MATHEMATICS

Maximum Marks: 80
Time Allotted: Three Hours
Reading Time: Additional Fifteen minutes

Instructions to Candidates

1. You are allowed an **additional fifteen minutes** for **only** reading the paper.
2. You must **NOT** start writing during reading time.
3. The question paper has **14 printed pages.**
4. The Question Paper is divided into **three** sections and has **22 questions** in all.
5. **Section A** is compulsory and has **fourteen** questions.
6. You are required to attempt **all** questions either from **Section B or Section C.**
7. **Section B** and **Section C** have **four** questions each.
8. Internal choices have been provided in **two** questions of **2 marks**, **two** questions of **4 marks** and **two** questions of **6 marks** in **Section A.**
9. Internal choices have been provided in **one** question of **2 marks** and **one** question of **4 marks** each in **Section B and Section C.**
10. While attempting **Multiple Choice Questions in Section A, B and C**, you are required to **write only ONE option as the answer.**
11. The intended marks for questions or parts of questions are given in the brackets [].
12. All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
13. Mathematical tables and graph papers are provided.

Instruction to Supervising Examiner

1. Kindly read aloud the instructions given above to all the candidates present in the examination hall.

Note: The Specimen Question Paper in the subject provides a realistic format of the Board Examination Question Paper and should be used as a practice tool. The questions for the Board Examination can be set from any part of the syllabus, though the format of the Board Examination Question Paper will remain the same as that of the Specimen Question Paper. The weightage allocated to various topics, as given in the syllabus, will be strictly adhered to.

SECTION A - 65 MARKS

Question 1

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) If A and B are square matrices of order 3, A is non-singular matrix and AB = **O**, then the matrix B is: **(Understanding)** **[1]**

(a) unit matrix

(b) Scalar matrix

(c) non-singular matrix

(d) null matrix

(ii) If m and n are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$ then the value of $(m - n)$ is: **(Recall)** **[1]**

(a) 0

(b) 1

(c) 2

(d) 3

(iii) The derivative of $xy = c^2$ with respect to x is: **(Understanding)** **[1]**

(a) $\frac{dy}{dx} = \frac{2c - y}{x}$

(b) $\frac{dy}{dx} = \frac{c^2}{x^2}$

(c) $\frac{dy}{dx} = \frac{-y}{x}$

(d) $\frac{dy}{dx} = \frac{y}{x}$

(iv) Consider $\Delta = \begin{vmatrix} 2a & 2b & 2c \\ 2e & f & g \\ 2i & j & k \end{vmatrix}$ **[1]**

Assertion: The value of $\Delta = 4 \times \begin{vmatrix} a & b & c \\ e & f & g \\ i & j & k \end{vmatrix}$

Reason: If all elements of one row or one column of a determinant are multiplied by a scalar, k then the value of the determinant is multiplied by k. **(Analysis)**

Which of the following is correct?

(a) Both Assertion and Reason are true, and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true, but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(v) Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3, ……., 18 and are arranged in the increasing order such that $x_1 < x_2 < x_3 < x_4 < x_5$. **[1]**

What is the probability that $x_2 = 7$ and $x_4 = 11$? **(Application)**

(a) $\frac{26}{51}$

(b) $\frac{3}{104}$

(c) $\frac{1}{68}$

(d) $\frac{1}{34}$

(vi) If $A = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$, then A^n equals to **(Understanding)** **[1]**

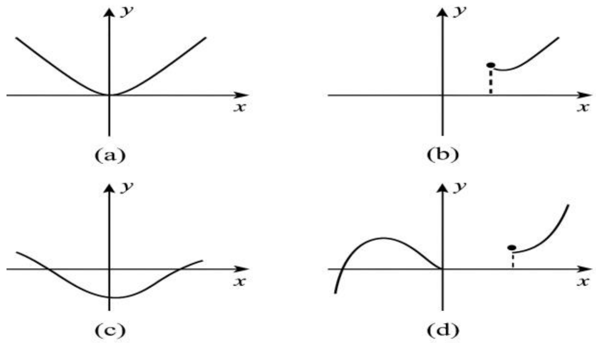
(a) $\begin{pmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{pmatrix}$

(b) $\begin{pmatrix} a & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{pmatrix}$

(c) $\begin{pmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a^n \end{pmatrix}$

(d) $\begin{pmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & a^n \end{pmatrix}$

(vii) Observe the following graphs (a), (b), (c), and (d), each representing different types of functions. **[1]**



Statement 1: A function which is continuous at a point may not be differentiable at that point.

Statement 2: Graph (c) is an example of a function that is continuous but not differentiable at the origin. **(Application)**

Which of the following is correct?

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(viii) If $f(x) = kx^2 + 7x - 4$ and $f'(5) = 97$ then what is the value of k? **[1]**

(Understanding)

(a) -4

(b) 0

(c) 4

(d) 9

(ix) **Statement 1**: If a relation R on a set A satisfies $R = R^{-1}$, then R is symmetric. **[1]**

Statement 2: For a relation R to be symmetric, it is necessary that $R = R^{-1}$

Which one of the following is correct? **(Understanding)**

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(x) **Assertion**: The equality $\tan(\cot^{-1} x) = \cot(\tan^{-1} x)$, is true for all $x \in R$ **[1]**

Reason: The identity $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, is true for all $x \in R$

Which of the following is correct? **(Application)**

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(xi) Given two events A and B such that $P\left(\frac{A}{B}\right) = 0.25$ and $P(A \cap B) = 0{\cdot}12$. **[1]**

The value $P(A' \cap B)$ is: **(Understanding)**

(a) 0·36

(b) 0·48

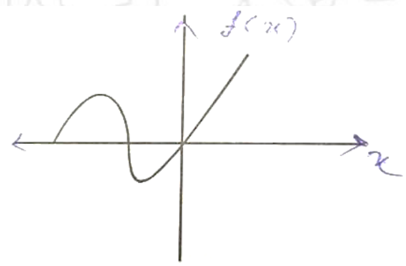
(c) 0·88

(d) 0·036

(xii) The value of the determinant of a matrix A of order 3 is 3. If C is the matrix of cofactors of the matrix A, then what is the value of determinant of C^2? **(Analysis)** **[1]**

