# MATHEMATICS

***Maximum Marks: 80***
***Time Allotted: Three Hours***
***Reading Time: Additional Fifteen minutes***

## Instructions to Candidates

- You are allowed an **additional 15 minutes** for **only** reading the paper.
- You must **NOT** start writing during reading time.
- The question paper has **12** printed pages.
- The Question Paper is divided into **three** sections and has **22 questions** in all.
- **Section A** is compulsory and has **fourteen** questions.
- You are required to attempt **all** questions either from **Section B or Section C.**
- **Section B** and **Section C** have **four** questions each.
- Internal choices have been provided in **two** questions of **2 marks**, **two** questions of **4 marks** and **two** questions of **6 marks** in **Section A.**
- Internal choices have been provided in **one** question of **2 marks** and **one** question of **4 marks** each in **Section B and Section C.**
- While attempting **Multiple Choice Questions in Section A, B and C**, you are required to **write only ONE option as the answer.**
- The intended marks for questions or parts of questions are given in the brackets [].
- All workings, including rough work, should be done on the same page as, and adjacent to, the rest of the answer.
- Mathematical tables and graph papers are provided.

## Instruction to Supervising Examiner

- Kindly read aloud the instructions given above to all the candidates present in the examination hall.

## SECTION A - 65 MARKS

**Question 1**

In subparts (i) to (xi) choose the correct options and in subparts (xii) to (xv), answer the questions as instructed.

(i) A matrix which is both symmetric and skew symmetric matrix is a / an: **[Understanding]**

(a) triangular matrix

(b) identity matrix

(c) diagonal matrix

(d) null matrix

(ii) The value of $\int a^x . e^x dx$ equals **[Recall]** **[1]**

(a) $( a^x . \log_e a)e^x + c$

(b) $\frac{a^x . e^x}{\log_e (ae)} + c$

(c) $\frac{a^x . e^x}{\log_{ae} e} + c$

(d) $\log_e (ae) (ae)^x + c$

(iii) The trigonometric equation $\tan^{-1} x = 3 \tan^{-1} a$ has solution for **[1]** **[Application]**

(a) $|a| \leq \frac{1}{\sqrt{3}}$

(b) $|a| > \frac{1}{\sqrt{3}}$

(c) $|a| < \frac{1}{\sqrt{3}}$

(d) all real value of a.

(iv) **Assertion:** Degree of the differential equation: $a\left(\frac{dy}{dx}\right)^2 + b\frac{dx}{dy} = c$ , is 3 **[1]**

**Reason:** If each term involving derivatives of a differential equation is a polynomial (or can be expressed as polynomial) then highest exponent of the highest order derivative is called the degree of the differential equation. **[Analysis]**

Which of the following is correct?

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(v) Five numbers $x_1, x_2, x_3, x_4, x_5$ are randomly selected from the numbers 1, 2, 3, ……., 18 and are arranged in the increasing order such that $x_1 < x_2 < x_3 < x_4 < x_5$. What is the probability that $x_2 = 7$ and $x_4 = 11$? **[1]**

**[Application]**

(a) $\frac{26}{51}$

(b) $\frac{3}{104}$

(c) $\frac{1}{68}$

(d) $\frac{1}{34}$

(vi) If $\begin{vmatrix} a & b & c \\ m & n & p \\ x & y & z \end{vmatrix} = k$, then what is the value of $\begin{vmatrix} 6a & 2b & 2c \\ 3m & n & p \\ 3x & y & z \end{vmatrix}$? **[1]**

**[Understanding]**

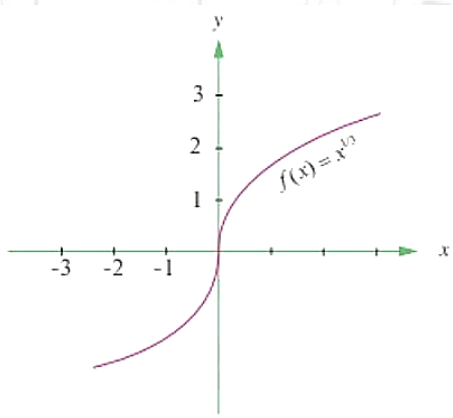
(a) $\frac{k}{6}$

(b) $2k$

(c) $3k$

(d) $6k$

(vii) Consider the graph $y = x^{\frac{1}{3}}$ **[1]**




**Statement 1**: The above graph is continuous at $x = 0$

**Statement 2**: The above graph is differentiable at $x = 0$

**[Application]**

Which of the following is correct?

(a) Statement 1 is true, and Statement 2 is false.

(b) Statement 2 is true, and Statement 1 is false.

(c) Both the statements are true.

(d) Both the statements are false.

(viii) The value of $\frac{dy}{dx}$ if $y = |x-1| + |x-4|$ at $x = 3$ is **[1]**

(a) $-2$

(b) 0

(c) 2

(d) 4

(ix) **Statement 1**: The intersection of two equivalence relations is always an equivalence relation. **[1]**

**Statement 2**: The Union of two equivalence relations is always an equivalence relation.

Which one of the following is correct? **[Understanding]**

(a) Statement 1 implies Statement 2.

(b) Statement 2 implies Statement 1.

(c) Statement 1 is true only if Statement 2 is true.

(d) Statement 1 and 2 are independent of each other.

(x) In a third order matrix $a_{ij}$ denotes the element of the i$^{th}$ row and the j$^{th}$ column. **[1]**

$$A = a_{ij} = \begin{cases} 0, for\ i = j \\ 1, for\ i > j \\ -1, for\ i < j \end{cases}$$

**Assertion**: Matrix 'A' is not invertible.

**Reason:** Determinant A = 0

Which of the following is correct? **[Application]**

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(xi) Given two events A and B such that $(A/B) = 0.25$ and $P(A \cap B) = 0.12$. **[1]**
The value $P(A \cap B')$ is: **[Understanding]**

(a) 0·36

(b) 0·48

(c) 0·88

(d) 0·036

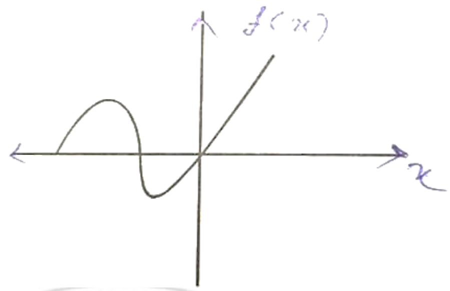
(xii) The value of the determinant of a matrix A of order 3 is 3. If C is the matrix of cofactors of the matrix A, then what is the value of determinant of $C^2$? **[1]**

**[Analysis]**

(xiii) If a relation R on the set $\{a, b, c\}$ defined by R = {(b,b)} , then classify the relation. **[1]**

**[Understanding]**

(xiv) **[1]**




The given function $f: R \to R$ is not 'onto' function. Give reason.

