Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admin Card)

Signature of Invigilator

Question Booklet Series **Y**

PAPER–II **Question Booklet No.**

(Identical with OMR Answer Sheet Number)

Subject Code : 16

PHYSICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D) , where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

PAPER II

(PHYSICAL SCIENCES)

1. The Fourier Series expansion of $f(x) = x^4$ within $-\pi \le x \le \pi$ is represented as

(A) $\frac{2\pi^4}{5} + \sum_{n=1}^{\infty} (-1)^n \left(\frac{8\pi^2}{n^2} - \frac{48}{n^4} \right) \cos nx$

(B) $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{n^2} \cos nx$

(C) $\frac{\pi^4}{5} + \sum_{n=1}^{\infty} (-1)^n \left(\frac{8\pi^2}{n^2} - \frac{48}{n^4} \right) \cos nx$

(D) $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} \cos nx$

2. The unit normal vector on the surface of the sphere $x^2 + y^2 + z^2 = 1$ at the point $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$ is

(A) $\hat{i} + \hat{j} + \hat{k}$

(B) $\frac{1}{\sqrt{3}} \hat{i} + \frac{1}{\sqrt{3}} \hat{j} + \frac{1}{\sqrt{3}} \hat{k}$

(C) $\frac{2}{\sqrt{3}} \hat{i} + \frac{2}{\sqrt{3}} \hat{j} + \frac{2}{\sqrt{3}} \hat{k}$

(D) $\sqrt{3}\,\hat{i} + \sqrt{3}\,\hat{j} + \sqrt{3}\,\hat{k}$

3. Electromagnetic wave of angular frequency ω is propagating in a medium in which over a band of frequencies the refractive index $n(\omega) \approx 1 - \left(\frac{\omega}{\omega_0} \right)^2$, where ω_0 is a constant.

The ratio of $\frac{v_g}{v_p}$ of the group velocity to the phase velocity at $\omega = \frac{\omega_0}{2}$ is

(A) $\frac{1}{4}$

(B) $\frac{2}{3}$

(C) 2

(D) 3

4. For Umklapp processes: $\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{G}$ [$G \rightarrow$ reciprocal lattice vector]. The energy of phonons associated with $\vec{k}_1$ and $\vec{k}_2$ suitable for Umklapp to occur is of the order of

[$\theta \rightarrow$ Debye temperature, k_B is the Boltzmann constant]

(A) $\frac{3}{2} k_B \theta$

(B) $k_B \theta$

(C) $\frac{1}{2} k_B \theta$

(D) $3 k_B \theta$

5. Consider atoms as hard, uniform spheres packed closely together so that they touch one another. The fraction of volume occupied in case of body centered and face centered cubic lattice is

(A) $\frac{\sqrt{3}}{8}\pi$ and $\frac{\pi}{6}$

(B) $\frac{\pi}{8}$ and $\frac{\pi}{6}$

(C) $\frac{\pi}{8}$ and $\frac{\sqrt{2}}{6}\pi$

(D) $\frac{\sqrt{3}}{8}\pi$ and $\frac{\sqrt{2}}{6}\pi$

6. A two dimensional lattice has the basis vectors $\vec{a} = 2\hat{x}; \vec{b} = \hat{x} + 2\hat{y}$. The reciprocal lattice vectors are respectively

(A) $\frac{1}{2}\hat{x} - \frac{1}{4}\hat{y}; \frac{1}{4}\hat{y}$

(B) $-\frac{1}{2}\hat{x} + \frac{1}{4}\hat{y}; \frac{1}{2}\hat{y}$

(C) $-\frac{1}{2}\hat{x} - \frac{1}{2}\hat{y}; \frac{1}{4}\hat{y}$

(D) $\frac{1}{2}\hat{x} - \frac{1}{4}\hat{y}; \frac{1}{2}\hat{y}$

7. Consider the free energy of a ferroelectric material as a function of polarization P and temperature T under an external uniform electric field $\vec{E}$ as $F(T,P,E)=F(T,0)+ \alpha_0 (T - T_c) P^2 + \alpha_4 P^4 - PE$ ($\alpha_0 > 0$; $\alpha_4 > 0$).

The inverse susceptibility $\left(\frac{1}{\chi}\right)$ will be given by

(A) $3\alpha_0(T_c - T)$

(B) $5\alpha_0(T_c - T)^2$

(C) $4\alpha_0(T_c - T)$

(D) $2\alpha_0(T_c - T)$

8. If the Gibbs' free energy per unit volume of a material in the superconducting and normal states at zero magnetic field are respectively $g_s(T)$ and $g_n(T)$, then the critical magnetic field strength is

(A) $H_C(T)=\left\{\frac{2}{\mu_0}[g_n(T)-g_s(T)]\right\}$

(B) $H_C(T)=\left\{\frac{2}{\mu_0}[g_n(T)-g_s(T)]\right\}^{1/2}$

(C) $H_C(T)=\left\{\frac{2}{\mu_0}[g_s(T)-g_n(T)]\right\}^{1/2}$

(D) $H_C(T)=\left\{\frac{2}{\mu_0}[g_s(T)-g_n(T)]\right\}$

[μ_0 = permeability of free space.]

9. The typical life time of an excited state of an atom is ~10^{-8} sec. If the energy difference between the excited and ground states of the atom is 1eV, the value of $\left(\frac{\Delta v}{v}\right)$ is

[Given: v is the frequency and Δv is the uncertainty in the frequency of the emitted photon]

(A) $3{\cdot}3 \times 10^{-8}$

(B) $3{\cdot}3 \times 10^{-7}$

(C) $3{\cdot}3 \times 10^{-7{\cdot}5}$

(D) $3{\cdot}3 \times 10^{-9}$

10. If a 6·4 V, 500 mW Zener diode is used in a regulator circuit, having maximum input voltage of 10 V, then the minimum value of resistance that should be connected in series to prevent any damage is

(A) 46·08 Ω

(B) 0·046 Ω

(C) 4608 Ω

(D) 4·608 Ω

11. What is the power developed in an AM wave in a load of 200 Ω when peak voltage of the carrier is 120 Volts and modulation factor is 0·5?