(xiii) If a relation R on the set $\{a, b, c\}$ defined by $R = \{(b, b)\}$, then classify the relation. **[1]**
(Understanding)

(xiv) **[1]**



The given function $f: R \to R$ is many to one function. Give reason.
(Understanding)

(xv) There are three machines and 2 of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. What is the probability that only two tests are needed to identify the faulty machines? **(Application)** **[1]**

Question 2 **[2]**

(i) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$ **(Understanding)**

OR

(ii) Find the values of $'a'$ for which the function $f(x) = b - ax + sin\, x$ is increasing on R. **(Understanding)**

Question 3 **[2]**

Find the point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the end points (2,0) and (4,4). **(Analysis)**

Question 4 **[2]**

Show that the general solution of the differential equation: $\frac{dy}{dx} = ycot2x$ is $\log y = \frac{1}{2}\log|sin2x| + C$ **(Evaluate)**

Question 5 **[2]**

(i) If $\int x^5 \cos(x^6)\, dx = k\sin(x^6) + C$, find the value of '$k$'. **(Evaluate)**

OR

(ii) Evaluate: $\int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\, dx$ **(Evaluate)**

Question 6 **[2]**

A music streaming app uses the function: $f(x) = \tan^{-1}\left(\frac{x}{10}\right)$ to assign a *mood score* based on the number of hours a user listens to music per week.
Let the listening times (in hours/week) of two users, User A and User B, be 6 and 8 respectively.
Compute the combined mood score of user A and user B, that is, $f(6) + f(8)$.
(Application)

Question 7 **[4]**

Solve: $\sin^{-1}(x) + \sin^{-1}(1-x) = \cos^{-1} x$. **(Application)**

Question 8 **[4]**

If $y = (A + Bx)e^{-2x}$, prove that: $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ **(Application)**

Question 9 **[4]**

(i) Solve the following differential equation:

$$x\,\frac{dy}{dx} = y - x\,tan\left(\frac{y}{x}\right)$$ **(Application)**

OR

(ii) Solve the following differential equation:

$$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$$ **(Application)**

Question 10 **[4]**

(i) Three friends go to a restaurant to have pizza. They decide who will pay for the pizza by tossing a coin. It is decided that each one of them will toss a coin and if one person gets a different result (heads or tails) than the other two, that person would pay. If all three get the same result (all heads or all tails), they will toss again until they get a different result. **(Analysis)**

(a) What is the probability that all three friends will get the same result (all heads or all tails) in one round of tossing?

(b) What is the probability that they will get a different result in one round of tossing?

(c) What is the probability that they will need exactly four rounds of tossing to determine who would pay?

OR

(ii) A school offers students the choice of three modes for attending classes:

- Mode A: Offline (in-person) – 40% of students
- Mode B: Online (live virtual classes) – 35% of students
- Mode C: Recorded lectures – 25% of students

After a feedback survey:

- 20% of students from Mode A reported the class as "Excellent"
- 30% from Mode B rated it as "Excellent"
- 50% from Mode C rated it as "Excellent"

A student is selected at random from the entire group, and it is found that they rated the class as "Excellent." **(Analysis)**

(a) Represent the data in terms of probability. Define the events clearly.

(b) Using Bayes' Theorem, find the probability that the student attended the Recorded lectures (Mode C), given that they rated the class as "Excellent."

(c) Interpret your result. Which mode has the highest likelihood of being chosen if a student says "Excellent"?

Question 11 **[6]**

To raise money for an orphanage, students of three schools A, B and C organised an exhibition in their residential colony, where they sold paper bags, scrap books and pastel sheets made by using recycled paper. Student of school A sold 30 paper bags, 20 scrap books and 10 pastel sheets and raised ₹ 410. Student of school B sold 20 paper bags, 10 scrap books and 20 pastel sheets and raised ₹ 290. Student of school C sold 20 paper bags, 20 scrap books and 20 pastel sheets and raised ₹ 440.

Answer the following question:

(i) Translate the problem into a system of equations.

(ii) Solve the system of equation by using matrix method.

(iii) Hence, find the cost of one paper bag, one scrap book and one pastel sheet.

(Application)

Question 12 **[6]**

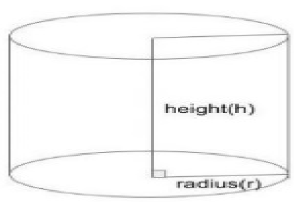
(i) Evaluate: $\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$ **(Evaluate)**

OR

(ii) Evaluate: $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x\, dx}{1+\sin x} dx$ **(Evaluate)**

Question 13 **[6]**

(i) A person has manufactured a water tank in the shape of a closed right circular cylinder. The volume of the cylinder is $\frac{539}{2}$ cubic units. If the height and radius of the cylinder be h and r. **(Application)**



(a) Express h in terms of radius r and given volume.

(b) Let the total surface area of the closed cylinder tank be S, express S in term of radius r.

(c) If the total surface area of the tank is minimum, then prove that radius r = $\frac{7}{2}$ units.

(d) Find the height of the tank.

OR

(ii) A Dolphin jumps and taken a path given by the equation $h(t) = \frac{1}{2}(-7t^2 + 3t + 2),\ (t \geq 0)$, $h(t)$ is the height of the Dolphin at any point of time. **(Application)**

(a) Is the function differentiable for $t \geq 0$? Justify.

(b) Find the instantaneous rate of change of height at $t = \frac{1}{14}$.

(c) $h(t)$ is increasing in $\left(-\infty, \frac{3}{14}\right)$. Is this true or false? Justify.

(d) Find the time at which the Dolphin will attains the maximum height. Also find the maximum height.

Question 14 **[6]**

In a school, three subject teachers English, Math, and Science sometimes give surprise tests on the same day. Based on past records:

- The English teacher gives a test 90% of the time
- The Math teacher gives a test 80% of the time
- The Science teacher gives a test 70% of the time

Each teacher decides independently. If the average number of surprise tests is less than 2.3 then the teachers should coordinate better to increase the performance of the students. Otherwise, no action is needed.
Let X be the number of surprise tests a student gets on a given day. So, $X \in \{0,1,2,3\}$. **(Application)**

(i) Find the probability for each possible number of surprise tests.