**[Understanding]**

(xv) There are three machines and 2 of them are faulty. They are tested one by one in a random order till both the faulty machines are identified. What is the probability that only two tests are needed to identify the faulty machines? **[Application]** **[1]**

**Question 2** **[2]**

(i) If $x^y = y^x$, then find $\frac{dy}{dx}$ **[Understanding]**

**OR**

(ii) Find the interval in which the function $f(x) = x^2e^{-x}$ is strictly increasing or decreasing. **[Understanding]**

**Question 3** **[2]**

Evaluate: $\int_0^{\sqrt{2}}[x^2]\,dx$ **[Evaluate]**

**Question 4** **[2]**

Find the equation to the tangent at (0,0) on the curve $y = 4x^2 - 2x^3$ **[Analysis]**

**Question 5** **[2]**

(i) Evaluate: $\int \frac{2x^3 - 1}{x^4 + x}\,dx$ **[Evaluate]**

**OR**

(ii) Evaluate: $\int e^x \operatorname{cosec} x\,(1 - \cot x)\,dx$ **[Evaluate]**

**Question 6** **[2]**

Find the value of: $\tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right)$ **[Application]**

**Question 7** **[4]**

Solve: $\sin^{-1}(x) + \sin^{-1}(1-x) = \cos^{-1} x$. **[Application]**

**Question 8** **[4]**

Evaluate: $\int \sqrt{sec\frac{x}{2} - 1}\, dx$ **[Application]**

**Question 9** **[4]**

(i) A kite is being pulled down by a string that goes through a ring on the ground 8 meters away from the person pulling it. If the string is pulled in at 1 meter per second, how fast is the kite coming down when it is 15 meters high? **[Application]**

**OR**

(ii) If $y = \left(x + \sqrt{a^2 + x^2}\right)^m$, prove that $(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2 y = 0$ **[Analysis]**

**Question 10** **[4]**

(i) Three friends go to a restaurant to have pizza. They decide who will pay for the pizza by tossing a coin. It is decided that each one of them will toss a coin and if one person gets a different result (heads or tails) than the other two, that person would pay. If all three get the same result (all heads or all tails), they will toss again until they get a different result. **[Analysis]**

(a) What is the probability that all three friends will get the same result (all heads or all tails) in one round of tossing?

(b) What is the probability that they will get a different result in one round of tossing?

(c) What is the probability that they will need exactly four rounds of tossing to determine who would pay?

**OR**

(ii) Students of under graduation submitted a case study on
"Understanding the Probability of Left-Handedness in Children Based on Parental Handedness". Following

Recent studies suggest that roughly 12% of the world population is left-handed. Depending on the parents' handedness, the chances of having a left-handed child vary as follows:

Scenario A:

Both parents are left-handed, with a 24% chance of the child being left-handed.

Scenario B:

The father is right-handed, and the mother is left-handed, with a 22% chance of child being left-handed.

Scenario C:

The father is left-handed, and the mother is right-handed, with a 17% chance of child being left-handed.

Scenario D:

Both parents are right-handed, with a 9% chance of having a left-handed child.

Assuming that scenarios A, B, C and D are equally likely, and L denotes the event that the child is left-handed, answer the following questions.

(a) What is the overall probability that a randomly selected child is left-handed?

(b) Given that exactly one parent is left-handed, what is the probability that a randomly selected child is left-handed?

(c) If a child is left-handed, what is the probability that both parents are left-handed?

**Question 11** **[6]**

To raise money for an orphanage, students of three schools A, B and C organised an exhibition in their residential colony, where they sold paper bags, scrap books and pastel sheets made by using recycled paper. Student of school A sold 30 paper bags, 20 scrap books and 10 pastel sheets and raised ₹ 410. Student of school B sold 20 paper bags, 10 scrap books and 20 pastel sheets and raised ₹ 290. Student of school C sold 20 paper bags, 20 scrap books and 20 pastel sheets and raised ₹ 440.

Answer the following question:

(i) Translate the problem into a system of equations.

(ii) Solve the system of equation by using matrix method.

(iii) Hence, find the cost of one paper bag, one scrap book and one pastel sheet.

**[Application]**

**Question 12** **[6]**

(i) Solve the differential equation:

$$(xdy - ydx)\, y \sin\left(\frac{y}{x}\right) = (ydx + xdy)\, x \cos\left(\frac{y}{x}\right).$$

Find the particular solution satisfying the condition that $y = \pi$ when $x = 1$.

**[Application]**

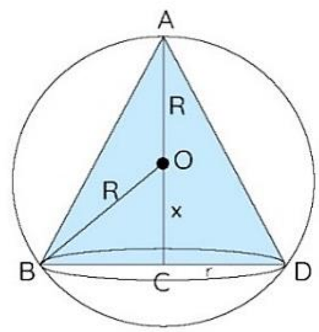
**OR**

(ii) Evaluate: $\int_0^{\pi} (sin^4 x + cos^4 x)\, dx$

Hence evaluate: $\int_{-2\pi}^{2\pi} \frac{sin^4 x + cos^4 x}{1 + e^x} dx$ **[Evaluate]**

**Question 13** **[6]**

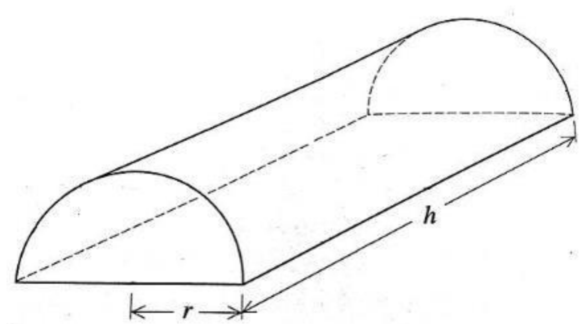
(i) A cone of maximum volume is inscribed in a given sphere. Then prove that ratio of the height of the cone to the diameter of the sphere is equal to $\frac{2}{3}$. **[Application]**