(A) 32 W

(B) 60·5 W

(C) 36 W

(D) 40·5 W

12. A crystal of MnO has NaCl structure. There is a paramagnetic to antiferromagnetic transition below ~ 120 K where the spins within a single [111] planes are parallel but the spins in adjacent [111] planes are anti-parallel. If the neutron scattering experiment is used to measure the lattice constants, d and d' respectively below and above the transition temperature of MnO, then

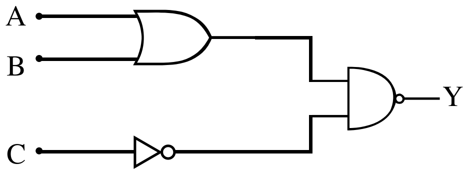
(A) $d = \frac{d'}{2}$

(B) $d = 2d'$

(C) $d = \frac{d'}{\sqrt{3}}$

(D) $d = \frac{d'}{\sqrt{2}}$

13. Find out 'Y' for the given circuit.



(A) $Y = \overline{A}B + \overline{C}$

(B) $Y = \overline{A}\,\overline{B} + \overline{C}$

(C) $Y = \overline{A + B} + \overline{C}$

(D) $Y = \overline{A}\,\overline{B} + C$

14. If $Z_{11} = 2\Omega$, $Z_{12} = 1\Omega$, $Z_{21} = 1\Omega$, $Z_{22} = 3\Omega$, then the determinant of admittance matrix will be

(A) 0

(B) 1

(C) 5

(D) $\frac{1}{5}$

15. An n-bit digital to analog converter has a resolution of about 0·1% of its full scale voltage. What is the value of n?

(A) 16

(B) 10

(C) 8

(D) 4

16. The binding energies of $^{41}Sc_{21}$ and $^{41}Ca_{20}$ nuclei are 343·143 MeV and 350·420 MeV, respectively. Given that $\frac{e^2}{4\pi\epsilon_0} = 1{\cdot}44$ MeV-fm, the prediction of the semi-empirical mass formula of the radius of these nuclei is

(A) 7·28 fm

(B) 4·09 fm

(C) 4·75 fm

(D) 3·79 fm

17. The magnetic moment of a proton is $\mu_p = 2{\cdot}79\ \mu_N$ and that of a neutron is $\mu_n = -1{\cdot}91\ \mu_N$, where μ_N is the nuclear magneton. The shell model prediction of the ground state magnetic moment of $^{33}S_{16}$ is

(A) $-1{\cdot}91\ \mu_N$

(B) $0{\cdot}88\ \mu_N$

(C) $2{\cdot}79\ \mu_N$

(D) $1{\cdot}15\ \mu_N$

18. A system of N particles can occupy discrete energy levels ε_r where $r = 0, 1, 2, 3$ ... etc. The system is in thermal equilibrium at an absolute temperature T. The energy levels are now shifted by an amount δ. As a result the specific heat at constant volume of the system will

(A) increase in magnitude.

(B) decrease in magnitude.

(C) remain unchanged.

(D) depend on δ.

19. Let there be a system of non-interacting particles in d-dimension obeying a dispersion relation like $\varepsilon \propto k^s$, where ε is the energy and k is the wave vector for each particle and s is an integer. For this system the density of states $P(\varepsilon) = dn/d\varepsilon$ is proportional to

(A) $\varepsilon^{\frac{s}{d}-1}$

(B) $\varepsilon^{\frac{d}{s}-1}$

(C) $\varepsilon^{\frac{d}{s}+1}$

(D) $\varepsilon^{\frac{s}{d}+1}$

20. A one dimensional gas system comprises of N non-interacting molecules contained within a length l. The Hamiltonian of each gas molecule of mass m, is given by

$$H = \frac{p_x^2}{2m} + \frac{1}{2}\alpha\left(x^2 + 4x\right)$$

At high temperature the specific heat of the gas is given by

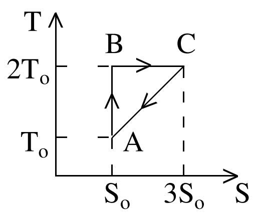
(A) $\frac{1}{2} Nk_B$

(B) $\frac{3}{2} Nk_B$

(C) $Nk_B - 2\alpha$

(D) Nk_B

21. Consider the following reversible cycle of an ideal Carnot engine represented by a right-angled triangle in a T-S diagram shown below:



The efficiency of the engine is

(A) 66%

(B) 50%

(C) 33%

(D) 25%

22. Consider a system of two atoms each having any 3 quantum states of energies 0, $\in$ and $2\in$. The system is in contact with a heat reservoir at temperature T. If the particles obey Bose-Einstein statistics, then the partition function in terms of $A = 1 + e^{-\beta\in} + e^{-2\beta\in}$ $(\beta = (k_B T)^{-1})$ will be

(A) A^2

(B) $e^{-\beta\in}A$

(C) $(1+ e^{-2\beta\in})\, A$

(D) $\frac{A}{2}$

23. The energy in terms of $k_B T$ and E_F at which the difference between the Boltzmann approximation and the Fermi-Dirac function is 5% of the Fermi function, will be

(A) $(1 + \ln 20) k_B T$

(B) $k_B T(\ln 20 - 1)$

(C) $\frac{k_B T}{2} \ln 20$

(D) $k_B T \ln 20$

24. A system is composed of a very large number N of non-interacting distinguishable particles, each of which is allowed three states of energy, viz., 0, $\in$, $2\in$ $(\in > 0)$. In the limit $N \to \infty$, let E_1 be the total energy when the system is not necessarily at equilibrium and let E_2 be the total energy when it is at equilibrium at some arbitrary positive temperature T. Then, [max(E_1), max(E_2)] are

(A) $2N\in, 2N\in/3$

(B) $2N\in, N\in/3$

(C) $N\in, N\in/2$

(D) $2N\in, N\in/2$

25. An electron is in the $2p_x$ state of the hydrogen atom. If F be the magnitude of the force acting on it, then the expectation value $\langle F \rangle$ in this state (here, E_0 is the energy of the ground state and a is the Bohr radius) is