(ii) Use the probabilities to build a distribution table.

(iii) Calculate the average number of surprise tests per day.

(iv) Based on your calculations, decide: Should the teachers coordinate better? Or is the current plan acceptable?

SECTION B - 15 MARKS

Question 15 **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Consider the following statements and choose the correct option:

Statement 1: If $\vec{a}$ and $\vec{b}$ represents two adjacent sides of a parallelogram then the diagonals are represented by $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

Statement 2: If $\vec{a}$ and $\vec{b}$ represents two diagonals of a parallelogram then the adjacent sides are represented by $2(\vec{a}+\vec{b}\,)$ and $2(\vec{a}-\vec{b})$.

Which of the following is correct? **(Recall)**

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(ii) A plane passes through three points A, B and C with position vectors $\hat{\imath}+\hat{\jmath}, \hat{\jmath}+\hat{k}$ and $\hat{k}+\hat{\imath}$ respectively. The equation of the line passing through the point P with position vector $\hat{\imath}+2\hat{\jmath}+2\hat{k}$ and normal to the plane is **(Application)**

(a) $\vec{r}=\left(\hat{\imath}+2\hat{\jmath}+2\hat{k}\right)+\lambda\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right), \lambda \in R$

(b) $\vec{r}=\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)+\lambda\left(\hat{\imath}+2\hat{\jmath}+2\hat{k}\right), \lambda \in R$

(c) $\vec{r}\cdot\left(\hat{\imath}-\hat{\jmath}-\hat{k}\right)=\hat{\imath}+2\hat{\jmath}+2\hat{k}$

(d) $x-1=y=z$

(iii) If the direction cosines of a line are $<\frac{1}{c},\frac{1}{c},\frac{1}{c}>$ then **(Understanding)**

(a) $c > 0$

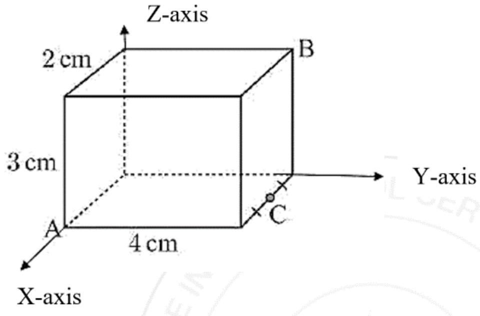
(b) $0 < c < 1$

(c) $c = \pm\sqrt{3}$

(d) $c > 2$

(iv) If $\vec{a}$ and $\vec{b}$ are unit vectors enclosing an angle θ and $\left|\vec{a}+\vec{b}\right| < 1$, then find the values between which θ lies. **(Understanding)**

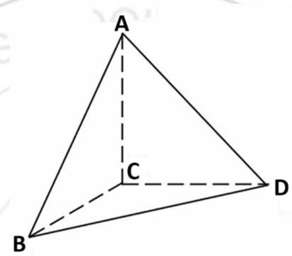
(v) Shown below is a cuboid. Find $\overrightarrow{BA}\,.\,\overrightarrow{BC}$



(Analysis)

Question 16 **[2]**

(i)



A building is to be constructed in the form of a triangular pyramid ABCD as shown in the figure. Let the angular points be $A\,(0,1,2), B\,(3,0,1), C\,(4,3,6)$ and $D\,(2,3,2)$ and let G be the point of intersection of the medians of ΔBCD.

Using the above information, answer the following **(Application)**

(a) What will be the length of vector $\overrightarrow{AG}$?

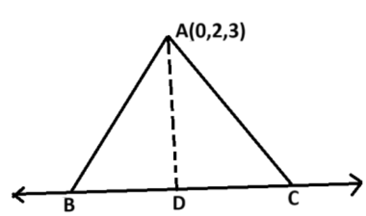
(b) Find the area of ΔABC.

OR

(ii) What are the values of x for which the angle between the vectors?
$2x^2\hat{\imath} + 3x\hat{\jmath} + \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + x^2\hat{k}$ is obtuse? **(Application)**

Question 17 **[4]**

(i)



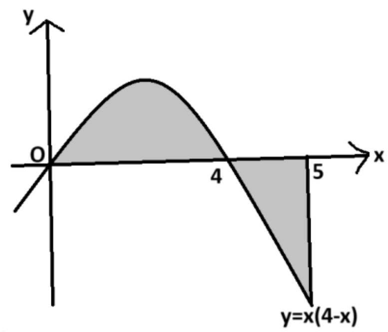
Given, B and C lie on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ and BC = 5 units find the area of ΔABC.

(Application)

OR

(ii) Find the equation of the plane containing the line $\frac{x}{-2} = \frac{y-1}{3} = \frac{1-z}{1}$ and the point $(-1, 0, 2)$. **(Application)**

Question 18 **[4]**



Find the area bounded by the curve, $y = x(4 - x)$ and the x-axis from $x = 0$ to $x = 5$ as shown in the figure given above. **(Application)**

SECTION C - 15 MARKS

Question 19 **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Which condition is true if Average Cost (AC) is constant at all levels of output? **(Recall)**

(a) MC > AC

(b) MC = AC

(c) MC < AC

(d) MC = $\frac{1}{2}$ AC

(ii) Which of the following statement(s) is/are correct with respect to regression coefficients?

Statement 1: It measures the degree of linear relationship between two variables.

Statement 2: It gives the value by which one variable changes for a unit change in the other variable.

Which of the following is correct? **(Recall)**

(a) Statement 1 is true and Statement 2 is false.

