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(Write Roll Number from left side exactly as in the Admit Card)

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Signature of Invigilator

Question Booklet Series

Y

PAPER-II

Question Booklet No.

Subject Code : 16

PHYSICAL SCIENCES

Time : 2 Hours

Maximum Marks: 200

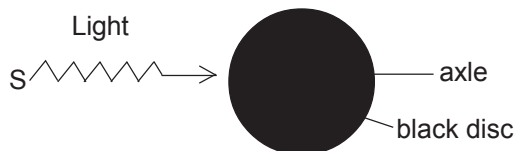
Instructions for the Candidates

- Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
- At the commencement of the examination, the Question Booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
 - To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
 - Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
 - Verify whether the Question Booklet Number is identical with OMR Sheet Number; if not, the full set is to be replaced.
 - After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
- This paper consists of One Hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
- Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.
Example: (A) (B) (C) (D) , where (C) is the correct response.
- Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
- Rough work is to be done at the end of this booklet.
- If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
- Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
- You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
- Use only **Black Ball point pen**.
- Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.
- There is no negative marks for incorrect answer.

**The Question Booklet
is encrypted with
QR code for
security purpose.**

PHYSICAL SCIENCES

1. A special visible light (6000 \AA) of 2 Watts power falls on a black disc, which can rotate on an axle parallel to the light beam as shown in figure:

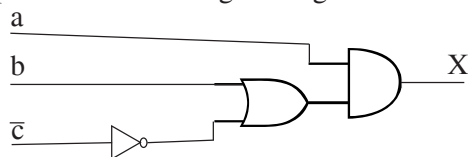


Consider that the black disc absorbs the light totally and the velocity of light is $c = 3 \times 10^8 \text{ m/s}$.

The torque exerted on the disc by the light is

- (A) $\frac{10^{-15}}{2\pi} \text{ N.m.}$
 (B) $\frac{2 \times 10^{-15}}{\pi} \text{ N.m.}$
 (C) $\frac{6 \times 10^{-7}}{\pi} \text{ N.m.}$
 (D) $4 \times 10^{-15} \text{ N.m.}$

2. Which Boolean expression describes the output 'X' in the arrangement given below?



- (A) $X = a + (b.c)$
 (B) $X = a.(b + c)$
 (C) $X = (a.b) + c$
 (D) $X = a + b + c$

3. Let U be an $n \times n$ unitary matrix. Which among the following cannot be a possible value of $\det U$?

- (A) $\frac{1 \pm i}{\sqrt{2}}$
 (B) ± 1
 (C) $e^{i\theta}$ (θ is a real no.)
 (D) $e^{\pm 1}$

4. Let $|p\rangle$ and $|n\rangle$ denote the isospin (I, I_z) states of proton and neutron respectively. Which of the following two-nucleon states corresponds to $I = 0$ and $I_z = 0$?

- (A) $\frac{1}{\sqrt{2}} [|n, n\rangle + |p, p\rangle]$
 (B) $\frac{1}{\sqrt{2}} [|n, p\rangle + |p, n\rangle]$
 (C) $\frac{1}{\sqrt{2}} [|n, n\rangle - |p, p\rangle]$
 (D) $\frac{1}{\sqrt{2}} [|n, p\rangle - |p, n\rangle]$

5. If a group is defined as

$$a * b = a + b - 1$$

then inverse of the group is

- (A) $2 - a$
 (B) a
 (C) a^{-1}
 (D) 0

6. A semiconductor specimen of Si has electron and hole mobilities of 0.13 and $0.05 \text{ m}^2/\text{V.s}$, respectively at 300 K . It is doped with P and Al with doping densities of $2.0 \times 10^{21}/\text{m}^3$ and $2.5 \times 10^{21}/\text{m}^3$, respectively. The conductivity of the doped Si sample at 300 K is

- (A) $7.5 \Omega\text{m}^{-1}$
 (B) $7.0 \Omega\text{m}^{-1}$
 (C) $7.25 \Omega\text{m}^{-1}$
 (D) $8.0 \Omega\text{m}^{-1}$

7. A simple pendulum is being operated in vacuum. Let θ_{\max} be the maximum amplitude such that its motion is described by simple harmonic oscillations to an accuracy of 5% . Then θ_{\max} is best approximated by

- (A) 30°
 (B) 10°
 (C) 5°
 (D) 45°

8. If the frequencies of vibrations of a solid are all equal and $\frac{h\nu}{K_B} = 100K$, then the molar specific heat of the solid at 500 K will be

(A) infinite
(B) 2R
(C) 3R
(D) 1.5 R

(R is the Universal gas constant.)

9. Consider a one-dimensional system with Hamiltonian $H = \frac{p^2}{2m} + \lambda x^6$ ($\lambda > 0$) in equilibrium with a heat bath kept at temperature T . The average thermal energy of the system is (K_B = Boltzmann constant)

(A) $\frac{3}{4} K_B T$
(B) $\frac{1}{6} K_B T$
(C) $\frac{5}{6} K_B T$
(D) $\frac{2}{3} K_B T$

10. After every second, a particle jumps one lattice point to the immediate right (probability 5/8) or left (probability 3/8) along a very long chain. If it starts at the origin at $t = 0$, what is the mean displacement after one hour?

(A) 900 units to the right
(B) 600 units to the right
(C) 200 units to the right
(D) 0 unit

11. χ is a spin -1 neutral particle. Consider the decays

(i) $\chi \rightarrow \gamma \gamma$
(ii) $\chi \rightarrow e^+ e^- \gamma$
(iii) $\chi \rightarrow \gamma \gamma \gamma$
(iv) $\chi \rightarrow \nu \bar{\nu} \gamma$

Which decays (is) are not allowed at all?