(A) $\frac{|E_0|}{a}$

(B) $\frac{|E_0|}{4a}$

(C) $\frac{|E_0|}{4\sqrt{2}\,a}$

(D) 0

26. A system is described by a density matrix ρ as given in a basis spanned by $\{|\psi_i\rangle\}$. The entire Hilbert space H is now projected to a subspace by an operator P: $|\psi_i\rangle \to |\chi_i\rangle \equiv P|\psi_i\rangle$. In terms of $|\chi_i\rangle$, the density matrix for the projected system is

(A) $P\rho$

(B) $P\rho P$

(C) $\frac{P\rho P}{tr(P\rho)}$

(D) $\frac{P\rho}{tr(P\rho)}$

27. For Lithium, the Fermi energy and the electron density are respectively 4·70 eV and $4{\cdot}6 \times 10^{28}\text{m}^{-3}$. Then, the electron density of a metal having Fermi energy 2·35 eV will be

(A) $2{\cdot}3 \times 10^{28}\,\text{m}^{-3}$

(B) $1{\cdot}6 \times 10^{28}\,\text{m}^{-3}$

(C) $1{\cdot}8 \times 10^{28}\,\text{m}^{-3}$

(D) $2{\cdot}2 \times 10^{28}\,\text{m}^{-3}$

28. Consider a particle of mass m in a two dimensional potential well, where,

$$V(x, y) = \frac{1}{2} m\omega^2(4x^2 + 9y^2).$$

What is the energy of 1st excited state?

(A) $\frac{3}{2}\hbar\omega$

(B) $\frac{7}{2}\hbar\omega$

(C) $\frac{9}{2}\hbar\omega$

(D) $\frac{15}{2}\hbar\omega$

29. Consider a particle moving in one dimension under the influence of a parity even potential function $V(x)$. Let $|\psi\rangle = \alpha|\psi_0\rangle + \beta|\psi_1\rangle$ be a normalized state, where α and β are two real constant coefficients, and $|\psi_0\rangle$ and $|\psi_1\rangle$ are respectively, the ground and first excited states of the system. The expectation value of the position operator x in the state $|\psi\rangle$ is given by

(A) $\alpha^2 + \beta^2$

(B) $\alpha^2\langle\psi_0|x|\psi_0\rangle + \beta^2\langle\psi_1|x|\psi_1\rangle$

(C) $\alpha\beta\left[\langle\psi_0|x|\psi_1\rangle + \langle\psi_1|x|\psi_0\rangle\right]$

(D) 0

30. Let $\hat{\vec{J}}$ be the angular momentum operator and let $\vec{a}$ and $\vec{b}$ be two constant vectors. Then, the commutator $\left[\vec{a}\cdot\hat{\vec{J}}, \vec{b}\cdot\hat{\vec{J}}\right]$ equals

(A) $i\hbar\left(\vec{a}\times\vec{b}\right)\cdot\hat{\vec{J}}$

(B) $-i\hbar\sum_k a_k b_k J_k$

(C) $i\hbar\left(\vec{b}\times\vec{a}\right)\cdot\hat{\vec{J}}$

(D) $i\hbar\sum_k a_k b_k J_k$

31. L_x, L_y and L_z are the components of angular momentum $\vec{L}$, then the correct Poisson Bracket relations among them is

(A) $[L_x, L_y] = -L_z, [L_y, L_z] = L_x, [L_z, L_x] = L_y$

(B) $[L_x, L_y] = L_z, [L_y, L_z] = L_x, [L_z, L_x] = L_y$

(C) $[L_y, L_x] = L_y, [L_z, L_y] = L_x, [L_z, L_y] = L_z$

(D) $[L_x, L_y] = L_z, [L_y, L_z] = -L_x, [L_z, L_y] = L_y$

32. The electric field of a uniform plane wave propagating in a dielectric non-conducting medium is given by $\vec{E} = 5\cos\left(2\pi \times 10^7 t - 0{\cdot}2\pi\, x\right)\hat{j}$. The phase velocity of the wave is

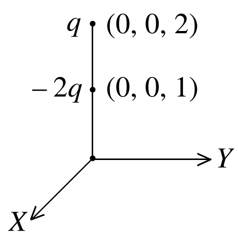
(A) 10^8 m/s

(B) 2×10^8 m/s

(C) 3×10^8 m/s

(D) $\frac{1}{2} \times 10^8$ m/s

33. Two point charges – 2q and q are located at (0, 0, 1) and (0, 0, 2) in front of a grounded infinite conducting plate that lies in the XY-plane as shown in figure.



The net force acting on charge q is $A\left(\frac{q^2}{4\pi \in_0}\right)\hat{k}$.

The value of A is

(A) $-\frac{265}{144}$

(B) $\frac{265}{144}$

(C) $\frac{137}{144}$

(D) $-\frac{137}{144}$

34. A plane electromagnetic wave travelling in free space is incident normally on a medium of refractive index 1·6. What is the transmittivity(T) of the medium? (Consider there is no absorption by the medium.)

(A) $\frac{3}{13}$

(B) $\frac{10}{13}$

(C) $\frac{9}{169}$

(D) $\frac{160}{169}$

35. An air filled rectangular waveguide has dimensions 6 cm. × 4 cm. The cut-off frequency for TE_{10} is

(A) 2·5 GHz

(B) 25 MHz

(C) 20·5 MHz

(D) 5 GHz

36. The number of independent elements of a real tensor of rank-4, symmetric with respect to only 2 indices and defined over an n-dimensional vector space, is