**OR**

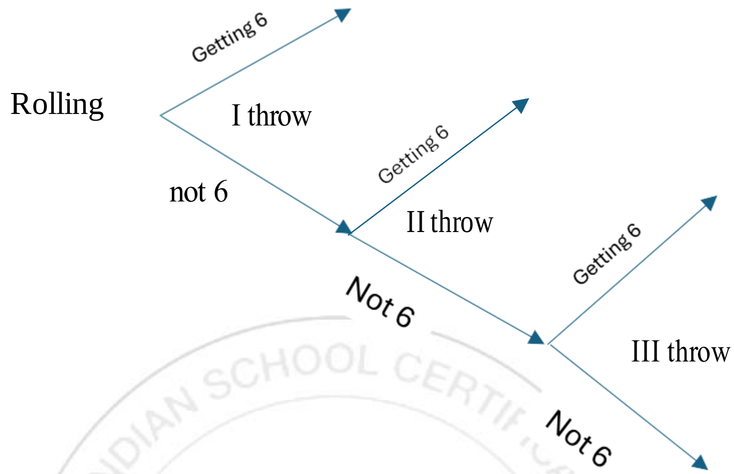
(ii) A given quantity of metal is to be cast into a solid half circular cylinder with a rectangular base and semi-circular ends. If the total surface is minimum then prove that the ratio of the length of cylinder to the diameter of semi-circular ends is $\pi : \pi + 2$

**[Application]**



**Question 14** **[6]**

Kiran plays a game of throwing a fair die 3 times but to quit as and when she gets a six. Kiran gets $+1$ point for a six and $-1$ for any other number. **[Analysis]**




(i) If X denotes the random variable "points earned" then what are the possible values X can take?

(ii) Find the probability distribution of this random variable X.

(iii) Find the expected value of the points she gets.

## SECTION B - 15 MARKS

**Question 15** **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Consider the following statements and choose the correct option:

**Statement 1:** If $\vec{a}$ and $\vec{b}$ represents two adjacent sides of a parallelogram then the diagonals are represented by $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ .

**Statement 2:** If $\vec{a}$ and $\vec{b}$ represents two diagonals of a parallelogram then the adjacent sides are represented by $2(\vec{a}+\vec{b})$ and $2(\vec{a}-\vec{b})$.

Which of the following is correct? **[Recall]**

(a) Only Statement 1

(b) Only Statement 2

(c) Both Statements 1 and 2

(d) Neither Statement 1 nor Statement 2

(ii) The distance of the plane through $(1,1,1)$ and perpendicular to the line $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ from the origin is **[Application]**

(a) $\frac{3}{4}$

(b) $\frac{4}{3}$

(c) $\frac{7}{5}$

(d) 1

(iii) If the direction cosines of a line are $< \frac{1}{c}, \frac{1}{c}, \frac{1}{c} >$ then **[Understanding]**

(a) $c > 0$

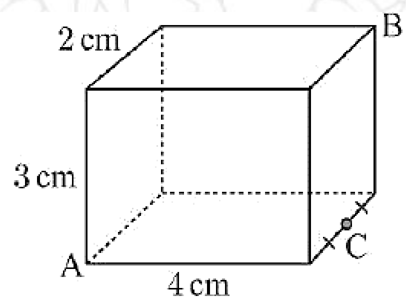
(b) $0 < c < 1$

(c) $c = \pm 3$

(d) $c > 2$

(iv) If $\vec{a}$ is a unit vector perpendicular to $\vec{b}$ and $(\vec{a} + 2\,\vec{b}).(3\,\vec{a} - \vec{b}) = -5$, find $|\vec{b}|$. **[Understanding]**

(v) Shown below is a cuboid. Find $\overrightarrow{BA}.\overrightarrow{BC}$




**[Analysis]**

**Question 16** **[2]**

(i) Find a vector of magnitude 9 units and perpendicular to the vectors.
$\vec{a} = 4\,\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = -2\,\hat{\imath} + \hat{\jmath} - 2\,\hat{k}$ **[Application]**

**OR**

(ii) What are the values of $x$ for which the angle between the vectors?
$2x^2\hat{\imath} + 3x\hat{\jmath} + \hat{k}$ and $\hat{\imath} - 2\hat{\jmath} + x^2\hat{k}$ is obtuse? **[Application]**

**Question 17** **[4]**

(i) Show that the line whose vector equation is $\vec{r} = (2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + 4\hat{k})$ is parallel to the plane whose vector equation is $\vec{r}.(\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 5$. Also find the distance between them. **[Application]**

**OR**

(ii) Find the equation of the plane containing the line $\frac{x}{-2} = \frac{y-1}{3} = \frac{1-z}{1}$ and the point$(-1, 0, 2)$. **[Application]**

**Question 18** **[4]**

(i) Sketch the region enclosed bounded by the curve, $y = x\,|x|$ and the ordinates $x = -1$ and $x = 1$

(ii) Evaluate: $\int_0^1 x^2\,dx$.

(iii) Hence find the area bounded by the curve, $y = x\,|x|$ and the ordinates $x = -1$ and $x = 1$.

## SECTION C - 15 MARKS

**Question 19** **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Which condition is true if Average Cost (AC) is constant at all levels of out put **[Recall]**

(a) MC > AC

(b) MC = AC

(c) MC < AC

(d) MC = $\frac{1}{2}$ AC

(ii) Read the following statements and choose the correct option:

(I) If r = 0, then regression lines are not defined.

(II) If r = 0, then regression lines are parallel.

(III) If r = 0, then regression lines are perpendicular.

(IV) If r = $\pm 1$ , then regression lines coincide. **[Recall]**

Which of the following is correct?

(a) Only IV is correct

(b) Only I and II are correct

(c) Only I and IV are correct

(d) Only III and IV are correct

(iii) Mean of $x = 53$, mean of $y = 28$ regression co-efficient $y \; on \; x = -1 \cdot 2$, regression co-efficient $x \; on \; y = -0 \cdot 3$. Find coefficient of correlation (r). **[Understanding]**

(iv) The total revenue received from the sale of x unit of a product is given by $R(x) = 3x^2 + 36x + +5$. Find the marginal revenue when $x = 5$.

**[Understanding]**

(v) A manufacturing company finds that the daily cost of producing $x$ item of product is given by $C(x) = 210x + 7000$. Find the minimum number that must be produced and sold daily, if each item is sold for ₹280. **[Understanding]**

**Question 20** **[2]**

(i) The cost function of a commodity is $C(x) = 200 + 20x - \frac{1}{2}x^2$ (in rupees). Find the range where AC falls. **[Application]**

**OR**

(ii) The demand function of a monopoly is given by $x = 100 - 4p$. Find the quantity at which the MR will be zero. **[Application]**

**Question 21** **[4]**

(i) A survey of 50 families to study the relationships between expenditure on accommodation in (₹ $x$) and expenditure on food and entertainment (₹ $y$) gave the following results:

$$\sum x = 8500, \; \sum y = 9600, \sigma_x = 60, \sigma_y = 20, r = 0.6$$

Estimate the expenditure on food and entertainment when expenditure on accommodation is ₹200. **[Application]**

**OR**

(ii) The random variables have regression lines $3x + 2y - 26 = 0$ and $6x + y - 31 = 0$

Calculate

(a) Mean value of $x \; and \; y$.