(A) Only (i)
(B) Only (i) and (iii)
(C) Only (iii) and (iv)
(D) All of (i), (iii) and (iv)

12. Consider two rockets 1 and 2 moving with a velocity $w = (w_x, 0, 0)$ and $-w = (-w_x, 0, 0)$, respectively, as measured from the frame S . Now the velocity of rocket 2 as measured from the frame of rocket 1 is

(A) $\left(\frac{-2w_x}{1 + \frac{w_x}{c}}, 0, 0 \right)$
(B) $\left(\frac{-2w_x}{1 + \frac{w_x^2}{c^2}}, 0, 0 \right)$
(C) $\left(\frac{-2w_x}{1 - \frac{w_x^2}{c^2}}, 0, 0 \right)$
(D) $\left(\frac{2w_x}{1 + \frac{w_x}{c}}, 0, 0 \right)$

13. Two bodies of mass $2m$ and $4m$ are connected by a spring of spring constant K . The frequency of normal mode is

(A) $\sqrt{\frac{K}{6m}}$
(B) $\sqrt{\frac{6K}{m}}$
(C) $\sqrt{\frac{3K}{4m}}$
(D) $\sqrt{\frac{4K}{3m}}$

14. Let $q_{Cl}(t)$ be a trajectory in the interval $[0, T]$ for a system defined by the Lagrangian $L = \frac{1}{2}(\dot{q}^2 - q^2)$. Let $\tilde{q}(t) \equiv q_{Cl}(t) + \epsilon \sin \frac{\pi t}{T}$ be another this is path infinitesimally close to $q_{Cl}(t)$. The second order deviation in the action,

$$\delta^2 S \equiv \delta S[\tilde{q}(t)] - \delta S[q_{Cl}(t)]$$

- (A) is positive for $T < \pi$ and negative for $T > \pi$
 (B) is always positive
 (C) vanishes
 (D) is negative for $T > \pi$ and positive for $T < \pi$

15. The number of ways by which N identical bosons can be distributed in 4 different energy levels is

- (A) $\frac{N!}{4!(N-4)!}$
 (B) $\frac{N(N-4)}{4!}$
 (C) $\frac{(N+3)!}{N! 3!}$
 (D) $\frac{N!}{4!}$

16. The spin-orbit interaction term for an electron moving in a central field is given by, $V(r) = f(r) \vec{L} \cdot \vec{S}$, where r is the radial distance of the electron from the origin. If the electron is moving inside a uniformly charged sphere, then

- (A) $f(r) = \frac{1}{r^3}$
 (B) $f(r) = \frac{1}{r^2}$
 (C) $f(r) = \frac{1}{r}$
 (D) $f(r) = \text{constant}$

17. The spin-parity of the ground state of a nucleus is $\frac{1}{2}^-$ and that of its first excited state is $\frac{5}{2}^-$. The polarity of the gamma radiation emitted by the excited nucleus is

- (A) E2 or E3
 (B) M2 or M3
 (C) E2 or M3
 (D) E3 or M2

18. Eigenfunctions of the Hamiltonian $\hat{H} = \hat{T} + \hat{V}$ of a harmonic oscillator are

- (A) eigenfunctions of \hat{T} as well as \hat{V}
 (B) eigenfunction of \hat{T} but not of \hat{V}
 (C) eigenfunction of \hat{V} but not of \hat{T}
 (D) eigenfunction of neither \hat{T} nor \hat{V}

19. Assume that free charges are present in a material of dielectric constant 11 and resistivity $10^{11} \Omega \text{ m}$. The time required for the charge density inside the material to decay by $\frac{1}{e}$ is closest to

- (A) 10^{-10} s
 (B) $11 \times 10^{-11} \text{ s}$
 (C) $11 \times 10^{11} \text{ s}$
 (D) 10 s

20. Given $Y = ABC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$. The simplified form of the given logic expression will be

- (A) $Y = A \text{ XOR } B$
 (B) $Y = A \text{ XNOR } C$
 (C) $Y = A \text{ XOR } C$
 (D) $Y = A \text{ XNOR } B$

21. The Hamiltonian of two spin- $\frac{1}{2}$ particles coupled by an interaction is described by $\hat{H} = A\hat{S}_1 \cdot \hat{S}_2$. Combination of these two spin- $\frac{1}{2}$ particles results in a joint entity with spin quantum number $S_{\text{tot}} = 0$ or 1. Then $\hat{S}_1 \cdot \hat{S}_2$ is equal to ($\hbar = 1$)

- (A) $\frac{1}{4}$ for $S_{\text{tot}} = 1$ and $-\frac{3}{4}$ for $S_{\text{tot}} = 0$
 (B) $\frac{3}{4}$ for $S_{\text{tot}} = 1$ and $-\frac{1}{4}$ for $S_{\text{tot}} = 0$
 (C) $-\frac{3}{4}$ for $S_{\text{tot}} = 1$ and $\frac{1}{4}$ for $S_{\text{tot}} = 0$
 (D) $-\frac{1}{4}$ for $S_{\text{tot}} = 0$ and $\frac{3}{4}$ for $S_{\text{tot}} = 0$

22. In a medium, an electromagnetic wave has a field given by

$$\vec{E} = 20e^{-0.02x} \sin[2 \times 10^8 t - 4x] \hat{z} \frac{V}{m}$$

where x, t are measured in SI units. The speed of light v in the medium and the skin depth δ are respectively given by