(A) $\frac{n!}{4!\,(n-4)!}$

(B) $n^2 \cdot \frac{n(n-1)}{2}$

(C) $n^2 \cdot \frac{n(n+1)}{2}$

(D) n^4

37. Find out the sum of the following series:

$b\sin\alpha + b^2\sin 2\alpha + b^3\sin 3\alpha + \dots$

(A) $\dfrac{b\sin\alpha}{1+b^2-2b\cos\alpha}$

(B) $\dfrac{b\sin\alpha}{1+b^2-2b\sin\alpha}$

(C) $\dfrac{b\cos\alpha}{1+b^2-2b\cos\alpha}$

(D) $\dfrac{b\cos\alpha}{1+b^2-2b\sin\alpha}$

38. Consider the matrix $M = \begin{bmatrix} -9 & 2 & 6 \\ 5 & 0 & -3 \\ -16 & 4 & 11 \end{bmatrix}$.

The eigenvalues of M are

(A) 2, 2, – 2

(B) 4, – 2, 0

(C) 0, 0, 2

(D) – 1, 1, 2

39. Consider $G = SU(2)\otimes SU(2)$. How many inequivalent 4-dimensional irreducible representations does G have?

(A) 3

(B) 2

(C) 1

(D) 4

40. The value of the integral $\int_0^\infty dx\, \delta(\cos x)e^{-x}$ is

(A) $\dfrac{1}{2}$

(B) $e^{-\pi/2}$

(C) $\dfrac{1}{2\sinh(\pi/2)}$

(D) $\dfrac{1}{2\cosh(\pi/2)}$

41. You are the quality control manager in a factory manufacturing a large number of electronic chips. While it is known that the probability of a chip being faulty is p, individual bottles must be checked. If the first *bod* chip is spotted in the N-th trial, what is the expected value of N?

(A) p^{-1}

(B) $(1-p)^{-1}$

(C) e/p

(D) e/p^2

42. The values of α and β for which the transformation $Q = q^{\alpha-1}\cos\beta p$ and $P = \sqrt{q}\sin\beta p$ is canonical, are

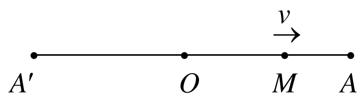
(A) $\alpha = 1, \beta = 1$

(B) $\alpha = 2, \beta = 1$

(C) $\alpha = 1, \beta = 2$

(D) $\alpha = \dfrac{1}{2}, \beta = 2$

43. A particle is under simple harmonic motion with a mean position at O and time period T. The particle passes the position M with speed v as shown in figure (Given $OM = d$). The time required to come back to position M is



(A) $\frac{T}{\pi}\tan^{-1}\frac{vT}{2\pi d}$

(B) $\frac{T}{\pi}\tan^{-1}\frac{vT}{\pi d}$

(C) $\frac{2T}{\pi}\tan^{-1}\frac{vT}{2\pi d}$

(D) $\frac{T}{2\pi}\tan^{-1}\frac{vT}{2\pi d}$

44. A linear chain is composed of three masses m, αm and m in that order. Consecutive masses are connected by springs of spring constant k each. For longitudinal small oscillations, the frequencies are $\left(\omega \equiv \sqrt{\frac{k}{m}}\right)$

(A) $0, \omega, \sqrt{\frac{\alpha+2}{\alpha}}\,\omega$

(B) $\omega, \omega, \sqrt{\frac{\alpha}{\alpha+2}}\,\omega$

(C) $0, \omega, \frac{\alpha+2}{\alpha}\,\omega$

(D) $\omega, \omega, \frac{\alpha}{\alpha+2}\,\omega$

45. A particle of mass m carrying charge q is at rest at the origin. An electric field $\vec{E} = \frac{a}{\rho}\hat{r}$ and a magnetic field $\vec{B} = B\hat{z}$ are applied on to it. Then, the constant of motion will be

(A) $mR^2\dot{\phi}$

(B) $mR^2\dot{\phi} - \frac{qB}{2}R^2$

(C) $\frac{qB}{2}R^2$

(D) $mR^2\dot{\phi} + \frac{qB}{2}R^2$

46. The half-life of a π-meson at rest is $1{\cdot}8 \times 10^{-8}$ sec. The half-life of the π-meson when it travels with a speed $c/2$ is

(A) $\frac{1}{\left(1-\frac{1}{4}\right)^{\frac{1}{2}}} \times 1{\cdot}8 \times 10^{-8}$ s

(B) $\frac{3}{\sqrt{\left(2-\frac{1}{4}\right)}} \times 1{\cdot}8 \times 10^{-8}$ s

(C) $\frac{1}{\left(1+\frac{1}{4}\right)^{\frac{1}{2}}} \times 1{\cdot}8 \times 10^{-8}$ s

(D) $\frac{1}{\left(1+\frac{1}{4}\right)} \times 1{\cdot}8 \times 10^{-8}$ s

47. Consider a particle in the region of potential $V(x) = kx^2 - \frac{nx^3}{2}$; where k and n are positive constants. The position of unstable equilibrium is at

(A) $x = 0$

(B) $x = \frac{4k}{3n}$

(C) $x = \frac{-4k}{3n}$

(D) $x = \frac{2k}{n}$

48. Consider, $V = 9x^2 - 2y^2 + F(z) - e^{3z}$ is the electrostatic potential (in volts) for a chargeless region. If z-component of electromagnetic field $E_z = 0\ V/m$ and $V = 0$ volt at the origin, then $F(z)$ can be written as

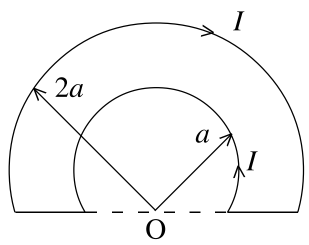
(A) e^{3z}

(B) $e^{3z} - 7z^2$

(C) $3e^{3z} - 14z$

(D) $3e^{3z} - 14z^2$

49. A steady current I flows through the loop as shown in figure.



The magnitude of the magnetic field induction at the point O is

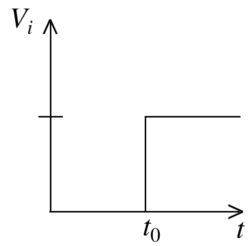
(A) 0

(B) $\frac{\mu_0 I}{2a}$

(C) $\frac{\mu_0 I}{4a}$

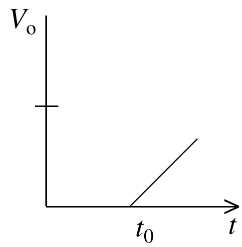
(D) $\frac{\mu_0 I}{8a}$

50. The input (V_i) given to an ideal OP-AMP integrator circuit is

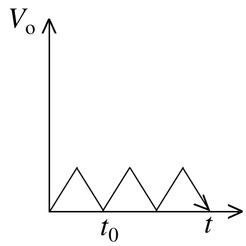


The correct output (V_o) of the integrator circuit is

(A)