(b) Co-efficient of correlations. **[Application]**

**Question 22** **[4]**

A linear programming problem is given by $Z = px + qy$ where $p, q > 0$ subject to the constraints:

$x + y \leq 60, \; 5x + y \leq 100, \; x \geq 0 \; and \; y \geq 0$ **[Analysis]**

(i) Solve graphically to find the corner points of the feasible region.

(ii) If Z = $px + qy$ is maximum at (0,60) and (10, 50) , find the relation of $p \; and \; q$. Also mention the number of optimal solution(s) in this case.

# MATHEMATICS

# ANSWER KEY

# SECTION A - 65 MARKS

**Question 1**

**In answering Multiple Choice Questions, candidates have to write either the correct option number or the explanation against it. Please note that only ONE correct answer should be written.**

(i) (d) a null matrix **[1]**

(ii) (b) $\frac{a^x . e^x}{\log_e(ae)} + c$ **[1]**

(iii) (c) $|a| < \frac{1}{\sqrt{3}}$ **[1]**

(iv) (d) Assertion is false and Reason is true. **[1]**

(v) (c) $\frac{1}{68}$ **[1]**

(vi) (d) $6k$ **[1]**

(vii) (a) Statement 1 is true, and Statement 2 is false. **[1]**

(viii) (b) 0 **[1]**

(ix) (c) Statement 1 is true only if Statement 2 is true. **[1]**

(x) (a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion. **[1]**

(xi) (a) 0·36 **[1]**

(xii) **[1]**

n = 3

$|A| = 3.$

$As$ per question, C=$(adj\ A)^T$

$\therefore |C| = |(adj\ A)^T| = |adj\ A|$

$\therefore\ |C| = |A|^{n-1} = 3^{3-1} = 3^2 = 9$

$|C^2| = |C| \cdot |C| = 9 \times 9 = 81$

**ANS : 81**

(xiii) The relation is symmetric, transitive but not reflexive. **[1]**

(xiv) Since each line in co-domain of the function parallel to x –axis, doesn't cuts the graph of function at least one point, therefore the $f(x)$ is not an onto function. **[1]**

(xv) Two tests will be required if first machine is faulty and second is good OR both machines are faulty. **[1]**

Probability that only two tests are needed = $\frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} = \frac{2}{3}$

ANS: $\frac{2}{3}$

**Question 2** **[2]**

(i) $x^y = y^x$

$y \, log \, x = x \, log \, y$

By differentiating on both sides,

$y \cdot \frac{1}{x} + \log x \frac{dy}{dx} = x \cdot \frac{1}{y}\frac{dy}{dx} + \log y$

$\frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \log y - \frac{y}{x}$

$\frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$

**OR**

(ii) $f(x) = x^2 e^{-x}$

$\frac{dy}{dx} = -x^2 e^{-x} + e^{-x} \cdot 2x$

$= \frac{1}{e^x}(2x - x^2) = \frac{1}{e^x}(2 - x)x = -e^{-x}[x(x-2)]$

$\because e^x \; is \; always \; positive$

$\therefore$ Setting $x(x-2) = 0 \Rightarrow x = 0 \; or \; x = 2$

For strictly decreasing function $\frac{dy}{dx} < 0 \;\Rightarrow x < 0 \; or \; x > 2$

$\Rightarrow x \in (-\infty, 0) \cup (2, \infty)$

For strictly increasing function $\frac{dy}{dx} > 0 \Rightarrow 0 < x < 2 \Rightarrow x \in (0,2)$

$\therefore$ $f(x)$ is increasing in $(0,2)$ and decreasing in $(-\infty, 0) \cup (2, \infty)$

**Question 3** **[2]**

$\int_0^{\sqrt{2}} [x^2] \, dx$

We know greatest integer function is discontinuous when $x$ is an integer.

$\therefore \int_0^{\sqrt{2}} [x^2] \, dx = \int_0^1 0 \, dx + \int_1^{\sqrt{2}} 1 \, dx = x|_1^{\sqrt{2}} = \sqrt{2} - 1.$

**Question 4** **[2]**

$y = 4x^2 - 2x^3$

$\frac{dy}{dx} = 8x - 6x^2$

Let $P(x_1, y_1)$ be the point of contact.

$\therefore$ Slope of the tangents = $8x_1 - 6x_1^2$

Tangent equation: $y - y_1 = (8x_1 - 6x_1^2)(x - x_1)$

$\because$ the tangent passes through the origin

$y_1 = (8x_1 - 6x_1^2)x_1$

Now, $y_1 = 4x_1^2 - 2x_1^3 = 8x_1^2 - 6x_1^3$

$\Rightarrow 4 - 2x_1 = 8 - 6x_1$

$\Rightarrow 4x_1 = 4$

$x_1 = 1 \Rightarrow y_1 = 2$

The point on the curve is (1,2)

**Question 5** **[2]**

(i) $\int \frac{2x^3 - 1}{x^4 + x} dx$

Dividing both nominator and denominator by $'x'$ we get

$= \int \frac{2x - \frac{1}{x^2}}{x^2 + \frac{1}{x}} dx$ Let $\boldsymbol{x^2 + \frac{1}{x} = t} \Rightarrow \left(\boldsymbol{2x - \frac{1}{x^2}}\right) dx = dt$

$= \int \frac{dt}{t}$

$= \log_e|t| + c$

$= \log_e \left|\boldsymbol{x^2 + \frac{1}{x}}\right| + \text{c}$

**OR**

(ii) $\int e^x \operatorname{cosec} x\,(1 - \cot x) dx$

$= \int e^x (\operatorname{cosec} x - \operatorname{cosec} x \,.\, \cot x) dx$

$= \int e^x (\operatorname{cosec} x + (-\operatorname{cosec} x \,.\, \cot x)) dx$

$[\because \int e^x (f(x) + f'(x)) dx = e^x f(x) + c]$

$= e^x \operatorname{cosec} x + \text{c}$

-----------------------------------------------------------------------------------------------------------------

