$

(d) $E\left(\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}_n\right)^2\right) = 4$ for $n \geq 2$

(A) (a) only

(B) (a), (b) only

(C) (b), (c), (d) only

(D) (a), (c), (d) only

20. Which of the following is *not* correct?

(A) Set of all rotations in a plane forms a group.

(B) Prime order groups are cyclic.

(C) Normal subgroups are of prime order.

(D) All abelian groups are not cyclic.

21. For the first order auto-regressive series $U_{t+1} = aU_t + \in_{t+1}, |a| < 1$, where $\in_t$'s are independent with mean zero, the correlogram is

(A) a^k

(B) a^{-k}

(C) $a^{1/k}$

(D) $a^{-1/k}$

22. You are given a random variable X with the following distribution function:

$$F(x)=\begin{cases}0, & \text{if} \quad x<0\\ \frac{1}{2}, & \text{if} \quad x=0\\ \frac{1}{2}+\frac{x}{2}, & \text{if} \quad 0<x<1\\ 1, & \text{if} \quad x\geq 1\end{cases}$$

The following claims are made:

Claim 1: F has a point of discontinuity at $x = 0$.

Claim 2: F is the cdf of a discrete r.v..

Claim 3: F is the cdf of a continuous r.v..

Claim 4: F is the cdf of a distribution that is neither continuous nor discrete.

Then which of the following is true?

(A) Only Claim 2.

(B) Only Claim 3.

(C) Only Claim 1.

(D) Claim 1 and Claim 4.

23. Suppose T_1 and T_2 be the linear operators on $\mathbb{R}^2$ such that $T_1(x_1, x_2) = (x_2, x_1)$ and $T_2(x_1, x_2) = (x_1, 0)$, then

(A) $T_1 T_2 \neq T_2 T_1$

(B) $T_1 T_2 = T_2 T_1$

(C) $(T_1 T_2)(x_1, x_2) = (x_1 + x_2, 0)$

(D) $(T_2 T_1)(x_1, x_2) = (0, x_1)$

24. Let A be a connected subset in the Euclidean space $\mathbb{R}^n, n\geq 1$ having at least two elements. Then number of elements in A is

(A) exactly 2

(B) finite but more than 2

(C) uncountable

(D) countably infinite

25. Which of the following is *not* true?

(A) Borel sets are Lebesgue measurable.

(B) Lebesgue measure of a countable set is zero.

(C) All subsets of real line are not Lebesgue measurable.

(D) Lebesgue measurable sets are Borel sets.

26. Consider the following statements:

Statement I : If f is a Lebesgue integrable function on $\mathbb{R}$ such that $\int f(x)dx = 0$, then $f = 0$ *a.e.*

Statement II : If g is a non-negative real valued function on $\mathbb{R}$ with finite range such that $\{x \in \mathbb{R} | g(x) = t\}$ is Legesgue measurable for every t in $\mathbb{R}$, then g is Lebesgue measurable.

Then,

(A) *Statement I* is false but *Statement II* is true

(B) Both *Statement I* and *Statement II* are true

(C) Both *Statement I* and *Statement II* are false

(D) *Statement I* is correct but *Statement II* is false

27. In order that the χ^2 test for goodness of fit is applicable, which of the following conditions must be satisfied?

(A) Observed frequency should be ≥ 5 for all groups/categories.

(B) Expected frequency should be < 5 for all groups/categories.

(C) Observed frequency should be < 5 for all groups/categories.

(D) Expected frequency should be ≥ 5 for all groups/categories.

28. Consider a Markov chain with state space $S = \{0, 1, 2\}$ and with transition probability matrix P given by,

$$\begin{pmatrix} \frac{2}{3} & 0 & \frac{1}{3} \\ 1 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Then,

(A) 1 is recurrent state

(B) 0 is recurrent state

(C) Both 0 and 1 are recurrent states

(D) 2 is recurrent state

29. Which of the following mathematical techniques is used to predict physical parameter in fluid flow?

(A) Temperature analysis

(B) Dimensional analysis

(C) Pressure analysis

(D) Combustion analysis

30. Which of the following is *not* correct?

(A) The set z of all integers is countable

(B) The set of all rational numbers in the closed interval [0, 1] is countable

(C) The cantor set c is countable

(D) The set $\left\{\left(n, \frac{1}{n^2}\right) \middle| n \in \mathbb{N}\right\}$ is countable, where $\mathbb{N}$ is the set of natural numbers