(b) Statement 2 is true and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(iii) Mean of $x = 53$, mean of $y = 28$ regression co-efficient y *on* $x = -1 \cdot 2$, regression co-efficient x *on* $y = -0 \cdot 3$. Find coefficient of correlation (r). **(Understanding)**

(iv) The total revenue received from the sale of x unit of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue when $x = 5$. **(Understanding)**

(v) A manufacturing company finds that the daily cost of producing x item of product is given by $C(x) = 210x + 7000$. Find the minimum number that must be produced and sold daily for break even, if each item is sold for ₹280. **(Understanding)**

Question 20 **[2]**

(i)

A real estate company is going to build a new residential complex. The land they have purchased can hold at the most 500 apartments. Also, if they make x apartments, then the monthly maintenance cost for the whole complex would be as follows:
Fixed cost = ₹ 4000
Variable cost = ₹ $(14x - 0 \cdot 04x^2)$
How many apartments should the complex have in order to minimize the maintenance costs? **(Application)**

OR

(ii) The demand function of a monopoly is given by $x = 100 - 4p$. Find the quantity at which the Marginal Revenue will be zero. **(Application)**

Question 21 **[4]**

A survey of 50 families to study the relationships between expenditure on accommodation in (₹ x) and expenditure on food and entertainment (₹ y) gave the following results:

$$\sum x = 8500,\ \sum y = 9600, \sigma_x = 60, \sigma_y = 20, r = 0.6$$

Estimate the expenditure on food and entertainment when expenditure on accommodation is ₹200. **(Application)**

Question 22 **[4]**

(i) A linear programming problem is given by $Z = px + qy$ where $p, q > 0$ subject to the constraints:

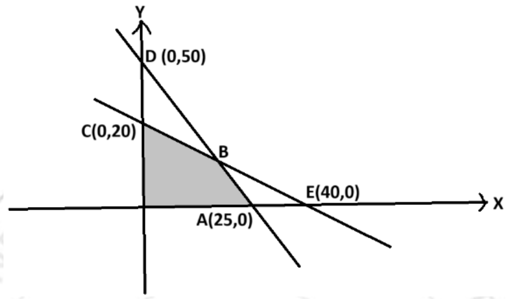
$x + y \leq 60,\ 5x + y \leq 100,\ x \geq 0 \text{ and } y \geq 0$ **(Analysis)**

(a) Solve graphically to find the corner points of the feasible region.

(b) If Z = $px + qy$ is maximum at (0,60) and (10, 50), find the relation of $p \text{ and } q$. Also mention the number of optimal solution(s) in this case.

OR

(ii) The feasible region for an L.P.P. is shown in the adjoining figure:



(Analysis)

Based on the given graph, answer the following questions.

(a) Write the constraints for the L.P.P.

(b) Find the co-ordinates of the point B.

(c) Find the maximum value of the objective function $Z = x + y$.

MATHEMATICS

ANSWER KEY

SECTION A - 65 MARKS

Question 1

In answering Multiple Choice Questions, candidates have to write either the correct option number or the explanation against it. Please note that only ONE correct answer should be written.

(i) (d) or a null matrix **[1]**

(ii) (b) or 1 **[1]**

(iii) (c) or $\frac{dy}{dx} = \frac{-y}{x}$ **[1]**

(iv) (d) or Assertion is false, and Reason is true. **[1]**

(v) (c) or $\frac{1}{68}$ **[1]**

(vi) (a) or $\begin{pmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{pmatrix}$ **[1]**

(vii) (a) or Statement 1 is true and Statement 2 is false. **[1]**

(viii) (d) or 9 **[1]**

(ix) (c) or Both the statements are true. **[1]**

(x) (a) or Both Assertion and Reason are true and Reason is the correct explanation for Assertion. **[1]**

(xi) (a) or 0·36 **[1]**

(xii) **[1]**

n = 3

$|A| = 3.$

As per question, C=$(adj\ A)^T$

$\therefore |C| = |(adj\ A)^T| = |adj\ A|$

$\therefore |C| = |A|^{n-1} = 3^{3-1} = 3^2 = 9$

$|C^2| = |C| \cdot |C| = 9 \times 9 = 81$

Ans: 81

(xiii) The relation is symmetric, transitive but not reflexive. **[1]**

(xiv) The curve crosses x -axis at three different points. That shows, for different value of x, the value of y is zero. That is more than one domain, mapped to the same point "zero" **[1]**

(xv) Two tests will be required if first machine is faulty and second is good OR both machines are faulty. **[1]**

Probability that only two tests are needed = $\frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} = \frac{2}{3}$

Ans: $\frac{2}{3}$

Question 2 **[2]**

(i) $x = e^{\frac{x}{y}}$

$\log x = \frac{x}{y} \log e = \frac{x}{y}$

$y \log x = x$

By differentiating on both sides,

$y \cdot \frac{1}{x} + \log x \frac{dy}{dx} = 1$

$\log x \frac{dy}{dx} = 1 - \frac{y}{x}$

$\frac{dy}{dx} = \frac{x - y}{x \log x}$

(ii) The values of $'a'$ for which the function $f(x) = b - ax + sin\ x$ is increasing on R.

$f'(x) = -a + cos\ x \ \geq 0$

It implies $a \leq cosx$. But $-1 \leq cosx \leq 1$

so $a \leq -1$.

Question 3 **[2]**

$y = (x - 2)^2$

$\frac{dy}{dx} = 2(x - 2)$

Slope of the chord = $\frac{4-0}{4-2} = 2$

Let $P(x_1, y_1)$ be the point on the curve.

Since the tangent is parallel to the chord. Therefore, their slopes are equal.

$\therefore\ 2(x_1 - 2) = 2 \Longrightarrow x_1$=3 and y_1=1

So, the point P is (3,1).

Question 4 **[2]**

$\frac{dy}{dx} = ycot2x$, by separation of variable

$\frac{dy}{y} = cot2x\ dx$

Integrating both sides

$$\int \frac{dy}{y} = \int cot2x\ \ dx$$

$\Longrightarrow \log y = \frac{1}{2}\log|sin2x| + C$

Question 5 **[2]**

(i) $\int x^5 \cos(x^6)\ dx$

Let $x^6 = t$

$6x^5 dx = dt$

$$\int \cos(t)\frac{dt}{6} = \frac{1}{6}\sin t + c = \frac{1}{6}\sin(x^6) + c$$

On comparing

$$k\sin(x^6) = \frac{1}{6}\sin(x^6)$$

$$k = \frac{1}{6}$$

OR

(ii) $I = \int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$

Applying,

$$\int_0^a f(x) = \int_0^a f(a-x)\, dx$$

$I = \int_0^5 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}} dx$

$2I = \int_0^5 \frac{\sqrt{5-x}\, dx}{\sqrt{5-x}+\sqrt{x}} + \int_0^5 \frac{\sqrt{x}\, dx}{\sqrt{5-x}+\sqrt{x}}$