- (A) $v = \frac{c}{6}, \quad \delta = 50 \text{ m}$
 (B) $v = c, \quad \delta = 0.02 \text{ m}$
 (C) $v = \frac{2}{3}c, \quad \delta = 0.02 \text{ m}$
 (D) $v = c, \quad \delta = 50 \text{ m}$

23. If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, then the value of $A^{25} + A^{102}$ will be

- (A) $\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$
 (B) $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$
 (C) $\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$
 (D) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

24. A Dirac spinor will change its sign under a spatial rotation by an angle

- (A) $\frac{\pi}{2}$
 (B) π
 (C) $\frac{3\pi}{2}$
 (D) 2π

25. If the mean square fluctuation in the energy of a system at an equilibrium temperature T is proportional to T^n , then the energy of the system is proportional to

- (A) T^{n-2}
 (B) T^{n-1}
 (C) $T^{n/2}$
 (D) T^n

26. For Dy^{3+} ion with electron configuration $[\text{Xe}]4f^9$, the values of S, L and J are respectively

- (A) $\frac{5}{2}, 3, \frac{11}{2}$
 (B) $\frac{7}{2}, 3, \frac{13}{2}$
 (C) $\frac{5}{2}, 5, \frac{15}{2}$
 (D) $\frac{5}{2}, 7, \frac{19}{2}$

27. Given the following results:

$$(i) \frac{1}{2} + \sum_{n=1}^n \cos nx = \frac{\sin\left(n + \frac{1}{2}\right)x}{2 \sin x}$$

$$(ii) \frac{1}{2\pi} \int_0^\pi \frac{\sin\left(n + \frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)} dx = \frac{1}{2}$$

Choose the correct option.

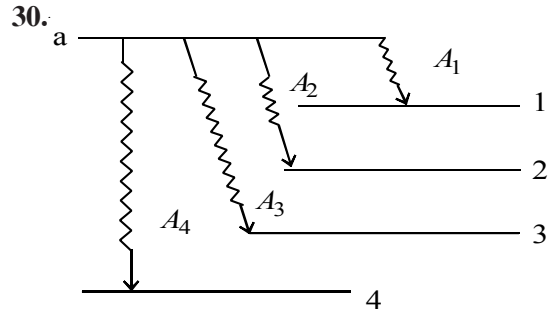
- (A) Only Statement (i) is correct
 (B) Only Statement (ii) is correct
 (C) Statements (i) and (ii) both are correct
 (D) Statements (i) and (ii) both are incorrect

28. Consider N electrons at $T = 0\text{K}$ in a $2d$ box of size $L \times L$. Its Fermi energy is E_F . Now, the box size is changed to $2L \times 2L$, then the new Fermi energy will be

- (A) $4E_F$
 (B) $\frac{E_F}{4}$
 (C) $\frac{E_F}{2}$
 (D) $2E_F$

29. Consider the band energies $E(k) = \pm V_0(1 + 2\cos ka)$, V_0 is a constant. The two bands will cross each other at the value ka equal to

- (A) $\frac{\pi}{4}$
 (B) $\frac{2\pi}{3}$
 (C) $\frac{\pi}{3}$
 (D) $\frac{\pi}{2}$



Let A_i be the transition rates shown in the above diagram for the atomic energy levels i from 'a' energy level. The life time of the atomic state 'a' is

- (A) $\frac{1}{A_1 + A_2 + A_3 + A_4}$
 (B) $\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}$
 (C) $\frac{1}{A_1 + A_2} + \frac{1}{A_3 + A_4}$
 (D) $\frac{1}{A_1 + A_3} + \frac{1}{A_2 + A_4}$

31. If the eccentricity of a planet's orbit is e , then the ratio of maximum to minimum speeds of the planet in its orbit is

- (A) $\frac{1+e}{1-e^2}$
 (B) $\frac{1+e^2}{1+e}$
 (C) $\frac{1+e}{1-e}$
 (D) $\frac{1+e^2}{1-e^2}$

32. The maximum attainable temperature of an ideal gas in a process given by pressure

$$p = p_0 - \alpha V^2 \quad (p_0, \alpha \text{ are constant})$$

- (A) $\frac{2}{3} p_0 \sqrt{\frac{p_0}{3\alpha}}$
 (B) $\sqrt{\frac{p_0}{3\alpha}}$
 (C) $\frac{1}{3} \sqrt{\frac{2p_0}{3\alpha}}$
 (D) ∞

33. The octal equivalent of the binary number 11111001 is

- (A) 761
 (B) 762
 (C) 371
 (D) 372

34. An electric field vector of an electromagnetic wave travelling in vacuum is represented as $\vec{E} = \vec{E}_0 (\cos(5x + 4y - wt))$ where w is a constant. The valid choice for the electric field vector \vec{E}_0 is

- (A) $\hat{i} + \frac{4}{5} \hat{j}$
 (B) $\hat{i} - \frac{4}{5} \hat{j}$
 (C) $\hat{i} + \frac{5}{4} \hat{j}$
 (D) $\hat{i} - \frac{5}{4} \hat{j}$

35. An electromagnetic wave is represented as $\vec{E}(x, t) = 2E_0 \cos(kx + wt) \hat{j} + E_0 \sin(kx + wt) \hat{k}$ where w, k are positive constants. This electromagnetic wave represents