31. By the method of successive approximation, the solution of the integral equation

$u(x) = e^x + \int_0^x e^{x-t} u(t)\,dt$ is

(A) $u(x) = 1$

(B) $u(x) = e^x$

(C) $u(x) = e^{2x}$

(D) $u(x) = e^{3x}$

32. Consider a non-empty set X with $|X|>1$ together with the discrete metric d on X given by,

$$d(x, y) = \begin{cases} 1, & \text{if } x \neq y \\ 0, & \text{otherwise} \end{cases}$$

Then for $a \in X$ and $0 < r < 1$, open sphere $S(a, r)$ and closed sphere $S[a, r]$ are given by

(A) $S(a, r) = \{a\}, S[a, r] = \{a\}$

(B) $S(a, r) = \{a\}, S[a, r] = X$

(C) $S(a, r) = X, S[a, r] = \{a\}$

(D) $S(a, r) = X, S[a, r] = X$

33. Suppose $\underset{\sim}{X} \sim N_p\left(\underset{\sim}{\mu}, \Sigma\right), \Sigma$ positive definite and consider the following statements:

(a) Every non-null linear combination of $\underset{\sim}{X}$ is univariate normal

(b) Each marginal distribution is univariate normal

(c) There exists a non singular matrix C that the components of the random vector $\underset{\sim}{Y} = C\underset{\sim}{X}$ are independent

(d) The joint moment generating function of $\underset{\sim}{X}$ is the product of the individual moment generating functions

Which of the following statements is/are *false*?

(A) (a) only

(B) (d) only

(C) (b), (d) only

(D) (c), (d) only

34. The principle of 'local control' is not used in 'Completely Randomised Design' because

(A) Each plot has just one observation

(B) Numbers of plots per treatment are unequal

(C) The experimental area is heterogenous

(D) The experimental area is homogenous

[Please Turn Over]

35. The series $\sum_{n=1}^{\infty}(-1)^n \frac{x^2+n}{n^2}$

(A) converges uniformly in [– 1, 1] and does not converge absolutely at $x = -1$

(B) does not converge uniformly in [– 1, 1] but converges absolutely at $x = -1$

(C) converges absolutely at $x = -1$ and also uniformly in [– 1, 1]

(D) neither converges uniformly nor absolutely in [– 1, 1]

36. Which of the following is *not* true?

(A) In finite dimensional vector spaces all the norms are equivalent

(B) Dual space of a Hilbert space is also a Hilbert space

(C) Bounded linear operators from a normed linear space to a normed linear space may not be continuous

(D) Linear operators on a finite dimensional normed linear space is continuous

37. For what choice of n the following transformation in phase space be canonical (q, p represent generalized coordinate and generalized momentum respectively).

$P = q^n \sin 2p, \qquad Q = q^n \cos 2p$?

(A) $n = 1$

(B) $n = 2$

(C) $n = \frac{1}{2}$

(D) $n = -1$

38. Which of the following statements is *true*?

(A) Set of singularities of a complex valued function can be uncountable

(B) $\lim_{z \to 0} e^{-\frac{1}{z^2}}$ exists

(C) The domain of an analytic function need not be open

(D) If $u(x, y)$ and $v(x, y)$ are real differentiable functions, then $f(z) = u(x, y) + i\, v(x, y)$ is also differentiable

39. The path taken by the smoke coming out of a chimney in concentric circles represents a

(A) Path line

(B) Streamline

(C) Streamtube

(D) Streakline

40. Consider the function

$$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & 1 < x \le 2 \end{cases}$$

and $F(x) = \int_0^x f(t)\,dt$ on [0, 2].

Which one of the following is correct?