**Question 6** **[2]**

$\tan^{-1}\left(\frac{x}{y}\right)+\ \tan^{-1}\left(\frac{y-x}{y+x}\right)\ =\tan^{-1}\left(\frac{x}{y}\right)+\ \tan^{-1}\left(\frac{1-\frac{x}{y}}{1+\frac{x}{y}}\right)$

$=\tan^{-1}\left(\frac{x}{y}\right)+\ \tan^{-1}(1)-\ \tan^{-1}\left(\frac{x}{y}\right)$

$=\tan^{-1}(1)=\ \frac{\pi}{4}$

**Question 7** **[4]**

$\sin^{-1}(x)+\sin^{-1}(1-x)=\cos^{-1}x$

$\Rightarrow \sin^{-1}(x)+\sin^{-1}(1-x)=\frac{\pi}{2}-\ \sin^{-1}x$

$\Rightarrow\ \sin^{-1}(1-x)=\frac{\pi}{2}-\ 2\sin^{-1}x$

$\Rightarrow (1-x)=\sin\left(\frac{\pi}{2}-\ 2\sin^{-1}x\right)$

$\Rightarrow (1-x)=\cos(\ 2\sin^{-1}x)$

$\Rightarrow (1-x)=\cos\left(\cos^{-1}\left(1-2x^2\right)\right)$

$\Rightarrow (1-x)=1-2x^2$

$\Rightarrow 2x^2-x=0$

$\therefore\ x=0,\frac{1}{2}$

**Question 8** **[4]**

$I=\int\sqrt{sec\frac{x}{2}-1}\,dx\ =\int\sqrt{\frac{1-\cos\frac{x}{2}}{\cos\frac{x}{2}}}dx=\ \int\sqrt{\frac{\left(1-\cos\frac{x}{2}\right)\left(1+\cos\frac{x}{2}\right)}{\cos\frac{x}{2}\left(1+\cos\frac{x}{2}\right)}}\ dx\ =\int\frac{\sin\frac{x}{2}}{\sqrt{cos^2\frac{x}{2}+\cos\frac{x}{2}}}dx$

Let $\cos\frac{x}{2}=t\ \Rightarrow\ -\sin\frac{x}{2}.\frac{1}{2}\ dx=dt\ \Rightarrow\ \sin\frac{x}{2}.dx=-2dt$

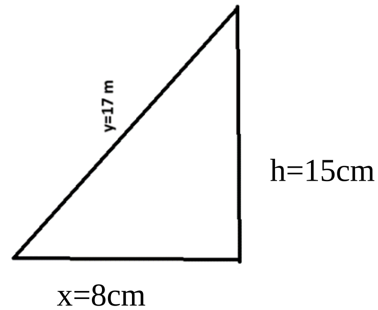
$\therefore\ I=-2\int\frac{dt}{\sqrt{t^2+t}}=-2\int\frac{dt}{\sqrt{\left(t+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}}=-2\log_e\left|\left(t+\frac{1}{2}\right)+\sqrt{t^2+t}\right|+c$

$=-2\log_e\left|\left(\ \cos\frac{x}{2}+\frac{1}{2}\right)+\sqrt{cos^2\frac{x}{2}+\ \cos\frac{x}{2}}\right|+c$

**Question 9** **[4]**

(i) $x^2 + h^2 = y^2 \Rightarrow x^2 + 225 = y^2$

$$Given, \frac{dy}{dt} = 1\, m/sec$$




Differentiating w.r.t 't'

$$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$$

$8 \times \frac{dx}{dt} = 17 \times 1,$

**OR**

(ii) **Method I** $y = \left[x + \sqrt{a^2 + x^2}\right]^m$

$$\frac{dy}{dx} = \frac{m\left[x + \sqrt{a^2 + x^2}\right]^m}{x + \sqrt{a^2 + x^2}} \cdot \left[1 + \frac{1 \cdot 2x}{2\sqrt{a^2 + x^2}}\right] = my\frac{1}{\sqrt{a^2 + x^2}}$$

$$\sqrt{a^2 + x^2}\frac{dy}{dx} = my$$

$$\sqrt{a^2 + x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1}{2} \cdot \frac{2x}{\sqrt{a^2 + x^2}} = m\frac{dy}{dx}$$

$$(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m\sqrt{a^2 + x^2}\frac{dy}{dx} = 0$$

$$(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

**Method II** $y = \left[x + \sqrt{a^2 + x^2}\right]^m$

$$\frac{dy}{dx} = \frac{m\left[x + \sqrt{a^2 + x^2}\right]^m}{x + \sqrt{a^2 + x^2}} \cdot \left[1 + \frac{1 \cdot 2x}{2\sqrt{a^2 + x^2}}\right] = my\frac{1}{\sqrt{a^2 + x^2}}$$

$$\sqrt{a^2 + x^2}\frac{dy}{dx} = my$$

Squaring both sides we get,

$$(a^2 + x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$$

Differentiating w.r.t '$x$' ,

$$2x\left(\frac{dy}{dx}\right)^2 + (a^2 + x^2)2\frac{dy}{dx}.\frac{d^2y}{dx^2} = 2m^2y\frac{dy}{dx}$$

$$\Rightarrow (a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

**Question 10** **[4]**

(i)

(a) P (no odd person) = P(HHH) + P(TTT) = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$

(b) P (odd person) = $1-\frac{1}{4}=\frac{3}{4}$

(c) P (odd person in 4$^{th}$ round) = $\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{3}{4}=\frac{3}{256}$

**OR**

(ii)

(a) Since events A, B, C, D are equally likely

$\Rightarrow P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$

As per question, $P(L/A)=\frac{24}{100}$, $P(L/B)=\frac{22}{100}$, $P(L/C)=\frac{17}{100}$ , $P(L/D)=\frac{9}{100}$

The probability that a randomly selected child is left-handed=

$$\mathrm{P(A)\times P(L/A)+P(B)\times P(L/B)+P(C)\times P(L/C)+P(D)\times P(L/D)\,P(A/L)}$$

$$\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}$$

(b) The probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed

$= P(L/B) + P(L/C)$

$=\frac{22}{100}+\frac{17}{100}=\frac{39}{100}$

(c) $P(A/L)$

$$=\frac{\mathrm{P(A)\times P(L/A)}}{\mathrm{P(A)\times P(L/A)+P(B)\times P(L/B)+P(C)\times P(L/C)+P(D)\times P(L/D)\,P(A/L)}}$$

$$=\frac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}}=\frac{1}{3}$$

**Question 11** **[6]**

(i) Let the cost of one paper bag, one scrap book and one pastel sheet be x, y and z respectively.

$$30x+20y+10z=410\Rightarrow 3x+2y+z=41$$

$$20x+10y+20z=290\Rightarrow 2x+y+2z=29$$

$$20x+20y+20z=440\Rightarrow x+y+z=22$$

(ii) Given system of equations is equivalent to AX = B

Where $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix}$

$|A| = -2 \neq 0 \Rightarrow A^{-1}$ exists.

adj A = $\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$

Thus $A^{-1} = \frac{1}{|A|} adj\ A = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B = -\frac{1}{2}\begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \\ 5 \end{bmatrix}$

∴ x = 2, y = 15, z = 5

(iii) The cost of one paper bag, one scrap book and one pastel sheet be Rs 2, Rs 15 and Rs 5 respectively.