- (A) a circularly polarised wave travelling in the negative x direction
 (B) an elliptically polarised wave travelling in the negative x direction
 (C) a linearly polarised wave travelling in the negative x direction
 (D) a circularly polarised wave travelling in the positive x direction

36. If the energy gap of a superconductor is 3.4×10^{-4} eV and the associated Fermi velocity of electrons is 2.02×10^6 m/s, then the intrinsic coherence length of the superconductor is

- (A) $3.9 \mu\text{m}$
 (B) 2.4 nm
 (C) $1.8 \mu\text{m}$
 (D) 1.8 nm

37. For a gas obeying $P(V-b) = RTe^{-a/RTV}$, the value of $\frac{\partial}{\partial V}(C_V)_T$ is

- (A) $\frac{Pa}{RVT^2}$
 (B) $\frac{2Pa^2}{RV^2T^2}$
 (C) $\frac{Pa^2}{R^2V^2T^3}$
 (D) $\frac{Pa^2}{R^2V^2T^2}$

38. Which of the following gates can be used as a parity checker?

- (A) An OR gate
 (B) A NOR gate
 (C) An XOR gate
 (D) An AND gate

39. A solid homogeneous cylinder of mass m and radius r rolls without slipping inside a hollow cylinder of radius $7r$. If the solid cylinder executes small oscillations about the stable point, what is the frequency?

- (A) $\frac{1}{3}\sqrt{\frac{g}{r}}$
 (B) $\sqrt{\frac{g}{r}}$
 (C) $\frac{1}{2}\sqrt{\frac{g}{r}}$
 (D) $2\sqrt{\frac{g}{r}}$

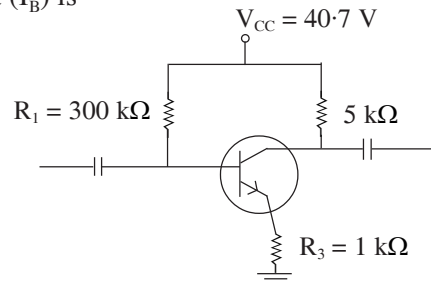
40. The mobility of the holes in a p -type semiconductor kept in a magnetic field of strength $B = 0.25$ T, having width and thickness equal to 5 mm each (Given, the measured values of the current and Hall voltage are $15\mu\text{A}$ and 75 mV respectively, and the resistivity of the material is 2.25×10^3 ohm-m) is

- (A) $0.0044 \text{ m}^2/\text{V-s}$
 (B) $0.044 \text{ m}^2/\text{V-s}$
 (C) $44 \text{ m}^2/\text{V-s}$
 (D) $0.44 \text{ m}^2/\text{V-s}$

41. Superconducting tin has a critical temperature 3.7 K at zero magnetic field and a critical field of 0.0306 Tesla at 0K. The critical field at 2K is

- (A) 0.0216 Tesla
 (B) 0.216 Tesla
 (C) 0.00216 Tesla
 (D) 0.025 Tesla

42. Consider the given circuit, where the common emitter forward amplification factor β of the transistor is 100 and $V_{BE} = 0.7$ V, the base current (I_B) is



- (A) 0.1 mA
 (B) 1 μA
 (C) 0.01 A
 (D) 1 mA

43. A class is composed of 60% boys and 40% girls. It is known that, in a given examination, 12% of the girls got an 'A' grade while only 6% of the boys did so. If a randomly selected student got an 'A' grade, the probability that it is a girl is closest to

- (A) 0.57
- (B) 0.60
- (C) 0.42
- (D) 0.67

44. An unperturbed two-level system has energy eigenvalues E_1 and E_2 and eigenfunctions

$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ respectively. When perturbed its

Hamiltonian is represented by

$$\begin{bmatrix} E_1 & A \\ A^* & E_2 \end{bmatrix}.$$

The first order correction to E_1 , is

- (A) 0
- (B) $2A$
- (C) A
- (D) $\frac{A}{2}$

45. An LED operates at 2 V and 16 mA in forward bias. Assuming a 90% external efficiency of the LED, the number of photons emitted per second is

- (A) 9×10^{14}
- (B) 2×10^{30}
- (C) 9×10^{16}
- (D) 2×10^{21}

46. An electromagnetic wave incident from a medium of $\epsilon = 3\epsilon_0$ and $\mu = \mu_0$ to its boundary with air, at $x = 0$. If the magnetic field of the medium is $\vec{H} = \hat{k} H_0 \cos\left(\omega t - kx - \frac{k}{\sqrt{3}}y\right)$ where, ω and k are positive constants, the angle of refraction is

- (A) 90°
- (B) 60°
- (C) 45°
- (D) 30°

47. A lattice is defined by the unit vectors

$$\vec{a}_1 = a\hat{i}, \vec{a}_2 = -\frac{a}{2}\hat{i} + \frac{a\sqrt{3}}{2}\hat{j} \quad \text{and} \quad \vec{a}_3 = a\hat{k}, \quad \text{where}$$

$a > 0$ is a constant. The spacing between

(100) planes of the lattice is

- (A) $\frac{\sqrt{3}}{2}a$
- (B) $\frac{a}{2}$
- (C) a
- (D) $\sqrt{2}a$

48. The interplanar angle θ between the plane $(h_1 \ k_1 \ l_1)$ with spacing d_1 and the plane $(h_2 \ k_2 \ l_2)$ with spacing d_2 for a cubic lattice is

$$(A) \cos \theta = \frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}$$

$$(B) \cos \theta = \frac{h_1^2 h_2^2 + k_1^2 k_2^2 + l_1^2 l_2^2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}$$

$$(C) \cos \theta = \frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2) + (h_2^2 + k_2^2 + l_2^2)}}$$

$$(D) \cos \theta = \frac{h_1^2 h_2^2 + k_1^2 k_2^2 + l_1^2 l_2^2}{\sqrt{(h_1^2 + k_1^2 + l_1^2) + (h_2^2 + k_2^2 + l_2^2)}}$$

49. The Lagrangian of a particle of mass m moving in one dimension is given by, $L = \frac{1}{2} m \dot{x}^2 - bx$, where b is a positive constant.