(B)



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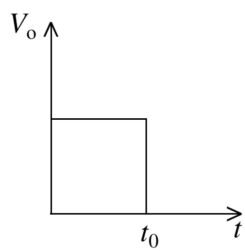

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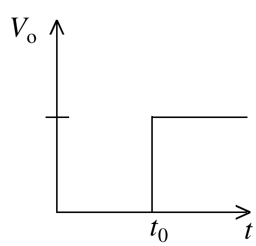


51. In an impure copper the collision time for phonon scattering and impurity scattering are respectively 2×10^{-10} s and 4×10^{-10} s. If for pure copper the effective collision time is 2×10^{-9} s, then the ratio in conductivities of pure to the impure copper is

(A) 30

(B) 15

(C) 20

(D) 25

52. A metallic ring of area 1 cm^2 and resistance 10Ω is placed in a time-varying magnetic field $B(\mathrm{t}) = 2e^{-0\cdot5\mathrm{t}}\cos(2\pi t)$, where B is measured in Tesla and t is in seconds. The net charge that flows past any point in the ring from $t = 0$ to $t = \infty$ is

(A) 1 μC

(B) 3 μC

(C) 5 μC

(D) 20 μC

53. The magnetic field inside a long solid cylinder is given as

$$\vec{H} = \frac{10^2}{r}\left[\frac{1}{\alpha}\sin(\alpha r) - \frac{r}{\alpha}\left(\cos(\alpha r)\right)\right]\hat{\phi}\ {}^{A}\!/_{m}$$

where, $\alpha = \frac{\pi}{a}$ and the radius $a = 0\cdot2$ m. What is the current [in ampere (A)] in the conductor?

(A) 16π

(B) 16

(C) 8

(D) $\frac{800}{\pi}$

54. The electric and magnetic field vector components corresponding to a plane electromagnetic field propagating in vacuum, respectively are

$$E_x = E_y = -E_z = \frac{1}{\sqrt{3}}|\vec{E}| \text{ and}$$

$$B_x = -B_y = \frac{1}{\sqrt{2}}|\vec{B}|,\ B_z = 0$$

The unit vector along the direction of propagation of the plane wave is

(A) $\frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$

(B) $-\frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$

(C) $\frac{2\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{3}}$

(D) $-\frac{2\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{3}}$

55. The Lagrangian for a particle moving in three dimensions is given by

$$L = m\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2}m\dot{z}^2 - \frac{k}{2}\left[\left(x + y\right)^2 + z^2\right]$$

How many independent additive constants of motion exist?

(A) 3

(B) 1

(C) 2

(D) 4

56. Consider the following relations defined in phase space:

(i) $\left(p^2 + a_1 x^2 - b_1\right)\left(p^2 + a_2 x^2 - b_2\right) = 0$

(ii) $x^2 p^2 = c^2$

(iii) $\left(p^2 + a_1 x^2 - b_1\right)\left(p - c_1 x\right) = 0$

where a_i, b_i, c_i are positive constants of appropriate dimension.

Which of the above are admissible as phase space orbits for some Hamiltonian?

(A) Both (i) and (ii) but not (iii)

(B) All three

(C) Both (i) and (iii) but not (ii)

(D) Both (ii) and (iii) but not (i)

57. A phase space represented by (q, p) is canonically transformed to a new one represented by (Q, P) by the relations $Q = q^2$ and $\mathrm{P} = \dfrac{p}{2q}$. Now the time independent generating function is

(A) $\dfrac{P}{q^2}$

(B) $q^2 P$

(C) $\dfrac{q^2}{P}$

(D) qP^2

58. The Lagrange's equations of motion for an electrical circuit comprising of an inductance (L) and capacitance (C') is [Given: The condensor is charged to q coulombs and the current flowing in the circuit is i amp.]

(A) $L\dfrac{d^2q}{dt^2} - \dfrac{q}{C'} = 0$

(B) $L\dfrac{d^2q}{dt^2} + \dfrac{q}{C'} = 0$

(C) $L\dfrac{d^2q}{dt^2} + \dfrac{q^2}{C'^2} = 0$

(D) $L\dfrac{d^2q}{dt^2} + \dfrac{q^2}{C'} = 0$

59. A mass m is attached, by means of two massless springs of relaxed length l and spring constant k, to two immovable objects located at coordinates (0, 0, 0) and $(2l, 0, 0)$. If the mass is given a small displacement from its equilibrium along the y-axis, then its subsequent motion is described by

(A) harmonic oscillations with frequency $\sqrt{\dfrac{k}{m}}$.

(B) harmonic oscillations with frequency $\sqrt{\dfrac{2k}{m}}$.

(C) harmonic oscillations with frequency $\sqrt{\dfrac{k}{2m}}$.

(D) anharmonic oscillations.

60. A particle of mass m moves under a force field $\vec{f} = -kr^4\ \hat{r},\ (k > 0)$ with angular momentum $\vec{L}$. If the path is circular, the radius of the circular path is

(A) $\left(-\frac{L^2}{mk}\right)^{\frac{1}{7}}$

(B) $\left(-\frac{L^2}{m^2k}\right)^{1/7}$

(C) $\left(-\frac{L}{mk}\right)^{+\frac{1}{7}}$

(D) $\left(-\frac{L}{mk^2}\right)^{\frac{1}{7}}$

61. A body is thrown vertically upwards with a velocity u from a point on the surface of the earth. It will fall back at another point displaced to the west by a distance

(A) $\frac{1}{3}\left(\frac{4h^3}{g}\right)^{\frac{1}{2}} \omega \cos\phi$

(B) $\frac{1}{3}\left(\frac{8h^3}{g}\right)^{\frac{1}{2}} \omega \cos\phi$

(C) $\frac{2}{3}\left(\frac{8h^3}{g}\right)^{\frac{1}{2}} \omega \cos\phi$

(D) $\frac{4}{3}\left(\frac{8h^3}{g}\right)^{\frac{1}{2}} \omega \cos\phi$

[symbols have their usual meanings]

62. If $S = \sum_{j=0}^{50} {}^{50}C_j\ {}^{150}C_{50-j}$, then S is best approximated by

(A) 10^{60}

(B) 10^{50}

(C) 10^{100}

(D) 10^{75}

63. The spherical unit vectors are expressed in terms of Cartesian unit vectors as

$$\hat{e}_\theta = \hat{e}_x \cos\theta\cos\phi + \hat{e}_y \cos\theta\sin\phi - \hat{e}_z \sin\theta$$

$$\hat{e}_\phi = -\hat{e}_x \sin\phi + \hat{e}_y \cos\phi$$

Which of the following statements is correct?