**Question 12** **[6]**

(i) **Method-I**

$(xdy - ydx)\, y \sin\left(\frac{y}{x}\right) = (ydx + xdy)\, x \cos\left(\frac{y}{x}\right).$

$\Rightarrow (xdy - ydx)\, \frac{\sin\left(\frac{y}{x}\right)}{\cos\left(\frac{y}{x}\right)} = \frac{(ydx+xdy)\, x}{y}$

$\Rightarrow \frac{(xdy - ydx)}{x^2} \tan\left(\frac{y}{x}\right) = \frac{(ydx + xdy)\, x}{x^2 y}$

$\Rightarrow \frac{(xdy - ydx)}{x^2} \tan\left(\frac{y}{x}\right) = \frac{(ydx + xdy)}{xy}$

$\Rightarrow \tan\left(\frac{y}{x}\right) . d\left(\frac{y}{x}\right) = \frac{d(xy)}{xy}$ ,

which is variable separable form where variables are $\frac{y}{x}$ and $xy$

Integrating both sides,

$\Rightarrow \int \tan\frac{y}{x} . d\left(\frac{y}{x}\right) = \int \frac{d(xy)}{xy}$

$\Rightarrow \log\left|\sec\left(\frac{y}{x}\right)\right| = \log(xy) + \log c$

$\Rightarrow \sec\left(\frac{y}{x}\right) = c\,(xy).$

Given, when $x = 1, y = \pi \Rightarrow c = -\frac{1}{\pi}$.

Hence required particular solution is: $\sec\left(\frac{y}{x}\right) = -\frac{1}{\pi}(xy).$

**Method II**

$(xdy - ydx)\, y \sin\left(\frac{y}{x}\right) = (ydx + xdy)\, x \cos\left(\frac{y}{x}\right).$

$\Rightarrow \frac{dy}{dx} = \frac{(xy)\cos\left(\frac{y}{x}\right) + y^2 \sin\left(\frac{y}{x}\right)}{(xy)\sin\left(\frac{y}{x}\right) - x^2 \cos\left(\frac{y}{x}\right)}$

$\Rightarrow \frac{dy}{dx} = \frac{1+\left(\frac{y}{x}\right)\tan\left(\frac{y}{x}\right)}{\tan\left(\frac{y}{x}\right)-\frac{1}{\left(\frac{y}{x}\right)}}$ ,

which is homogenous differential equation of type $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$.

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v\tan v}{\tan v-\frac{1}{v}}$

$\Rightarrow \left(\tan v - \frac{1}{v}\right) dv = 2\frac{dx}{x}$ ,

which is variable separable form where variables are $v$ and $x$

Integrating both sides,

$\Rightarrow \int \left(\tan v - \frac{1}{v}\right) dv = 2\int \frac{dx}{x}$

$\Rightarrow \log|\sec v| - \log|v| = 2\log|x| + \log c$

$\Rightarrow \log|\sec v| = \log|v.x^2.c|$

$\Rightarrow \sec v = c\,(xy)$

$\Rightarrow \sec\left(\frac{y}{x}\right) = c\,(xy)$.

Given, when $x = 1, y = \pi \Rightarrow c = -\frac{1}{\pi}$.

Hence required particular solution is: $\sec\left(\frac{y}{x}\right) = -\frac{1}{\pi}(xy)$.

**OR**

(ii) $\int_0^{\pi}(sin^4x + cos^4x)\,dx$ $[\because sin^4x + cos^4x = 1 - \frac{1}{2}sin^2 2x = \frac{1}{4}(3+\cos 4x)]$

$= \frac{1}{4}\int_0^{\pi}(3+\cos 4x)\,dx$

$= \frac{1}{4}\left[3x + \frac{\sin 4x}{4}\right]_0^{\pi} = \frac{3\pi}{4}$

$\mathrm{I} = \int_{-2\pi}^{2\pi}\frac{sin^4x + cos^4x}{1+e^x}dx$

$= \int_{-2\pi}^{2\pi}\frac{sin^4(-x) + cos^4(-x)}{1+e^{-x}}dx$ $\quad [\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx]$

$= \int_{-2\pi}^{2\pi}\frac{sin^4x + cos^4x}{1+e^{-x}}dx$

$= \int_{-2\pi}^{2\pi}\frac{e^x(sin^4x + cos^4x)}{1+e^x}dx$

$\mathrm{I} + \mathrm{I} = \int_{-2\pi}^{2\pi}\frac{sin^4x + cos^4x}{1+e^x}dx + \int_{-2\pi}^{2\pi}\frac{e^x(sin^4x + cos^4x)}{1+e^x}dx$

$\therefore 2\mathrm{I} = \int_{-2\pi}^{2\pi}(sin^4x + cos^4x)dx$

$= 2\int_0^{2\pi}(sin^4x + cos^4x)dx$

$[\because \int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx \text{ if } f(-x) = f(x)]$

$\therefore \mathrm{I} = \int_0^{2\pi}(sin^4x + cos^4x)dx$

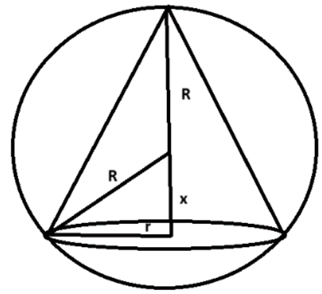
$= 2\int_0^{\pi}(sin^4x + cos^4x)dx$

$[\because \int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx \text{ if } f(2a-x) = f(x)]$

$= 2.\frac{3\pi}{4} = \frac{3\pi}{2}$

**Question 13** **[6]**

(i) Let the given radius of sphere be R.
Consider radius of cone be r and height be = h = R + x