The co-ordinate of the particle $x(t)$ at time t is given as

$$(A) \ c_1 t + c_2$$

$$(B) \ -\frac{b}{2m} t^2 + c_1 t + c_2$$

$$(C) \ c_1 \cos\left(\frac{bt}{m}\right) + c_2 \sin\left(\frac{bt}{m}\right)$$

$$(D) \ c_1 \cosh\left(\frac{bt}{m}\right) + c_2 \sinh\left(\frac{bt}{m}\right)$$

50. For a gas of non-interacting massive bosons, Bose Condensation can take place in

- (A) one-dimension only
(B) three-dimensions only
(C) three or higher dimensions
(D) all dimensions

51. A LASER of 5 kW contains high intensity parallel beams. It is concentrated into a 10^{-4} cm^2 area. What is the magnitude of the Poynting vector?

- (A) $5 \times 10^7 \text{ W/m}^2$
(B) $5 \times 10^9 \text{ W/m}^2$
(C) $5 \times 10^{10} \text{ W/m}^2$
(D) $5 \times 10^{11} \text{ W/m}^2$

52. Let Ψ and $\bar{\Psi}$ represent respectively the 4-component Dirac spinor and its conjugate for a free electron. Which among the following quantities is a Lorentz scalar?

- (A) $\Psi \bar{\Psi}$
(B) $\bar{\Psi} \Psi$
(C) $\Psi^\dagger \Psi$
(D) $\bar{\Psi} \gamma^\mu \Psi$

53. The static dielectric constant of Ge is 15.8 and the effective mass $m_e = 0.1 m$ [$m \rightarrow$ isolated electron mass]. Then the donor ionization energy is

- (A) 5 meV
(B) 9 meV
(C) 12 meV
(D) 15 meV

54. For atomic hydrogen, the effective range of atom-atom force is about 0.4 nm. For such a gas in equilibrium, the maximum temperature below which the scattering is s-wave dominated is approximately

- (A) 2 K
(B) 10 K
(C) 20 K
(D) 40 K

55. In a Geiger-Muller detector the central anode wire is surrounded by a metallic cylindrical cathode tube. If the polarities of the wire and the tube are interchanged, then the system will

- (A) continue to detect in the same way as the original
- (B) continue to detect with an increasing efficiency compared to the original
- (C) continue to detect with a decreasing efficiency compared to the original
- (D) cease to detect

56. In a thermocouple pressure gauge, the temperature of heater element is a function of pressure for pressure range

- (A) above atmospheric pressure
- (B) below 10^{-5} mm of Hg
- (C) above 1 mm of Hg
- (D) below 10^{-3} mm of Hg

57. For the transformation $u = x + y, v = x / y$

with $x \geq 0$ and $y \geq 0$, the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ is given by

- (A) 0
- (B) 1
- (C) $-\frac{u}{(v+1)^2}$
- (D) $\frac{v}{(u+1)^2}$

58. The value of the integral

$$\int_{-\infty}^{+\infty} dx \delta(x^2 - \pi^2) (\sin x + \cos x) \text{ will be}$$

- (A) π
- (B) $-\frac{1}{2\pi}$
- (C) $-\frac{1}{\pi}$
- (D) 0

59. A particle of mass m is confined in the ground state of a one-dimensional box extending from $x = -2L$ to $x = 2L$. The wavefunction of the particle in this state is $\Psi(x) = \Psi_0 \cos \frac{\pi x}{4L}$, where Ψ_0 is constant. The energy eigenvalue corresponding to this state is

- (A) $\frac{\hbar^2 \pi^2}{2mL^2}$
- (B) $\frac{\hbar^2 \pi^2}{4mL^2}$
- (C) $\frac{\hbar^2 \pi^2}{16mL^2}$
- (D) $\frac{\hbar^2 \pi^2}{32mL^2}$

60. Two particles of mass m are confined in a one-dimensional length L . Their self-interaction is described by a hard-core potential of the form $V_0 \delta(x_1 - x_2)$. The first order shift in the ground state energy is

- (A) $\frac{3V_0}{2L}$
- (B) $\frac{V_0}{L}$
- (C) $\frac{2V_0}{L}$
- (D) $\frac{V_0}{2L}$

61. In which counter for the detection of thermal neutrons, boron is introduced in the form of BF_3 ?

- (A) Ionization chamber
- (B) Proportional counter
- (C) GM counter
- (D) Scintillation counter

62. Which of the following is an analytic function of the complex variable $Z = x + iy$ in the

domain $|Z| < 3$?