$2I = \int_0^5 1dx = [x]_0^5$

$I = \frac{5}{2} = 2.5$

Question 6 **[2]**

$\tan^{-1}\left(\frac{6}{10}\right) + \tan^{-1}\left(\frac{8}{10}\right) = \tan^{-1}\frac{\frac{7}{5}}{1-\frac{12}{25}}$

$= \tan^{-1}\left(\frac{35}{13}\right)$

Question 7 **[4]**

$\sin^{-1}(x) + \sin^{-1}(1-x) = \cos^{-1} x$

$\Rightarrow \sin^{-1}(x) + \sin^{-1}(1-x) = \frac{\pi}{2} - \sin^{-1} x$

$\Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} - 2\sin^{-1} x$

$\Rightarrow (1-x) = \sin\left(\frac{\pi}{2} - 2\sin^{-1} x\right)$

$\Rightarrow (1-x) = \cos(2\sin^{-1} x)$

$\Rightarrow (1-x) = \cos(\cos^{-1}(1-2x^2))$

$\Rightarrow (1-x) = 1-2x^2$

$\Rightarrow 2x^2 - x = 0$

$\therefore x = 0, \frac{1}{2}$

Question 8

Given, $y = (A+Bx)e^{-2x}$ **[4]**

Differentiating w.r.t 'x'

$\Longrightarrow \frac{dy}{dx} = (A+Bx)(-2e^{-2x}) + e^{-2x}B$

$\Longrightarrow \frac{dy}{dx} = (-2y) + e^{-2x}B$

$\Longrightarrow \frac{dy}{dx} + 2y = e^{-2x}B$ (i)

Differentiating w.r.t 'x'

$\Longrightarrow \frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -2e^{-2x}B$ (ii)

(i) and (ii) $\Longrightarrow$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -2\left(\frac{dy}{dx} + 2y\right)$

$\Longrightarrow \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$, Hence Proved.

Question 9 **[4]**

(i) Differential equation

$$x\,\frac{dy}{dx} = y - x\,tan\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \frac{y}{x} - \ tan\left(\frac{y}{x}\right)$$

Let $y\ =\ vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v - tanv$

$\Rightarrow \frac{dv}{\tan v} = -\frac{dx}{x}$

Integrating both sides

$\Rightarrow \int \cot v\ dv = -\int \frac{dx}{x}$

$\Rightarrow \log|\sin v| = -\log x + \log c$

$\Rightarrow \log|\sin\frac{y}{x}| = -\log x + \log c = \log\frac{c}{x}$

$\Rightarrow x\sin\frac{y}{x} = c$

OR

(ii) $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$

$\frac{dy}{dx} + \frac{2xy}{(1+x^2)} = \frac{4x^2}{(1+x^2)}$

Comparing with $\frac{dy}{dx} + Py = Q$

$P = \frac{2x}{(1+x^2)}\ Q = \frac{4x^2}{(1+x^2)}$

I.F. $= e^{\int\frac{2x}{1+x^2}dx}$

I.F. $= e^{\log(1+x^2)}$

I.F. $= (1+x^2)$

Thus, the solution of the differential equation is,

$y\ (1+x^2) = \int \frac{4x^2}{(1+x^2)}(1+x^2)dx$

$\Longrightarrow\quad y(1+x^2) = \frac{4x^3}{3} + c$

Question 10 **[4]**

(i) (a) P (no odd person) = P(HHH) + P(TTT) = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(b) P (odd person) = $1 - \frac{1}{4} = \frac{3}{4}$

(c) P (odd person in 4^{th} round) = $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{256}$

OR

(ii) (a) Let:

- A: student chose Mode A → P(A) = 0.40
- B: student chose Mode B → P(B) = 0.35
- C: student chose Mode C → P(C) = 0.25

Let E: student rated the class as Excellent
Given:

- P(E/A) = 0.20, P(E/B) = 0.30, P(E/C) = 0.50

(b) P(C/E)

$$= \frac{P(C) \times P(E/C)}{P(A) \times P(E/A) + P(B) \times P(E/B) + P(C) \times P(E/C)}$$

$$= \frac{0.25 \times 0.50}{(0.40 \times 0.20) + (0.35 \times 0.30) + (0.25 \times 0.50)}$$

$$= \frac{0.25 \times 0.50}{(0.08) + (0.105) + (0.125)}$$

$$= 0.403$$

(c) There's about a 40.3% chance that a student who rated the class as "Excellent" attended Recorded lectures.
Now, numerator of, P(A/E) = $P(A) \times P(E/A) = 0.40 \times 0.20 = 0.08$
and
Numerator of P(B/E) = $P(B) \times P(E/B) = 0.35 \times 0.30 = 0.105$

By checking the numerators of P(C/E), P(B/E) and P(A/E) we observed 0.125 > 0.105 > 0.08. Therefore, recorded lectures (Mode C) have the highest likelihood of being the chosen mode among students who gave an excellent rating.

Question 11 **[6]**

(i) Let the cost of one paper bag, one scrap book and one pastel sheet be x, y and z respectively.

$$30x + 20y + 10z = 410 \Rightarrow 3x + 2y + z = 41$$

$$20x + 10y + 20z = 290 \Rightarrow 2x + y + 2z = 29$$

$$20x + 20y + 20z = 440 \Rightarrow x + y + z = 22$$

(ii) Given system of equations is equivalent to AX = B

Where $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix}$

$|A| = -2 \neq 0 \Rightarrow A^{-1}$ exists.

adj A = $\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$

Thus $A^{-1} = \frac{1}{|A|} adj\ A = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$

$$AX = B \Rightarrow X = A^{-1}B = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \\ 5 \end{bmatrix}$$

$\therefore x = 2, y = 15, z = 5$

(iii) The cost of one paper bag, one scrap book and one pastel sheet be Rs 2, Rs 15 and Rs 5 respectively.