(A) Both $\hat{e}_\theta$ and $\hat{e}_\phi$ have odd parity.

(B) $\hat{e}_\theta$ has even parity while $\hat{e}_\phi$ has odd one.

(C) Both $\hat{e}_\theta$ and $\hat{e}_\phi$ have even parity.

(D) $\hat{e}_\theta$ has odd parity while $\hat{e}_\phi$ has even one.

64. A system can be either in the ground state or in the first excited state. The probability that it is in the ground state is given by p_g. The entropy would be maximum if p_g equals to

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

65. What are the eigenvalues of the operator $\vec{\sigma}\cdot\vec{p}$, where $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin$-\frac{1}{2}$ matrices and $\vec{p}$ is a vector?

(A) $\pm(p_x, p_y, p_z)$

(B) $p_x \pm ip_y + p_z$

(C) $p_x + p_y$ and p_z

(D) $\pm\left(p_x^2 + p_y^2 + p_z^2\right)^{\frac{1}{2}}$

66. Which among the following is a good quantum number for the Dirac Hamiltonian for a free electron?

(A) Spin angular momentum

(B) Orbital angular momentum

(C) Helicity

(D) Chirality

67. Two particles of mass m each are subjected to a potential $V(x)=\frac{1}{2}m\omega^2x^2$. If a interaction potential of the form

$$V_{\text{int}}(x_1,x_2)=V_0\left(x_1^2+x_2^2\right)\delta(x_1-x_2)$$

is switched on, the shift in the ground state energy, to the first order in V_0 is

(A) $V_0\sqrt{\frac{\hbar}{m\omega}}$

(B) $2V_0\sqrt{\frac{\hbar}{m\omega}}$

(C) $V_0\sqrt{\frac{\hbar}{2m\omega}}$

(D) $V_0\sqrt{\frac{2\hbar}{m\omega}}$

68. Consider the Hamiltonian of a system $\hat{H}=-\in_0\hat{h}_1\otimes\hat{h}_2$; where $\in_0$ is a constant, $\hat{h}_1=\begin{pmatrix}1&1\\1&1\end{pmatrix}=\hat{h}_2$. The symbol $\otimes$ denotes direct product. Which of the following statements is correct?

(A) The ground state energy is $-2\in_0$ and is two-fold degenerate.

(B) The ground state energy is $-2\in_0$ and is non-degenerate.

(C) The ground state energy is $-4\in_0$ and is three-fold degenerate.

(D) The ground state energy is $-4\in_0$ and is non-degenerate.

69. The Hamiltonian for a spinless particle is given by $H=\frac{5\alpha}{\hbar}\hat{L}_x-\frac{\alpha}{\hbar^2}\left(\hat{L}_y^2+\hat{L}_z^2\right)$, where α is a positive constant. If the orbital angular momentum of the particle is given to be $l=3$, then the ground state energy is

(A) $-18\,\alpha$

(B) $-16\,\alpha$

(C) $-20\,\alpha$

(D) $-14\,\alpha$

70. If the vector potential $\vec{A}=4x\hat{i}+my\hat{j}-6z\hat{k}$, satisfies the Coulomb gauge condition, the value of m is

(A) -2

(B) 2

(C) 10

(D) -10

71. The electric field for an electromagnetic wave is given as

$$E_x=5E_0\cos(7x+5y-300t)$$
$$E_y=7E_0\cos(7x+5y-300t+\pi)$$
$$E_z=0$$

Find the angle of polarization of the electric field vector.

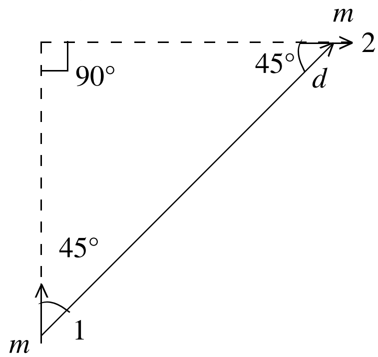
(A) $\tan^{-1}\left(-\frac{5}{7}\right)$

(B) $\tan^{-1}\left(\frac{7}{5}\right)$

(C) $\tan^{-1}\left(-\frac{7}{5}\right)$

(D) $\tan^{-1}\left(\frac{5}{7}\right)$

72. Two magnetic dipoles of magnitude m each are placed in a plane as shown in the figure. The energy of interaction is given by



(A) 0

(B) $\frac{\mu_0 m^2}{4\pi d^3}$

(C) $\frac{-3\mu_0 m^2}{8\pi d^3}$

(D) $\frac{-3\mu_0 m^2}{4\pi d^3}$

73. A certain state $|\psi\rangle$ satisfies $\hat{\vec{L}}^2|\psi\rangle = 6\hbar^2|\psi\rangle$ and $\hat{L}_x|\psi\rangle = 2\hbar|\psi\rangle$, where $\hat{\vec{L}}$ is the angular momentum operator. What is the uncertainty ΔL_z in the z-component?

(A) $\hbar$

(B) 0

(C) $2\hbar$

(D) $\frac{3}{2}\hbar$

74. Among the following, which one is a polar vector?

(A) Magnetic field

(B) Angular momentum

(C) Poynting vector

(D) Magnetic dipole moment due to a circular current loop

75. A polymer chain is composed of 1000 molecules (infinitesimally small), held by inter-molecular links of length d each. The angle between two consecutive links can only be 0° or 180° with equal probability. If the length of the polymer is $100d$, what is the number of possible microstates?