- (A) $(6 + x - iy)^9$
- (B) $(x + iy - 1)^{\frac{1}{2}}$
- (C) $(4 - 2x - iy)^4 (3 - x - iy)^3$
- (D) $(1 + x + iy)^3 (7 - x - iy)$

63. A one-dimensional system is governed by Hamiltonian $H = \frac{p^2}{2m} + \lambda |x|^3$ where $\lambda > 0$ is a constant. If $E_0(\lambda)$ is the energy of the ground state for a given λ , then the ratio $E_0(32) / E_0(1)$ is

- (A) $\sqrt{2}$
- (B) 4
- (C) $\sqrt{32}$
- (D) 32

64. For any vector \vec{A} , the value of

$|\vec{A} \times \vec{i}|^2 + |\vec{A} \times \vec{j}|^2 + |\vec{A} \times \vec{k}|^2$ will be

- (A) $|\vec{A}|^2$
- (B) $2|\vec{A}|^2$
- (C) $3|\vec{A}|^2$
- (D) $4|\vec{A}|^2$

65. Let $\vec{L} = (L_x, L_y, L_z)$ denote the orbital angular momentum operators of a particle and let, $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$. The particle is in an eigen state of L^2 and L_z with eigenvalues $\hbar^2 l(l+1)$ and $\hbar l$, respectively. The expectation value of $L_+ L_-$ in this state is

- (A) \hbar^2
- (B) $2\hbar^2$
- (C) 0
- (D) \hbar

66. If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\frac{K_B T}{\hbar \omega}$, the free-energy of N such independent oscillator is

- (A) $\frac{3}{2} N K_B T$
- (B) $K_B T \ln \frac{\hbar \omega}{K_B T}$
- (C) $N K_B T \ln \frac{\hbar \omega}{K_B T}$
- (D) $N K_B T \ln \frac{\hbar \omega}{2 K_B T}$

67. Classically estimated electronic heat capacity of a substance is $\frac{3}{2} N K_B$. If the Fermi temperature $T_F = 5 \times 10^4$ K, then at room temperature, the ratio of the electronic heat capacity to the classical value is

- (A) 1.050
- (B) 0.004
- (C) 0.002
- (D) 0.003

68. A particle of rest mass $m_0 = 9 \times 10^{-3}$ kg is moving with a velocity $\frac{4}{5}c$. The de-Broglie wavelength of the particle is

- (A) $\frac{4}{5} \frac{h}{m_0 c}$
 (B) $\frac{3}{4} \frac{h}{m_0 c}$
 (C) $\frac{4}{3} \frac{h}{m_0 c}$
 (D) $\frac{5}{4} \frac{h}{m_0 c}$

[Consider h is Planck's constant and c is the velocity of light in vacuum.]

69. A particle governed by the one-dimensional

Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2 \hat{x}^2$ is in a

state $|\psi\rangle$ such that $\langle\psi|\hat{\Pi}|\psi\rangle = 0$ where

$\hat{\Pi}$ is the parity operator. What is the

lowest possible value of $\langle\psi|\hat{H}|\psi\rangle = 0$?

- (A) $\hbar\omega$
 (B) $\frac{3}{2}\hbar\omega$
 (C) $2\hbar\omega$
 (D) $\frac{5}{2}\hbar\omega$

70. The Hamiltonian for a particle in

the $l = 3$ state is given by

$H = \frac{1}{\hbar^2} [2\hat{L}_x^2 + 2\hat{L}_z^2 + 2\hat{L}_y^2] + \frac{1}{20\hbar} \hat{L}_x$. To 1% accuracy, the energy of the ground state is

- (A) 15.0
 (B) 14.5
 (C) 15.6
 (D) 16.5

71. At normal incidence, a beam of light propagating in vacuum reflects off an interface with a medium of refractive index $n = 4$. The fraction of energy reflected R is given by

$$R = \left(\frac{n-1}{n+1} \right)^2.$$

If the fractional error in the value of n is 2%, then the fractional error in the estimation of R is

- (A) 0.53%
 (B) 1.4%
 (C) 1.54%
 (D) 2%

72. 1 mole of an ideal gas with initial volume V_1 is first expanded to volume V_2 and then to volume V_3 . The temperature T is kept constant throughout the process. The net work done is

(A) $RT \ln \frac{V_3}{V_1}$

(B) $RT \ln \left(\frac{V_2}{V_1} + \frac{V_3}{V_2} \right)$

(C) $RT \ln \frac{V_1}{V_3}$

(D) $RT \ln \left(\frac{V_1}{V_2} + \frac{V_2}{V_3} \right)$

73. An electron in an electron microscope with an initial velocity $v_0 \hat{i}$ enters a region of a stray transverse electric field $E_0 \hat{j}$. The time taken for the de Broglie wavelength to change from an initial value of λ to a final value of $\lambda/3$ is proportional to

- (A) E_0
 (B) $\frac{1}{E_0}$
 (C) $\frac{1}{\sqrt{E_0}}$
 (D) $\sqrt{E_0}$

74. For some event, observer X measures $\vec{E} = (\alpha, 0, 0)$ and $\vec{B} = (\alpha, 0, 2\alpha)$ in units of $c = 1$ and observer Y measures $\vec{E}' = (E'_x, \alpha, 0)$ and $\vec{B}' = (\alpha, B'_y, \alpha)$. Then, the value of E'_x and B'_y will be respectively

- (A) $2\alpha, -\alpha$
 (B) $-\alpha, 2\alpha$
 (C) $\alpha, -2\alpha$
 (D) $-\alpha, -2\alpha$

75. The canonical transformation from the phase space variables (q, p) to a new one (Q, P) is described by $P = \frac{p^2 + q^2}{2}$, $Q = \tan^{-1}\left(\frac{q}{p}\right)$.