(A) $F(x)$ is continuous on [0, 2]

(B) $F(x)$ is differentiable on [0, 2]

(C) $F(x)$ is strictly increasing on [0, 2]

(D) $F(x)$ is not continuously bounded on [0, 2]

41. Which one of the followings is a basic assumption in ANOVA?

(A) Observations follow normal distribution

(B) All observations are highly correlated

(C) Observations satisfy Markov chain property

(D) Observations are heteroscadastic

42. Each compact Hausdorff space is

(A) regular but not normal

(B) normal but not regular

(C) regular and normal

(D) neither regular nor normal

43. Given that

$f(x)+f\left(x+\frac{1}{2}\right)=5,\ 0<x<\frac{1}{2}$ the value of $\int_0^1 f(x)dx$ is

(A) 1

(B) 2

(C) 3

(D) $\frac{5}{2}$

44. Consider the equation

$$\frac{dy}{dt}=\left(1+f^2(t)\right)y(t),\ y(0)=1,\ t\geq 0$$

where $f(t)$ is a bounded continuous function on $[0, \infty)$. Then this equation admits

(A) a unique solution $y(t)$ and further $\lim_{t\to\infty} y(t)$ exists and is finite

(B) two linearly independent solutions

(C) a bounded solution for which $\lim_{t\to\infty} y(t)$ does not exist

(D) a unique solution $y(t)$ and further $\lim_{t\to\infty} y(t)=\infty$

45. Consider the differential equation $\frac{dy}{dt}+ay=e^{-bt}$ with the initial condition $y(0) = 0$. Then the Laplace transform $Y(s)$ of the solution $y(t)$ is

(A) $\frac{1}{(s+a)(s+b)}$

(B) $\frac{1}{b(s+a)}$

(C) $\frac{1}{a(s+b)}$

(D) $\frac{e^{-a}-e^{-b}}{b-a}$

46. Which of the following conditions imply that a function $f : [0, 1] \to \mathbb{R}$ is necessarily of bounded variation?

(A) f is Riemann integrable

(B) f is continuous

(C) f has a derivative at each $x \in (0, 1)$

(D) $x<y \Rightarrow f(x)>f(y)$ for all x, y in $[0, 1]$

47. Consider the sequence $a_n=\left(1+\frac{(-1)^n}{n}\right)^n$. Then

(A) $\limsup a_n = e$; $\liminf a_n = \frac{1}{e}$

(B) $\limsup a_n = \liminf a_n = e$

(C) $\limsup a_n = \liminf a_n = \frac{1}{e}$

(D) $\limsup a_n = \liminf a_n = e+1$

48. Consider $G = GL(n, \mathbb{Z}_p)$, the general linear group of degree n over $\mathbb{Z}_p$ where p is prime. The centre of G is a cyclic group of order

(A) $p-1$

(B) 1

(C) p

(D) n

49. The mean curvature of the surface $e^z \cos x = \cos y$ is

(A) 0

(B) 1

(C) 2

(D) – 1

50. In a 2^5 experiment, the treatments in the control block are

(1), *cd*, *a*, *acd*, *bce*, *bde*, *abce*, *abde*.

Then the following interactions are confounded:

(A) *CDE*, *ABCDE*, *AB*

(B) *CDE*, *DE*, *C*

(C) *BCD*, *CDE*, *BE*

(D) *BCD*, *ACD*, *AB*

51. The Fourier transform of $f(x) = e^{-x^2}$ with k as the transformation variable is given by

(A) πe^{-k}

(B) $\sqrt{\pi}\, e^{-\frac{k^2}{4}}$

(C) $\frac{\sqrt{\pi}}{2} e^{-\frac{k^2}{2}}$

(D) $\sqrt{2\pi}\, e^{-k^2}$

52. Let X be a random variable with moment generating function

$$M_X(t) = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{e^{\frac{t^2}{2n}}}{n^2},\ t \in R$$

Then $P(X \in Q)$, where Q is the set of rational numbers, equals

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{3}{4}$

53. Let $(X_1, X_2, ..., X_n)$ be a random sample of size n from a Poisson distribution with parameter θ (> 0). The Cramer-Rao lower bound for the variance of any unbiased estimator of $g(\theta) = \theta e^{-\theta}$ equals