**Method -I**

From the diagram $r^2 + x^2 = R^2$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(R + x) = \frac{1}{3}\pi(R + x)(R^2 - x^2)$$

$$\frac{dV}{dx} = \frac{1}{3}\pi[(R^2 - x^2)(1) + (R + x)(-2x)]$$

Setting, $\frac{dV}{dx} = 0$

$\Rightarrow \frac{1}{3}\pi[(R^2 - x^2)(1) + (R + x)(-2x)] = 0$

$\Rightarrow (R + x)(R - 3x) = 0$

$\Rightarrow R = 3x \text{ or } x = \frac{R}{3}$

$$\frac{d^2V}{dx^2} = \frac{1}{3}\pi[-2R - 6x] < 0 \text{ at } x = \frac{R}{3}$$

Volume is maximum at $x = \frac{R}{3}$

Height of cone = $R + x = R + \frac{R}{3} = \frac{4R}{3}$

Ratio of height of cone to diameter of sphere = $\frac{h}{2R} = \frac{\frac{4R}{3}}{2R} = \frac{2}{3}$

**Method –II**

From the diagram $R^2 = r^2 + x^2 = r^2 + (h - R)^2$

$\therefore R^2 = r^2 + h^2 + R^2 - 2Rh$

$\therefore r^2 = 2Rh - h^2$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2Rh - h^2)h = \frac{1}{3}\pi(2Rh^2 - h^3)$

$\therefore \frac{dV}{dh} = \frac{1}{3}\pi(4Rh - 3h^2)$

Setting $\frac{dV}{dh} = 0$ , we get $h = \frac{4R}{3}$

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4R - 6h) = \frac{1}{3}\pi(4R - 8R) < 0$$

$\therefore$ Volume is maximum at $h = \frac{4R}{3}$

Ratio of height of cone to diameter of sphere = $\frac{h}{2R} = \frac{\frac{4R}{3}}{2R} = \frac{2}{3}$

Setting $\frac{dV}{dh} = 0$ , we get $h = \frac{4R}{3}$

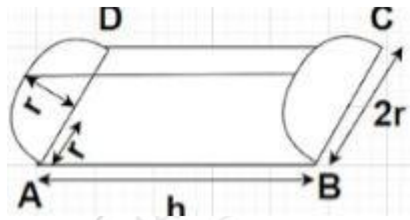
$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4R - 6h) = \frac{1}{3}\pi(4R - 8R) < 0$$

$\therefore$ Volume is maximum at $h = \frac{4R}{3}$

Ratio of height of cone to diameter of sphere = $\frac{h}{2R} = \frac{\frac{4R}{3}}{2R} = \frac{2}{3}$

**OR**

(ii)




Volume of half cylinder = $\frac{1}{2}\pi r^2 h = V \Rightarrow h = \frac{2V}{\pi r^2}$

$$S = \pi r^2 + \pi r h + 2rh = \pi r^2 + \pi r \cdot \frac{2V}{\pi r^2} + 2r \cdot \frac{2V}{\pi r^2} = \pi r^2 + \frac{1}{r}\left(2V + \frac{4V}{\pi}\right)$$

$$\frac{dS}{dr} = 2\pi r - \frac{1}{r^2}\left(2V + \frac{4V}{\pi}\right)$$

Setting $\frac{dS}{dr} = 0$ $\quad\Rightarrow 2\pi r - \frac{1}{r^2}\left(2V + \frac{4V}{\pi}\right) = 0$

$$\Rightarrow 2\pi r = \frac{1}{r^2}\left(2V + \frac{4V}{\pi}\right) \Rightarrow \pi r^3 = V + \frac{2V}{\pi} \Rightarrow \pi r^3 = V\left(\frac{\pi + 2}{\pi}\right)$$

$$\pi r^3 = \frac{1}{2}\pi r^2 h \cdot \frac{\pi + 2}{\pi} \Rightarrow \frac{2r}{h} = \frac{\pi + 2}{\pi}$$

$\therefore$ Ratio of the length of cylinder to the diameter of semi-circular ends $= \frac{\pi}{\pi + 2}$

$$\frac{d^2S}{dr^2} = 2\pi + \frac{2}{r^3}\left(2V + \frac{4V}{\pi}\right) > 0$$

$\therefore$ Surface area is minimum.

**Question 14** **[6]**

(i) X = $-3, -1, 0, 1$

(ii) $P(X = -3) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$

$P(X = -1) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$

$P(X = 0) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$

$P(X = 1) = \frac{1}{6}$

Required probability distribution :

| **X** | $-3$ | $-1$ | 0 | 1 |
|---|---|---|---|---|
| **P(X)** | $\frac{125}{216}$ | $\frac{25}{216}$ | $\frac{5}{36}$ | $\frac{1}{6}$ |

(iii) Expected value = $\sum px = (-3)\left(\frac{125}{216}\right) + (-1)\left(\frac{25}{216}\right) + 0 + \left(\frac{1}{6}\right)$

$= -\frac{91}{54} = -1\frac{37}{54} \approx -1.69$

## SECTION B - 15 MARKS

**Question 15** **[5]**

(i) (a) 1 only

(ii) (c) $\frac{7}{5}$

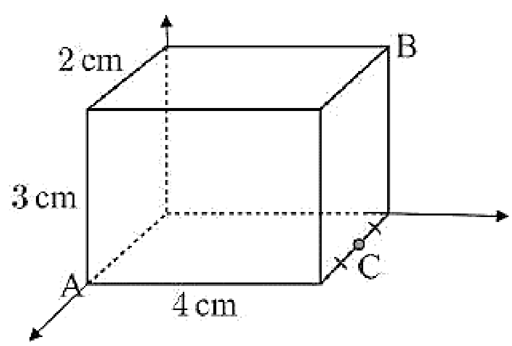
(iii) As per question $|\vec{a}| = 1$, $\vec{a}.\vec{b} = 0$

$\therefore (\vec{a} + 2\vec{b}).(3\vec{a} - \vec{b}) = -5$

$\Rightarrow 3|\vec{a}|^2 + 5(\vec{a}.\vec{b}) - 2|\vec{b}|^2 = -5$

$\Rightarrow 2|\vec{b}|^2 = 8 \Rightarrow |\vec{b}| = 2$

(iv) Placing the coordinate axes as illustrated , coordinates of A is (2,0,0) , B (0,4,3 ) and C (1,4,0 ).