Now the generating function is

- (A) $F = \frac{p^2}{2} \cdot \tan Q$
 (B) $F = -\frac{p^2}{2} \cdot \tan Q$
 (C) $F = \frac{q}{2} \cdot \tan Q$
 (D) $F = -\frac{q^2}{2} \cdot \tan Q$

76. The potential of a diatomic molecule as a function of the distance r between the atoms is given as $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$.

Now the value of the potential at equilibrium separation between the atoms is

- (A) $-\frac{4a^2}{b}$
 (B) $-\frac{2a^2}{b}$
 (C) $-\frac{a^2}{2b}$
 (D) $-\frac{a^2}{4b}$

77. The ground state spin-parity of ^{209}Bi - nucleus is $\frac{9}{2}^-$, and its radius is 7.12 fm. The shell model prediction of the quadrupole moment of ^{209}Bi - nucleus is

- (A) -0.11 barn
 (B) -0.22 barn
 (C) -0.33 barn
 (D) -0.44 barn

78. Given $f(x) = \begin{cases} 1 & |x| < 2 \\ -1 & 2 \leq |x| \leq 5 \\ 0 & |x| > 5 \end{cases}$

The Fourier transform of $f(x)$ is denoted by $F(k)$. For small k , $F(k)$ is given by

- (A) $-2 + \frac{109}{3}k^2$
 (B) $-2 + \frac{80}{3}k^2$
 (C) $k - \frac{40}{3}k^3$
 (D) $k + \frac{20}{3}k^3$

79. A spin $\frac{1}{2}$ particle is in the state $\frac{1}{\sqrt{6}} \begin{pmatrix} 1+i \\ 2 \end{pmatrix}$.

What is the probability of getting $\frac{\hbar}{2}$ when we measure S_x ?

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) $\frac{5}{6}$
- (D) $\frac{3}{8}$

80. Imagine a world with 4 spatial dimensions.

For a monoatomic gas, the ratio of specific heats

C_P/C_V is

- (A) $\frac{3}{2}$
- (B) $\frac{5}{3}$
- (C) 2
- (D) $\frac{4}{3}$

81. The magnitude of the directional derivative

of $\phi(x, y) = \frac{x}{x^2 + y^2}$ at (0,3) along a line making

an angle 45° with the positive x -axis is

- (A) $\frac{\sqrt{2}}{9}$
- (B) $\frac{1}{9\sqrt{2}}$
- (C) $\frac{1}{\sqrt{2}}$
- (D) $\frac{1}{81\sqrt{2}}$

82. Consider a sample of radiation contained in a volume V undergoing a quasistatic adiabatic change. If S denotes the entropy, then $T(S, V)$ will vary with volume V as

- (A) $\frac{C(S)}{V^{4/3}}$
- (B) $\frac{C(S)}{V^{1/3}}$
- (C) $\frac{C(S)}{V}$
- (D) $\frac{C(S)}{V^{2/3}}$

$C(S)$ is a constant depending on entropy.

83. The corresponding magnetic field due to the magnetic vector potential $\vec{A} = (\vec{F} \times \vec{r}) + 3y\hat{i}$, is (where \vec{F} is a constant vector and \vec{r} is the position vector)

- (A) $\vec{F} + 3\hat{k}$
- (B) $2\vec{F} - 3\hat{k}$
- (C) $2\vec{F} + 3\hat{k}$
- (D) $\vec{F} - 3\hat{k}$

84. The ratio of the decay lifetimes $T(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) : T(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$ is approximately

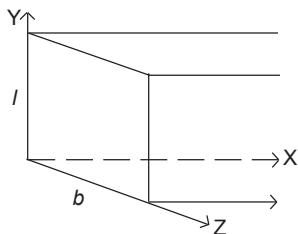
- (A) 7×10^{-7}
- (B) 1.4×10^6
- (C) 8×10^4
- (D) 1×10^{-5}

85. The doublet-splitting of first excited state,

$2^2P_{3/2} - 2^2P_{1/2}$ of He^+ is 5.84 cm^{-1} . The corresponding separation for H is

- (A) 1.460 cm^{-1}
- (B) 0.730 cm^{-1}
- (C) 0.365 cm^{-1}
- (D) 0.182 cm^{-1}

86. Consider a rectangular waveguide of dimension $(l \times b)$ as shown in the figure. The X -component of the magnetic field corresponding to the TE_{11} mode is given by $H_x = H_0 \cos(2\pi y) \cos(6\pi z)$. y & z are in cm. The area of cross-section of the waveguide is



- (A) $\frac{1}{12} \text{ cm}^2$
- (B) $\frac{1}{6} \text{ cm}^2$
- (C) 6 cm^2
- (D) 12 cm^2

87. The input impedance of a lossless transmission line is 100Ω when terminated in a short circuit and 64Ω when terminated in an open circuit. The characteristic impedance of the line is

- (A) 80Ω
- (B) 164Ω
- (C) 36Ω
- (D) 82Ω

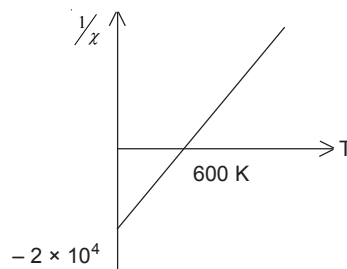
88. The change in collector current for a BJT connected in the CE configuration, when the base current changes by 0.6 mA (Given the current amplification factor $\alpha = 0.8$) is