(A) $\frac{1}{n}\theta(1-\theta)^2 e^{-2\theta}$

(B) $\theta(1-\theta)^2 e^{-2\theta}$

(C) $\theta(1-\theta)\, e^{-\theta}$

(D) $\frac{1}{n}\theta(1-\theta)\, e^{-\theta}$

54. Force of mortality at age x, μ_x is

(A) $\frac{1}{l_x} \cdot \frac{dl_x}{dx}$

(B) $\frac{dl_x}{dx}$

(C) $-\frac{dl_x}{dx}$

(D) $-\frac{1}{l_x} \cdot \frac{dl_x}{dx}$

55. Consider $J[y]=\int_0^1\left[2y+(y')^2\right]dx$ subject to $y(0)=1$, $y(1)=1$. Then inf $(J[y])$ is

(A) $\frac{23}{12}$

(B) $\frac{21}{24}$

(C) $\frac{18}{25}$

(D) $\frac{5}{3}$

56. Let $\beta=e^{i\frac{\pi}{2}}$ and let the matrix

$$M=\begin{bmatrix}1 & \beta & \beta^2\\ 0 & \beta & \beta^2\\ 0 & 0 & \beta^2\end{bmatrix}.$$

Then the trace of I_3+M+M^2 is

(A) $8+i$

(B) $8+2i$

(C) i

(D) $4+i$

57. Crude Death Rate (CDR) is considered to be a

(A) probability measure

(B) ratio measure

(C) Lebesgue measure

(D) growth measure

58. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with common p.d.f.

$$f(x|\theta)=\begin{cases}e^{-x+\theta}, & x\geq\theta\\ 0, & \text{otherwise.}\end{cases}$$

Write $\overline{X}_n=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $Y_n=\min(X_1,X_2,\ldots,X_n)$.

Then as $n\to\infty$ $(\overline{X}_n-Y_n)$ converges in probability to

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) e^{-1}

59. Let f be twice differentiable on $\mathbb{R}$ such that $f''(x)>0$ for all $x\in\mathbb{R}$. Then

(A) $f(x)=0$ has exactly two solutions

(B) $f(x)=0$ has a positive solution if $f(0)=0$ and $f'(0)=0$

(C) $f(x)=0$ has no positive solution if $f(0)=0$ and $f'(0)>0$

(D) $f(x)=0$ has no positive solution if $f(0)<0$ and $f'(0)<0$

60. You are allowed to choose four whole numbers from 1 to 10 (both inclusive, without repeats). Which of the following is false?

(A) The numbers 4, 5, 6 and 7 have the smallest possible standard deviation.

(B) The numbers 1, 2, 3 and 4 have the smallest possible standard deviation.

(C) The numbers 1, 5, 6 and 10 have the largest possible standard deviation.

(D) The numbers 1, 2, 9 and 10 have the largest possible standard deviation.

61. Items from two categories, n_1 from category 1 and $n_2 = n - n_1$ from category 2, are arranged in an array such that the total number of runs is R. Then under the assumption of randomness, Var(R) is given by

(A) $\frac{2n_1n_2(n_1n_2 - n)}{n(n-1)}$

(B) $\frac{2n_1n_2(2n_1n_2 - n)}{n(n-1)}$

(C) $\frac{2n_1n_2(2n_1n_2 - n)}{n^2(n-1)}$

(D) $\frac{2n_1n_2(2n_1n_2 - n)}{n^2(n+1)}$

62. A spring is stretched by applying a load to its free end. The strain produced in the spring is

(A) volumetric

(B) shear but not longitudinal

(C) longitudinal and shear

(D) longitudinal but not shear

63. The resolvent kernel for the Volterra integral equation $\varphi(x) = 5x^3 + \lambda\int_a^x \varphi(s)\,ds$ is

(A) $e^{\lambda(x-t)}$

(B) $e^{\lambda(x+t)}$

(C) $e^{-\lambda xt}$

(D) $e^{\lambda xt}$

64. The number of Sylow p-subgroups of the symmetric group S_p, where p is prime, is

(A) 1

(B) p

(C) $(p-2)!$

(D) $(p-1)!$

65. If the polynomial

$$p(x) = \alpha + \beta(x+2) + \gamma(x+2)(x+1) + \delta(x+2)(x+1)x$$ interpolates the data

x	– 2	– 1	0	1	2
$f(x)$	1	– 1	5	7	– 15

then $\alpha + \beta + \gamma + \delta$ is equal to

(A) 0

(B) 2

(C) 1

(D) – 1

66. Let V be the real vector space of all (3×3) real matrices and let

$$A = \begin{bmatrix} a & b & 0 \\ 0 & a & 0 \\ 0 & 0 & c \end{bmatrix},$$

where a, b, c are non-zero real numbers. Which of the following is not subspace of V?