Question 12 **[6]**

(i) $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)}\ dx$

Let: $\frac{x^2 + x + 1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx + C}{(x^2+1)}$

Equating the coefficients of x^2, x and constant respectively, we get

A + B = 1

2B + C = 1

A + 2C = 1

Solving for A, B, C we get,

C = $\frac{1}{5}$, B = $\frac{2}{5}$ and A = $\frac{3}{5}$

$\therefore \int \frac{x^2 + x + 1}{(x+2)(x^2+1)}\ dx$

$$= \int \left[\frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)}\right] dx$$

Integrating, we get

$=\frac{3}{5}\ln|x+2|+\frac{1}{5}[\ln|x^2+1|+tan^{-1}(x)]+C$

OR

(ii) $I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{x}{(1+\sin x)}dx$

Here, $a=\frac{\pi}{4}$ and $b=\frac{3\pi}{4}$, so $a+b=\pi$.

Using, $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{\pi-x}{(1+\sin(\pi-x))}dx$

$=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{\pi-x}{(1+\sin x)}dx$

Adding, both integrals,

$I+I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{x+(\pi-x)}{(1+\sin x)}dx=\pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{1}{(1+\sin x)}dx$

$\Rightarrow 2I=\pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\frac{1-\sin x}{(1-\sin^2 x)}dx=\pi\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}(sec^2x-\sec x\tan x)dx$

$\Rightarrow 2I=\pi[tanx-secx]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$

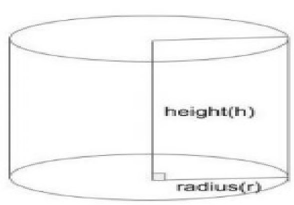
$I=(\sqrt{2}-1)\pi$

Question 13 **[6]**

(i) (a) express h in terms of radius r and given volume.

volume V = $\pi r^2 h$

$\frac{539}{2\pi r^2}=h$



(b) Let the total surface area of the closed cylinder tank be S. Expressing S in term of radius r.

$S=2\pi rh+2\pi r^2$

$S=2\pi r\frac{539}{2\pi r^2}+2\pi r^2$

$S=2\pi r^2+\frac{539}{r}$

(c) $\frac{ds}{dr}=4\pi r-\frac{539}{r^2}$

Setting, $\frac{ds}{dr}=0$ for stationary point, we get

$539=4\pi r^3$

$r = \frac{7}{2}$ unit

Now, $\frac{d^2s}{dr^2} = 4\pi + \frac{539\times2}{r^3} > 0$, when r = $\frac{7}{2}$ units.

Therefore, the total surface area (S) of the tank is minimum when r = $\frac{7}{2}$ units.

(d) Finding the height of the tank:

$h = \frac{539}{2\pi r^2} = \frac{539\times4\times7}{2\times22\times7\times7} = 7\ units$.

OR

(ii) (a) $h(t) = \frac{1}{2}(-7t^2 + 3t + 2)$, is a polynomial function, and all polynomial functions are continuous and differentiable everywhere on R.

$\therefore$ The function is differentiable for $t \geq 0$.

(b) Given, $h(t) = \frac{1}{2}(-7t^2 + 3t + 2)$

Differentiating w.r.t 't' we get,

$\frac{dh}{dt} = \frac{1}{2}(-14t + 3)$

$= -7\left(t - \frac{3}{14}\right)$.

$\therefore$ The instantaneous rate of change of height at t = $\frac{1}{14}$

$= \left[\frac{dh}{dt}\right]_{t=\frac{1}{14}} = \frac{3}{2} - \frac{1}{2} = 1$ unit.

(c) $\frac{dh}{dt} = -7\left(t - \frac{3}{14}\right)$.

Given, $t \geq 0$.

In $\left(0, \frac{3}{14}\right)$, $\frac{dh}{dt} > 0$. i.e., $h(t)$ is increasing in $\left(0, \frac{3}{14}\right)$.

$\therefore$ $h(t)$ is increasing in $\left(-\infty, \frac{3}{14}\right)$. is false.

(d) Setting $\frac{dh}{dt} = 0$, for stationary point, we get

$-7\left(t - \frac{3}{14}\right) = 0$

$\Rightarrow t = \frac{3}{14}$.

Now, $\frac{d^2h}{dt^2} = -7 < 0$.

$\therefore$ $h = f(t)$ has a local maximum at t = $\frac{3}{14}$.

$\therefore$ Maximum height $= [h(t)]_{\frac{3}{14}}$ = 1.161 units.

Question 14 **[6]**

(i) Let $P(E) = \frac{9}{10}, P(M) = \frac{4}{5}$ and $P(S) = \frac{7}{10}$

$\Longrightarrow P(\bar{E}) = \frac{1}{10}, P(\bar{M}) = \frac{1}{5}$ and $P(\bar{S}) = \frac{3}{10}$

$$P(X=0) = \frac{1}{10} \times \frac{1}{5} \times \frac{3}{10} = \frac{3}{500}$$

$$P(X=1) = \left(\frac{9}{10} \times \frac{1}{5} \times \frac{3}{10}\right) + \left(\frac{1}{10} \times \frac{4}{5} \times \frac{3}{10}\right) + \left(\frac{1}{10} \times \frac{1}{5} \times \frac{7}{10}\right) = \frac{46}{500}$$

$$P(X=2) = \left(\frac{9}{10} \times \frac{4}{5} \times \frac{3}{10}\right) + \left(\frac{9}{10} \times \frac{1}{5} \times \frac{7}{10}\right) + \left(\frac{1}{10} \times \frac{4}{5} \times \frac{7}{10}\right) = \frac{199}{500}$$

$$P(X=3) = \frac{9}{10} \times \frac{4}{5} \times \frac{7}{10} = \frac{252}{500}$$

(ii)

X	0	1	2	3
P(X)	$\frac{3}{500}$	$\frac{46}{500}$	$\frac{199}{500}$	$\frac{252}{500}$

(iii) Average number of surprise tests

$$= E(X) = 0 + \left(1 \times \frac{46}{500}\right) + \left(2 \times \frac{199}{500}\right) + \left(3 \times \frac{252}{500}\right) = \frac{1200}{500} = 2.4$$

(iv) Average number of surprise tests = 2.4 > 2.3

Students are getting a good average of surprise tests. So, the current system is balanced, and the teachers do not need to change anything.

SECTION B - 15 MARKS

Question 15 **[5]**

(i) (a) or Statement 1 is true and Statement 2 is false.