- (A) 3.6 mA
- (B) 1.2 mA
- (C) 2.8 mA
- (D) 2.4 mA

89. How many Flips-Flops are required to build a binary counter circuit to count from 0 to 1023?

- (A) 1
- (B) 16
- (C) 10
- (D) 24

90. Inverse susceptibility $\left(\frac{1}{\chi}\right)$ as a function of temperature T for a material undergoing paramagnetic to ferromagnetic transition is shown in the figure:



The values of the Curie constant C and the Weiss molecular field constant λ in CGS unit are

- (A) $C = 5 \times 10^{-5}$, $\lambda = 3 \times 10^{-2}$
- (B) $C = 3 \times 10^{-2}$, $\lambda = 5 \times 10^{-5}$
- (C) $C = 3 \times 10^{-2}$, $\lambda = 2 \times 10^4$
- (D) $C = 2 \times 10^4$, $\lambda = 3 \times 10^{-2}$

91. X-ray of wavelength $\lambda = a$ is reflected from the [111] plane of a simple cubic lattice. If the lattice constant is a , the corresponding Bragg angle (in radian) is

- (A) $\frac{\pi}{6}$
 (B) $\frac{\pi}{4}$
 (C) $\frac{\pi}{8}$
 (D) $\frac{\pi}{3}$

92. The wave function of a particle moving under the influence of a spherically symmetric potential $V(r)$ is given by, $\Psi(r, \theta, \phi) = z f(r)$.

The eigenvalue of the L^2 operator for this state is

- (A) 0
 (B) $\hbar^2/4$
 (C) $2\hbar^2$
 (D) $\hbar^2/2$

93. For two ultrarelativistic particles constrained to move in a one-dimensional segment of length R , the Hamiltonian is given by

$H(p_1, p_2) = c[|p_1| + |p_2|]$. The volume of the phase space enclosed by a surface of energy E is given by

- (A) $2R^2 E^2 / c^2$
 (B) $4R^2 E^2 / c^2$
 (C) $R^2 E^2 / c^2$
 (D) $R^2 E^2 / 2c^2$

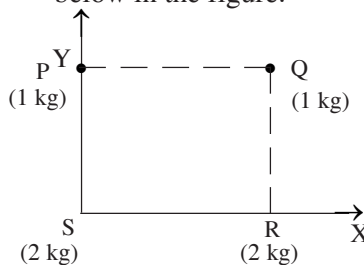
94. Audio signals are not transmitted by EM waves because

- (I) Antennas used will be very long.
 (II) Audio signals do not radiate.
 (III) Interference is due to simultaneous transmissions.
 (IV) Usable frequency range is too low and narrow.

Which of the following is correct?

- (A) (I) and (II)
 (B) (II) and (III)
 (C) (I) and (III)
 (D) (I), (III) and (IV)

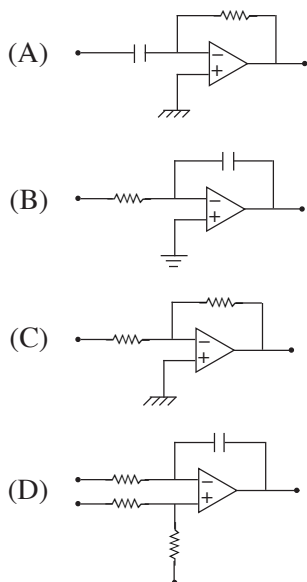
95. Particle of masses 1kg, 1kg, 2kg and 2kg are placed respectively at the corners P, Q, R, S of a square of side L as shown below in the figure.



The centre of mass is at

- (A) $\frac{L}{2} \hat{i} + \frac{L}{2} \hat{j}$
 (B) $\frac{L}{2} \hat{i} + \frac{L}{3} \hat{j}$
 (C) $\frac{L}{3} \hat{i} + \frac{L}{2} \hat{j}$
 (D) $\frac{L}{3} \hat{i} + \frac{L}{3} \hat{j}$

96. Which of the following represents a differentiator?



97. The eigenvalues of the Matrix

$$M = \begin{bmatrix} 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \end{bmatrix} \text{ is}$$

- (A) 0, 1, -1
 (B) 0, 0, 0
 (C) 0, $1 + i$, $-1 - i$
 (D) 0, i , $-i$

98. At time $t = 0$, the normalized wavefunction for a particle confined to $0 \leq x \leq L$ is given by $\Psi(x) = N \sum_{n=1}^{\infty} 2^{-n/2} \Psi_n(x)$ where $\Psi_n(x)$ are the normalized energy eigenstates. Then $N(\theta \in [0, 2\pi])$ is given by

- (A) $e^{i\theta}$
 (B) $2e^{i\theta}$
 (C) $\sqrt{\frac{2}{L}} e^{i\theta}$
 (D) $2\sqrt{\frac{2}{L}} e^{i\theta}$

99. A disk of radius $4m$ is kept at a plane perpendicular to the z -axis. An electromagnetic wave travelling in free space crosses the disk. The electric field of the wave is represented as $\vec{E}(z, t) = E_0 \cos(\omega t - kz) \hat{i}$, where $E_0 = 45 \text{ V/m}$. Find the value of the average power crossing the disk along z -direction.

- (A) 60 W
 (B) 90 W
 (C) 135 W
 (D) 180 W

100. An unstable neutral particle decays into two charged particles of kinetic energies 190 MeV and 30 MeV and corresponding momenta 300 MeV/c and 240 MeV/c, respectively. If the angle between the decay particles is 45° , then the rest mass of the neutral particle is

- (A) 450 MeV/c
 (B) 500 MeV/c
 (C) 550 MeV/c
 (D) 600 MeV/c

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work