(A) $\{X \in V \mid XA = AX\}$

(B) $\{X \in V \mid X + A = A + X\}$

(C) $\{X \in V \mid \text{trace}(AX) = 0\}$

(D) $\{X \in V \mid \det(AX) = 0\}$

67. Which of the following is *not* true?

(A) Eigen vectors of distinct eigen values of a Hermitian matrix are orthogonal to each other

(B) Unitary matrices have eigen values of the form $e^{i\theta}$, where θ is real

(C) Eigen values of a real symmetric matrix are real

(D) Eigen values of a real symmetric matrix are not necessarily real

68. Which one of the following statements is *true*?

(A) The geodesic between two points in a plane is a semi-circle

(B) The geodesics on a sphere are its great circles

(C) The path of a particle (in absence of friction) between two points in the shortest time under gravity is a parabola

(D) The geodesics on a sphere are small circles

69. Given data on x and y_x for several equidistant values of x, if Δy_x tends to decrease by a constant percentage, then the most appropriate curve to be fitted through these points is

(A) Quadratic

(B) Modified Exponential

(C) Gompertz

(D) Logistic

70. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T(a, b, c) = (a + 2b - c, b + c, a + b - 2c)$. Then the nullity of T is

(A) 1

(B) 2

(C) 3

(D) 0

71. Let A be a 2 × 2 matrix such that trace $(A) = \det(A) = a$ $(a \neq 0)$. Then the trace of A^{-1} is

(A) a

(B) 1

(C) $\frac{1}{a}$

(D) $-\frac{1}{a}$

72. In Newton-Cotes formula, if $f(x)$ is interpolated at equally spaced nodes by a polynomial of degree three, then it represents

(A) Trapezoidal rule

(B) Simpson rule

(C) Three-eight rule

(D) Booles rule

73. For the exponential distribution

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

the p.d.f. of the 1st order statistic (with n independent observations) is

(A) $f_1(x) = \begin{cases} n e^{-nx}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

(B) $f_1(x) = \begin{cases} n\left(1 - e^{-x}\right)^n, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

(C) $f_1(x) = \begin{cases} \underline{|n} e^{-nx}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

(D) $f_1(x) = \begin{cases} \underline{|n}\left(1 - e^{-x}\right)^n, & x > 0 \\ 0, & \text{otherwise} \end{cases}$

74. Let $\{e_n\}$ be an orthonormal set in a Hilbert space H with the property:

if $x \in H$ is such that $< x, e_n >= 0 \forall n$, then $x = 0$.

Which of the following statements is not correct?

(A) For every h in H, there exists a sequence $\{\alpha_n\}_{n=1}^{\infty}$ of scalers such that $h = \sum_{n=1}^{\infty} \alpha_n e_n$

(B) $\{e_n\}$ is an orthonormal basis for H

(C) There exists at least one h in H for which the series $\sum_{n=1}^{\infty} |< h, e_n >|^2$ is not convergent

(D) For any h in H, $< h, e_n > \to 0$ as $n \to \infty$

75. Which of the following integrals is/are absolutely convergent?

I. $\int_1^{\infty} \frac{\sin x}{x^2} dx$

II. $\int_0^{\infty} e^{-x} \sin x \, dx$

(A) Only I

(B) Only II

(C) Both I and II

(D) Neither I nor II

76. Suppose $X_1, X_2, ...,$ are *i.i.d.* observations on Bernoulli(p) distribution where $p \in \left\{\frac{1}{4}, \frac{3}{4}\right\}$. Let R denote the number of 1's in the first n observations. The sequential probability ratio test of strength $(0{\cdot}1, 0{\cdot}1)$ for testing $H_0 : p = \frac{1}{4}$ against $H_1 : p = \frac{3}{4}$ terminates with the rejection of H_0 at the nth stage if