(ii) (a) or $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) + \lambda(\hat{\imath} + \hat{\jmath} + \hat{k}), \lambda \in \mathbb{R}$

$$\overrightarrow{AB} = (\hat{\jmath} + \hat{k}) - (\hat{\imath} + \hat{\jmath}) = -\hat{\imath} + \hat{k}$$

$$\overrightarrow{AC} = (\hat{k} + \hat{\imath}) - (\hat{\imath} + \hat{\jmath}) = -\hat{\jmath} + \hat{k}$$

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

$$\vec{r} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} + \hat{\jmath} + \hat{k})$$

(iii) (c) or $c = \pm\sqrt{3}$

Using, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1 \Rightarrow \frac{3}{c^2} = 1$

$\Rightarrow c^2 = 3$

$\Rightarrow c = \pm\sqrt{3}$

(iv) $|\vec{a}+\vec{b}| < 1 \Rightarrow |\vec{a}+\vec{b}\,|^2 < 1 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}.\vec{b} < 1$

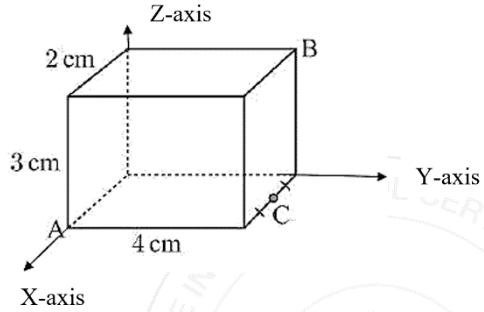
$\Rightarrow 1 + 1 + 2\,\vec{a}.\vec{b} < 1 \Rightarrow \vec{a}\cdot\vec{b} < -\frac{1}{2} \Rightarrow |\vec{a}||\vec{b}|\cos\theta < -\frac{1}{2}$

$\Rightarrow \cos\theta < -\frac{1}{2} \Rightarrow -1 \le \cos\theta < -\frac{1}{2} \Rightarrow \frac{2\pi}{3} < \theta \le \pi$ i.e., $\theta \in \left(\frac{2\pi}{3}, \pi\right]$

(v) Placing the coordinate axes as illustrated, coordinates of A is (2,0,0), B (0,4,3) and C (1,4,0).

$\therefore \overrightarrow{BA} = 2\hat{\imath}$ - $4\hat{\jmath}$ -3 $\hat{k}$ and $\overrightarrow{BC} = \hat{\imath}$ -3 $\hat{k}$

$\therefore \overrightarrow{BA}\,.\,\overrightarrow{BC} = 2 + 9 = 11$



Question 16 **[2]**

(i) (a) G is the centroid of ΔBCD. The coordinates are

$$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right) = (3,2,3)$$

$$\overrightarrow{AG} = (3-0)\hat{\imath} + (2-1)\hat{\jmath} + (3-2)\hat{k} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$$

$$|\overrightarrow{AG}| = \sqrt{3^2 + 1^2 + 1^2} = \sqrt{11} \text{ units}$$

(b) $\overrightarrow{AB} = (3-0)\hat{\imath} + (0-1)\hat{\jmath} + (1-2)\hat{k} = 3\hat{\imath} - \hat{\jmath} - \hat{k}$

$\overrightarrow{AC} = (4-0)\hat{\imath} + (3-1)\hat{\jmath} + (6-2)\hat{k} = 4\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

Area of ΔABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix} = -2\hat{\imath} - 16\,\hat{\jmath} + 10\,\hat{k} \text{ and } |\overrightarrow{AB} \times \overrightarrow{AC}| = 6\sqrt{10}$$

Hence area of ΔABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = 3\sqrt{10}$ sq units

OR

(ii) Let $\vec{a} = 2x^2\hat{\imath} + 3x\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + x^2\hat{k}$

$\because$ angle between the vectors is obtuse

$\Rightarrow \cos\theta < 0$

$\Rightarrow \frac{\vec{a}\,.\vec{b}}{|\vec{a}\,||\,\vec{b}|} < 0$

$\Rightarrow \vec{a}\,.\,\vec{b} < 0$

$\Rightarrow 2x^2 - 6x + x^2 < 0$

$\Rightarrow 3x(x-2) < 0$

$\Rightarrow 0 < x < 2$. i.e. $x \in (0,2)$

Question 17 **[4]**

(i) Let $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$.

$\therefore D(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

Dr of AD $\langle 5\lambda - 3, 2\lambda - 1, 3\lambda - 7\rangle$ and Dr of given line L $\langle 5,2,3\rangle$

AD ⊥ L, so $5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0 \Rightarrow \lambda = 1$

Coordinates of D are $(2, 3, -1)$

AD = $\sqrt{(2-0)^2 + (3-2)^2 + (-1-3)^2} = \sqrt{21}$

Area of ΔABC = $\frac{1}{2} \times BC \times AD = \frac{1}{2} \times 5 \times \sqrt{21} = \frac{5}{2}\sqrt{21}$ sq. units

OR

(ii) **Method-I**

Let the equation of the plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ is passing through $(-1, 0, 2)$

$\Rightarrow a(x + 1) + b\,(y - 0) + c\,(z - 2) = 0$………………………..(1)

Given line $\frac{x-0}{-2} = \frac{y-1}{3} = \frac{z-1}{-1}$ passing through (0, 1, 1) and having d.r. $\langle -2, 3, -1\rangle$

Since the plane contains the line and the point

$\Rightarrow a\,(1) + b\,(1) + c\,(1 - 2) = 0 \Rightarrow a + b - c = 0$……(2)

Also the line and normal to the plane are perpendicular

$\Rightarrow -2a + 3\,b - c = 0$…………….(3)

Solving (2) and (3) $\frac{a}{2} = \frac{b}{3} = \frac{c}{5} = k$

Hence required equation of the plane is

$\Rightarrow 2(x + 1) + 3(y - 0) + 5(z - 2) = 0$

$\Rightarrow 2x + 3y + 5z - 8 = 0$

Method –II

Let the equation of the plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ is passing through $(-1, 0, 2)$

$\Rightarrow a(x + 1) + b\,(y - 0) + c\,(z - 2) = 0$………………………..(1)