(A) $\frac{2\times 1000!}{550!\,450!}$

(B) $\frac{1000!}{550!\,450!}$

(C) $\frac{2\times 1000!}{600!\,400!}$

(D) $\frac{1000!}{600!\,400!}$

76. The entropy of a gas is determined as

$$S(E,V,N) = \left(\frac{k_B}{T_0}\right)^{1/2}\left(NE + \frac{k_B T_0 V^2}{v_0^2}\right)^{1/2}$$

where the constant T_0 and v_0 are positive intensive quantities with the dimensions of temperature and volume respectively. Then, the value of $C_V(T, V, N)$ for this gas will be

(A) 0

(B) $\frac{Nk_BT}{T_0}$

(C) $\frac{2Nk_BT}{T_0}$

(D) $\frac{Nk_BT}{2T_0}$

77. The wavefunction of a particle trapped in space between $x = 0$ and $x = L$ is expressed as $\Psi(x) = A\sin\left(\frac{2\pi x}{L}\right)$, where A is a constant. For which value(s) of x will the probability of finding the particle be the maximum?

(A) $\frac{L}{4}$

(B) $\frac{L}{2}$

(C) $\frac{L}{6}$ and $\frac{L}{3}$

(D) $\frac{L}{4}, \frac{3L}{4}, \frac{5L}{4}$ etc.

78. Which of the following processes is not allowed by strong interaction but is allowed by weak interaction?

(A) $P + n \rightarrow d + p + \overline{p}$

(B) $p + \Delta^{+} \rightarrow \overline{n} + \Delta^{++}$

(C) $p + n \rightarrow \Delta^{+} + K^{0}$

(D) $K^{0} + \pi^{0} \rightarrow \overline{K}^{0} + \pi^{+} + \pi^{-}$

79. Consider the Hamiltonian $H = J\sum_{i} S_i S_{i+1}$ for a one-dimensional chain of spins ($S_i = \pm 1$) with $J > 0$. At zero temperature, the ground state is

(A) ferromagnetic

(B) antiferromagnetic

(C) paramagnetic

(D) ferrimagnetic

80. If the scattering amplitude $f(\theta) = e^{+(\theta+\frac{\pi}{4})i}$, then total scattering cross-section is given by

(A) $\frac{\sqrt{2}\,\pi}{k}$

(B) $\frac{\pi}{2\sqrt{2}\,k}$

(C) $\frac{2\sqrt{2}\,\pi}{k}$

(D) $\frac{2\pi}{\sqrt{2}\,k}$

81. A monatomic ideal gas is contained in a vessel from which it leaks through a fine hole. The average kinetic energy of molecules leaving through the hole at a given temperature T will be

(A) $\frac{3}{2}k_B T$

(B) $\frac{1}{2}k_B T$

(C) $3k_B T$

(D) $2k_B T$

82. Consider the ratios of the branching fraction

$$\alpha \equiv \frac{Br\left(\rho^{0} \rightarrow \pi^{0}\pi^{0}\right)}{Br\left(\rho^{0} \rightarrow \pi^{+}\pi^{-}\right)} \text{ and } \beta \equiv \frac{Br\left(K^{+} \rightarrow \pi^{+}\pi^{-}e^{-}\overline{\nu}_e\right)}{Br\left(K^{-} \rightarrow \pi^{+}\pi^{-}e^{-}\overline{\nu}_e\right)}$$

Identify the correct combination.

(A) $\alpha \approx O$ and $\beta \approx O$

(B) $\alpha \sim O(1)$ and $\beta \sim O(1)$

(C) $\alpha \approx O$ and $\beta \sim O(1)$

(D) $\alpha \sim O(1)$ and $\beta \approx O$

83. Consider the three decays—

1. $\pi^{+} \rightarrow \mu^{+} + \nu_{\mu}$
2. $\pi^{+} \rightarrow e^{+} + \nu_{e}$
3. $\pi^{0} \rightarrow \gamma\gamma$
4. $\pi^{0} \rightarrow \gamma\gamma\gamma$

If the corresponding decay lifetimes are T_i, then

(A) $T_1 << T_2 << T_3 << T_4$

(B) $T_1 \sim T_2 << T_3 \sim T_4$

(C) $T_1 << T_2 \sim T_4 << T_3$

(D) $T_4 << T_3 << T_2 << T_1$

84. It is given that the decay $\Sigma^- \rightarrow \Lambda e^- \overline{\nu}_e$ proceeds through a dynamics that is identical to that of $n \rightarrow pe^- \overline{\nu}_e$. Given that the masses are (in MeV/c²)

$m_\Sigma \approx 1197{\cdot}4$ $m_\Lambda = 1116{\cdot}1$

$m_p \approx 938{\cdot}3$ $m_n \approx 939{\cdot}6$ $m_e \approx 0{\cdot}51$

the lifetime for this decay is approximately

(A) 10^{-7}s

(B) 10^{-6}s

(C) 10^{-8}s

(D) 10^{-5}s

85. Which of the following processes is ordinarily not allowed?

(A) $\pi^+ + p \rightarrow K^+ + \Sigma^+$

(B) $\pi^- + p \rightarrow K^- + \Sigma^+$

(C) $\pi^+ \rightarrow \mu^+ + \nu_\mu$

(D) $K^0 \rightarrow \pi^+ + \pi^- + \pi^0$

86. The Δ^0 baryon is in the $\left|\mathrm{I}, \mathrm{I}_3\right\rangle = \left|\frac{3}{2}, \frac{-1}{2}\right\rangle$ state.