(A) $R \geq \frac{n}{2} + 1$

(B) $R \geq \frac{n}{2} - 1$

(C) $R \geq \frac{n+1}{2}$

(D) $R \geq \frac{n-1}{2}$

77. The covariance matrix Σ of a 3-component random vector $\underset{\sim}{X}$ is given by

$$\sum = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

It is known that the smallest eigenvalue of Σ is $0{\cdot}17$. The percentage of total variability of X that is accounted for by the first two principal components is

(A) 75%

(B) 78%

(C) 96%

(D) 98%

78. If the cost matrix for an assignment problem is given by

$$\begin{bmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{bmatrix},$$

where $a, b, c, d > 0$, then the value of the assignment problem is

(A) $a + b + c + d$

(B) $\min\{a, b, c, d\}$

(C) $\max\{a, b, c, d\}$

(D) $4 \min\{a, b, c, d\}$

79. Harmonic analysis is a method of determining which of the following components of a time series?

(A) Trend

(B) Seasonal

(C) Cyclical

(D) Irregular

80. A general solution of the partial differential equation $2\frac{\partial^2 z}{\partial x^2}+5\frac{\partial^2 z}{\partial x \partial y}+2\frac{\partial^2 z}{\partial y^2}=0$ is

(A) $z = \varphi\,(2y - x)$

(B) $z = \varphi\,(y - 2x)$

(C) $z = \varphi\,(2y - x) + \psi\,(y - 2x)$

(D) $z = \varphi\,(x - 5y) + \psi\,(x + 2y)$

81. In 3σ-control charts, the probability that the subgroup quality characteristics will lie between two limits under control situation is

(A) less than $\frac{8}{9}$

(B) greater than $\frac{8}{9}$

(C) less than $\frac{3}{4}$

(D) greater than $\frac{3}{4}$

82. The joint density of X_1, X_2 and X_3 is given by

$$f(x_1, x_2, x_3) = \begin{cases} (x_1 + x_2)e^{-x_3} & \text{for } 0 < x_1 < 1,\ 0 < x_2 < 1,\ x_3 > 0 \\ 0, & \text{elsewhere.} \end{cases}$$

Then the regression of X_2 on X_1 and X_3 is

(A) $\dfrac{x_1 + \frac{2}{3}}{2x_1 + 1}$

(B) $\dfrac{x_1}{x_1 + 1}$

(C) $x_1 + x_3$

(D) $\dfrac{x_1}{x_3}$

83. Given a system of linear equations $Ax = b$, if x_B is one of the basic solutions for given basis B, then

(A) $x_B = B^{-1} b^{-1}$

(B) $x_B b = B^{-1}$

(C) $x_B = B^{-1} b$

(D) $x_B = b\, B^{-1}$

84. We have a two component series system consisting of independent and identical components following exponential distribution with mean 2. The failure rate of the system is

(A) 4

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

85. A distribution is indexed by a k-component vector of unknown parameters $\underset{\sim}{\theta} = (\theta_1, \theta_2, \ldots, \theta_k)$ and a r-component vector of nuisance parameters $\underset{\sim}{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_r)$ where $r < k$. For testing $H_0 : \underset{\sim}{\theta} = \underset{\sim}{\theta}_0$ (specified) against $H_1 : \underset{\sim}{\theta} \neq \underset{\sim}{\theta}_0$ based on n i.i.d observations, if λ be the likelihood ratio test statistic, then under H_0, as $n \to \infty$, the asymptotic distribution of $-21n\,\lambda$ is

(A) χ^2_{k-r}

(B) χ^2_r

(C) χ^2_{k-r-1}

(D) χ^2_k

86. If $(2x - x^2 + ay^2)$ is a harmonic function, then the value of a is

(A) 1

(B) 2

(C) 3

(D) 0

87. Suppose X is distributed as Binomial (n, p) and it is required to estimate p under squared-error loss. If the prior distribution is uniform over $0 < p < 1$ the Bayes estimator is

(A) $\frac{X+1}{n+1}$

(B) $\frac{X}{n+2}$

(C) $\frac{X}{n+1}$

(D) $\frac{X+1}{n+2}$

88. Consider the model $Y_i = i\beta + \in_i$, $i = 1, 2, 3$, where $\in_1$, $\in_2$ and $\in_3$ are independent with mean 0 and variances σ^2, $2\sigma^2$ and $3\sigma^2$ respectively. Which of the following is the best linear unbiased estimator of β?