Given line $\frac{x-0}{-2} = \frac{y-1}{3} = \frac{z-1}{-1}$ passing through (0, 1, 1) and having d.r. $\langle -2, 3, -1\rangle$

Since the plane contains the line and the point

$\Rightarrow a\,(1) + b\,(1) + c\,(1 - 2) = 0 \Rightarrow a + b - c = 0$……(2)

Also, the line and normal to the plane are perpendicular

$\Rightarrow -2a + 3\,b - c = 0$…………….(3)

Hence required equation of the plane is

$$\begin{vmatrix} x+1 & y & z-2 \\ 1 & 1 & -1 \\ -2 & 3 & -1 \end{vmatrix} = 0$$

$\Rightarrow 2(x+1)+3(y-0)+5(z-2)=0$
$\Rightarrow 2x+3y+5z-8=0$

Question 18 **[4]**

The required area is in two parts – one part above the x -axis and the other below the x -axis.

$$A=\int_0^4 y\,dx+\int_4^5 -y\,dx=\int_0^4 x(4-x)\,dx+\int_4^5 x(x-4)\,dx$$
$$=\left[2x^2-\frac{x^3}{3}\right]_0^4+\left[\frac{x^3}{3}-2x^2\right]_4^5=\left(32-\frac{64}{3}\right)+\left(\frac{125}{3}-50\right)-\left(\frac{64}{3}-32\right)$$
$$=\frac{32}{3}-\frac{25}{3}+\frac{32}{3}=13 \text{ sq units}$$

SECTION C - 15 MARKS

Question 19 **[5]**

(i) (b) or MC = AC

(ii) (b) or Both statements are true but Statement 2 is not the correct explanation of Statement 1.

(iii) Given: $\bar{x}=53,\ \bar{y}=28,\ b_{yx}=-1.2,\ b_{xy}=-0.3$

Using, $r^2 = b_{yx}\times b_{xy}$

$= -1.2\times -0.3 = 0.36$

$\Rightarrow r=-0.6$

Ans : $-0\cdot 6$

(iv) Given , $R(x)= 36x+3x^2+5$

$\Rightarrow MR=\dfrac{d}{dx}(36x+3x^2+5)$

$= 36+6x$

$\therefore MR(x=5)= 66$

Ans: 66

(v) Given $C(x) = 210x + 7000$, $R(x) = 280x$

Minimum number must be sold daily when $R(x) = C(x)$

$\Rightarrow 210x + 7000 = 280x$

$\Rightarrow 70x = 7000$

$\Rightarrow x = 100.$

$\therefore$Minimum number that must be sold is 100.

Ans: 100

Question 20 **[2]**

(i) $C(x) = 4000 + 14x - 0.04x^2$

$\Rightarrow C'(x) = 14 - 0.08x$

Now, $C'(x) = 0 \Rightarrow 14 = 0.08x \Rightarrow x = 175$

$C''(x) = -0.08 < 0$

$\therefore$C (x) will be maximum at x = 175.

As per question, we have to minimise the maintenance cost.

Since x can be 0 to 500 apartments,

$C(0) = 4000 + 14 \times 0 - 0.04 \times (0)^2 = 4000$

$C(500) = 4000 + 14 \times 500 - 0.04 \times (500)^2 = 1000$

$\therefore$The complex must have 500 apartments to minimise the maintenance cost.

OR

(ii) Given, $x = 100 - 4p$

$\Rightarrow p = \frac{100 - x}{4}$

$\therefore R(x) = px = \frac{100x - x^2}{4}$

$\Rightarrow MR = \frac{d}{dx}\left(\frac{100x - x^2}{4}\right) = \frac{100 - 2x}{4}$

$\because$MR = 0

$\therefore x = 50$

Question 21 **[4]**

$\bar{x} = \frac{\sum x}{n} = 170, \qquad \bar{y} = \frac{\sum y}{n} = 192,$

$b_{yx} = r\,\frac{\sigma_y}{\sigma_x} = 0.6 \times \frac{20}{60} = 0.2$

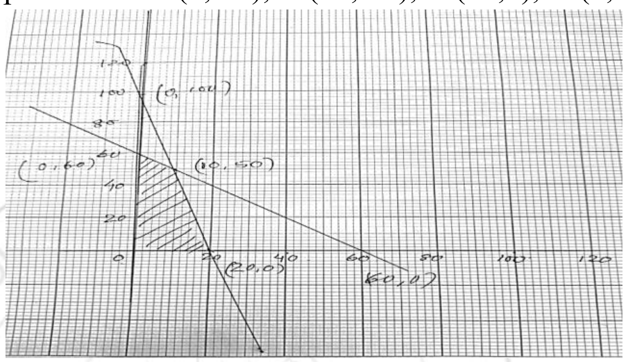
Regression equation y on x is

$y - 192 = 0.2(x - 170)$

$\Rightarrow y - 192 = 0.2x - 34$
$\Rightarrow y = 0.2x + 158$
Putting, $x = 200$
$\Rightarrow y = 0.2(200) + 158 = 198$
∴Expenditure on food and entertainment = ₹ 198.

Question 22 **[4]**

(i) (a) From graph, corner points are: A (0, 60), B (10, 50), C (20,0), D (0, 0)



(b) $Z = px + qy$
Given, Z is maximum at (0, 60) and (10, 50)
∴$0.\,p + 60.\,p = 10.\,p + 50.\,q$
$\Rightarrow 10p = 10q$
$\Rightarrow p = q$
So, there can be infinite number of optimal solutions.

OR

(ii) (a) Equation of line AD:

$$\frac{x}{25} + \frac{y}{20} = 1 \Rightarrow 2x + y = 50$$

Equation of line EC:

$$\frac{x}{40} + \frac{y}{20} = 1 \Rightarrow x + 2y = 40$$

As origin lies in the region, $2x + y \le 50$ and $x + 2y \le 40$
Therefore, the constraints are,
$2x + y \le 50$
$x + 2y \le 40$
$x \ge 0, y \ge 0$

(b) Solving simultaneously, the coordinate of B is (20, 10).

(c) The corner points are $O(0,0), C(0,20), B(20,10)$ and $A(25,0)$.
At O, Z = 0
At C, Z = 20
At B, Z = 30
At A, Z = 25
∴Maximum value of Z is 30.