The ratio of the branching fraction $\dfrac{Br\left(\Delta^0 \rightarrow \pi^0 + n\right)}{Br\left(\Delta^0 \rightarrow \pi^- + p\right)}$ is

(A) 2

(B) 1

(C) $1/2$

(D) $1/3$

87. The minimum number of NAND gates required to construct an OR gate is

(A) 6

(B) 2

(C) 3

(D) 4

88. The voltage resolution of a 8-bit digital to analog converter whose output varies from –5V to +5V is

(A) $\dfrac{5}{2^8 - 1}$

(B) $\dfrac{10}{2^8 - 1}$

(C) $\dfrac{5}{8}$

(D) $\dfrac{10}{8}$

89. For a particle moving in one-dimension, the probability density is given by

$$P(x) = \begin{cases} c \ , & |x| < b \\ 0 \ , & |x| \geq b \end{cases}$$

where c is a positive constant. Then, the minimum uncertainty in its momentum is

(A) $\dfrac{\sqrt{3}\hbar}{b}$

(B) $\dfrac{\hbar}{2b}$

(C) $\dfrac{\hbar}{b}$

(D) $\dfrac{\sqrt{3}\hbar}{2b}$

90. Neutrons of 1keV kinetic energy, when incident on a carbon target at rest, lead to an inelastic cross-section of 400 barns. If δ be the phase shift, then

(A) $\left|e^{2i\delta}\right| \approx \sqrt{\dfrac{3}{7}}$

(B) $\left|e^{2i\delta}\right| \approx \dfrac{3}{7}$

(C) $\left|e^{2i\delta}\right| \approx \sqrt{\dfrac{4}{7}}$

(D) $\left|e^{2i\delta}\right| \approx \sqrt{\dfrac{2}{7}}$

91. The co-ordinate of equivalent point of $\left(x, y+\frac{1}{2}, z\right)$, with the operation of $2_1(00z)$ diad screw is given by

(A) $\bar{x}, y+\frac{1}{2}, z+\frac{1}{2}$

(B) $x, \frac{1}{2}-y, z$

(C) $\bar{x}, \frac{1}{2}-y, z+\frac{1}{2}$

(D) $\bar{x}, y, \frac{1}{2}-z$

92. If the dispersion relation of electrons in graphene, which is considered as planar monatomic layer of carbon atoms, is taken as $\varepsilon(k) = \alpha k$ (where α is constant) over the entire k-space, then the Fermi energy ε_F depends on the number density of electrons ρ as

(A) $\varepsilon_F \propto \rho^{\frac{1}{2}}$

(B) $\varepsilon_F \propto \rho$

(C) $\varepsilon_F \propto \rho^{\frac{2}{3}}$

(D) $\varepsilon_F \propto \rho^{\frac{1}{3}}$

93. If the velocity of sound and interatomic spacing in a linear monatomic lattice are 3×10^3 ms^{-1} and 3×10^{-10} m respectively, then the cut-off frequency will be

(A) 10^{11} Hz

(B) 2×10^{11} Hz

(C) 3×10^{10} Hz

(D) 4×10^{11} Hz

94. The interaction between an electric field $\vec{E}$ acting in the z-direction and the first excited state (n = 2) of a H-atom, is represented by $H_{int} = -eEz$. If Ψ_{nlm} represents the hydrogenic wavefunction, which among the following matrix elements is non-vanishing?

(A) $\langle \psi_{200} | eEz | \psi_{211} \rangle$

(B) $\langle \psi_{200} | eEz | \psi_{210} \rangle$

(C) $\langle \psi_{200} | eEz | \psi_{21-1} \rangle$

(D) $\langle \psi_{200} | eEz | \psi_{200} \rangle$

95. If the specific heat of a material possesses both electronic and phononic contributions with corresponding coefficients α and β respectively, then the specific heat (C_v) at $t = -153$°C is given by

(A) $C_v = -153\alpha - (153)^3\beta$

(B) $C_v = 120\alpha + (120)^3\beta$

(C) $C_v = -(153)^3\alpha - 153\beta$

(D) $C_v = (120)^3\alpha + 120\beta$

96. The mean position of the first pair of lines of the principal series of sodium is 16960 cm^{-1}. If the convergence limit of the sharp series lines is at 24490 cm^{-1}, then the ionization potential of sodium is

[1eV = 8066 cm^{-1}]

(A) 2·10 eV

(B) 3·04 eV

(C) 0·94 eV

(D) 5·14 eV

97. A particle governed by the Hamiltonian $H = \frac{1}{\hbar^2}\hat{\vec{L}}\cdot\hat{\vec{S}}$ has angular momentum $L = 3$ and spin $S = 2$. The energy of the first excited state is

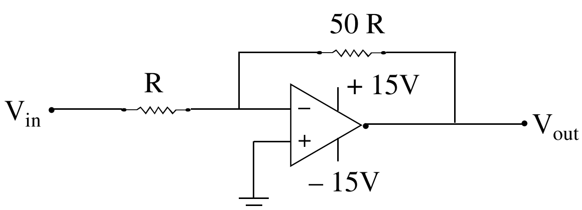
(A) – 6

(B) – 8

(C) – 3

(D) – 4

98. The magnitude of V_{out} (if $V_{in} = 2$ V) as per given diagram, is



(A) 0·1 V

(B) 0·01 V

(C) 10 V

(D) 100 V

99. Simplify the given 4-variable Boolean expression using the Karnaugh map.

F(A, B, C, D) = Σ (0, 2, 5, 7, 8, 10, 13, 15)

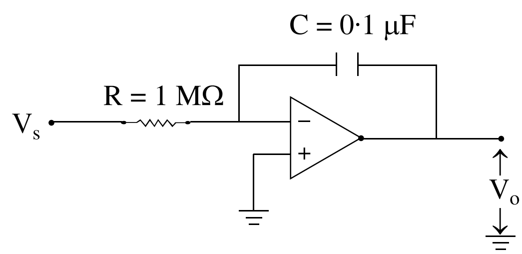
(A) $BD + \overline{B}\,\overline{D}$

(B) $A\overline{B} + \overline{C}D$

(C) $A\overline{B} + CD$

(D) $A\overline{B} + \overline{C}\,\overline{D}$

100.



If $V_s = 10 \sin(1000\,\pi t)$ mV is supplied to the OPAMP circuit, given in the figure, then the value of the output V_o is

(A) $\frac{1}{10\pi}\left[\cos\left(1000\,\pi t - 1\right)\right]$mV

(B) $\frac{1}{10\pi}\left[1 - \cos\left(1000\,\pi t\right)\right]$mV

(C) $\frac{\cos\left(1000\pi t\right)}{10\pi}$mV

(D) $\frac{\cos\left(1000\pi t\right)}{\pi}$mV

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work