(A) $\frac{Y_1 + 2Y_2 + 3Y_3}{6}$

(B) $\frac{6}{11}\left(Y_1 + \frac{Y_2}{2} + \frac{Y_3}{3}\right)$

(C) $\frac{Y_1 + Y_2 + Y_3}{6}$

(D) $\frac{3Y_1 + 2Y_2 + Y_3}{6}$

89. Let (X, d) be a compact metric space. Which of the following is *not* true?

(A) X has Bolzano-Weierstrass property.

(B) Every Cauchy sequence in X is convergent.

(C) A closed subset of X is bounded.

(D) A bounded subset of X is complete.

90. Let $f(x) = x + \left(1 + e^x\right)^{-1}, x \in \mathbb{R}$.

Then f has

(A) infinitely many fixed points.

(B) no fixed point.

(C) exactly one fixed point.

(D) more than one but finitely many fixed points.

91. If K and K_n are curvatures of oblique and normal section through the same tangent line and θ be the angle between these sections, then we have

(A) $K_n = K^2 \cos\theta$

(B) $K_n = K^2 \sin\theta$

(C) $K_n = 2K \cos\theta$

(D) $K_n = K \cos\theta$

92. If $\sin x$ and $x \sin x$ are two linearly independent solutions of a linear homogeneous ordinary differential equation with constant coefficients, then the minimum order of the differential equation is

(A) 2

(B) 3

(C) 4

(D) 5

93. An example of a sampling scheme which is partly probabilistic and partly non-probabilistic is

(A) Purposive sampling

(B) Cluster sampling

(C) Two stage sampling

(D) Systematic sampling

94. Let A be a non-empty subset of a topological space X. Then which of the following statements is *true*?

(A) If A is connected, then its closure $\overline{A}$ is not necessarily connected

(B) If A is path connected, then its closure $\overline{A}$ is path connected

(C) If A is path connected, then its interior is connected

(D) If A is connected, then its interior is not necessarily connected

95. Consider the group S_9 of all permutations on a set with 9 elements. What is the largest order of a permutation in S_9?

(A) 21

(B) 20

(C) 30

(D) 14

96. In a split plot design, which of the following is *true*?

(A) Only split plot treatment is confounded

(B) Both split plot and whole plot treatments are confounded

(C) Only whole plot treatments are confounded

(D) Nothing is confounded

97. Which of the following is *true*?

(A) Separable Hilbert spaces may not be isometrically isomorphic to a l_2-space

(B) Normed linear spaces are not necessarily topological spaces

(C) Hilbert spaces are reflexive Banach spaces

(D) Banach spaces are always reflexive

98. The number of zeroes (counting multiplicity) of $P(Z) = 3\,Z^5 + 2\,i\,Z^2 + 7\,i\,Z + 1$ in the annular region $\{Z \in \mathbb{C} : 1 < |Z| < 7\}$ is

(A) 1

(B) 2

(C) 3

(D) 4

99. Which of the following is *false*?

(A) An entire function maps bounded set to bounded set

(B) A non-constant entire function is unbounded

(C) A non-constant entire function maps unbounded set to unbounded set

(D) There exists an entire function having ∞ as its regular point

100. For any two events A and B, write

$$\Delta = \{P(A \cap B^c)\}^2 + \{P(A \cap B)\}^2 + \{P(A^c)\}^2 .$$

Which of the following statements always holds?

(A) $\Delta \geq \frac{1}{3}$

(B) $\Delta \neq \frac{1}{3}$

(C) $\Delta = 1$

(D) $\Delta < \frac{1}{3}$

Space for Rough Work

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