$\therefore \overrightarrow{BA} = 2\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ and $\overrightarrow{BC} = \hat{\imath} - 3\hat{k}$

$\therefore \overrightarrow{BA}.\overrightarrow{BC} = 2 + 9 = 11$

(v) Using, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1 \Rightarrow \frac{3}{c^2} = 1$

$\Rightarrow c^2 = 3$

$\Rightarrow c = \pm\sqrt{3}$

**Question 16** **[2]**

(i) A vector of magnitude 9 units and perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is $9\left(\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}\right)$

Given, $\vec{a} = 4\,\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = -\,2\,\hat{\imath} + \hat{\jmath} - 2\,\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -1 & 1 \\ -2 & 1 & -2 \end{vmatrix} = \hat{\imath} + 6\,\hat{\jmath} + 2\,\hat{k}$ and $|\vec{a} \times \vec{b}| = \sqrt{41}$

Hence required vector is $\frac{9}{\sqrt{41}}(\hat{\imath} + 6\,\hat{\jmath} + 2\,\hat{k})$

**OR**

(ii) Let $\vec{a} = 2x^2\hat{\imath} + 3x\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} + x^2\hat{k}$

$\because$ angle between the vectors is obtuse

$\Rightarrow \cos\theta < 0$

$\Rightarrow \frac{\vec{a}\,.\vec{b}}{|\vec{a}\,|\,|\vec{b}|} < 0$

$\Rightarrow \vec{a}\,.\vec{b} < 0$

$\Rightarrow 2x^2 - 6x + x^2 < 0$

$\Rightarrow 3x(x-2) < 0$

$\Rightarrow 0 < x < 2.$ i.e. $x \in (0,2)$

**Question 17** **[4]**

(i) P.V. of the point on the line $2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

Direction vector $\hat{\imath} - \hat{\jmath} + 4\hat{k}$

Given plane $\vec{r} \cdot (\hat{\imath} + 5\hat{\jmath} + \hat{k}) - 5 = 0$

$\therefore$ normal vector of the plane $\hat{\imath} + 5\hat{\jmath} + \hat{k}$

If straight line and plane are parallel to each other

$\Rightarrow$Direction vector of line and normal vector of the plane are perpendicular to each other

Now we get $(\hat{\imath} - \hat{\jmath} + 4\hat{k})(\hat{\imath} + 5\hat{\jmath} + \hat{k}) = 0$

$\because$ Since the dot product is zero, they are perpendicular to each other.

$\Rightarrow$ straight line and plane are parallel to each other.

Required distance = $\frac{(2\hat{i}-2\hat{j}+3\hat{k})\cdot(i+5j+k)-5}{\sqrt{1+25+1}} = \frac{10}{\sqrt{27}} = \frac{10}{3\sqrt{3}}\ units.$

**OR**

(ii) **Method-I**

Let the equation of the plane be $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ is passing through $(-1, 0, 2)$

$\Rightarrow a(x + 1) + b(y – 0) + c(z – 2) = 0$………………………..(1)

Given line $\frac{x-0}{-2}=\frac{y-1}{3}=\frac{z-1}{-1}$ passing through (0, 1, 1) and having d.r. $\langle -2, 3, -1\rangle$

Since the plane contains the line and the point

$\Rightarrow$ a(1) + b(1) + c(1 – 2) =0 $\Rightarrow$ a + b – c = 0……(2)

Also the line and normal to the plane are perpendicular

$\Rightarrow - 2a + 3b - c = 0$…………….(3)

Solving (2) and (3) $\quad \frac{a}{2}=\frac{b}{3}=\frac{c}{5}=k$

Hence required equation of the plane is

$\Rightarrow 2(x+1)+3(y-0)+5(z-2)=0$

$\Rightarrow 2x+3y+5z-8=0$

**Method –II**

Let the equation of the plane be $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ is passing through $(-1, 0, 2)$

$\Rightarrow a(x + 1) + b(y – 0) + c(z – 2) = 0$………………………..(1)

Given line $\frac{x-0}{-2}=\frac{y-1}{3}=\frac{z-1}{-1}$ passing through (0, 1, 1) and having d.r. $\langle -2, 3, -1\rangle$

Since the plane contains the line and the point

$\Rightarrow$ a(1) + b(1) + c(1 – 2) =0 $\Rightarrow$ a + b – c = 0……(2)

Also the line and normal to the plane are perpendicular

$\Rightarrow - 2a + 3b - c = 0$…………….(3)

Hence required equation of the plane is

$$\begin{vmatrix} x+1 & y & z-2 \\ 1 & 1 & -1 \\ -2 & 3 & -1 \end{vmatrix} = 0$$

$\Rightarrow 2(x+1)+3(y-0)+5(z-2)=0$

$\Rightarrow 2x+3y+5z-8=0$

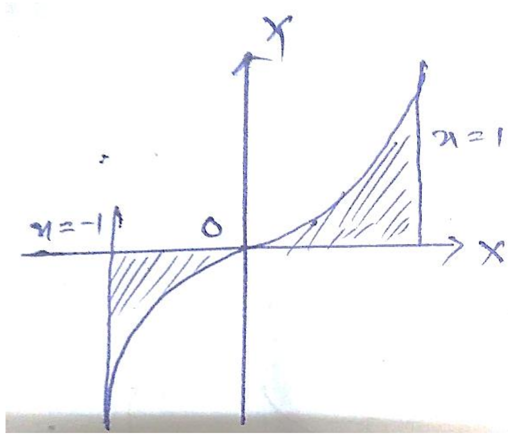
**Question 18** **[4]**

(i) Given, the equation of the curve: $y = x\,|x|$

$\therefore y = \begin{cases} -x^2, x < 0 \\ x^2, x \ge 0 \end{cases}$

Hence sketch of the region enclosed bounded by the curve, $y = x\,|x|$ and the ordinates $x = -1$ and $x = 1$ is as follows.

(ii)




(iii) $\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$

$\therefore$ Area of the shaded region = $2\int_0^1 x^2\,dx = \frac{2}{3}$ sq. units.

## SECTION C - 15 MARKS

**Question 19** **[5]**

(i) (b) MC = AC

(ii) (d) III and IV are correct

(iii) Given: $\bar{x} = 53,\ \bar{y} = 28,\ b_{yx} = -1.2,\ b_{xy} = -0.3$

Using, $r^2 = b_{yx} \times b_{xy}$

$= -1.2 \times -0.3 = 0.36$

$\Rightarrow r = -0.6$

**ANS :** $\mathbf{-0.6}$

(iv) Given , $R(x) = 36x + 3x^2 + 5$

$$\Rightarrow MR = \frac{d}{dx}(36x + 3x^2 + 5)$$

= 36 + 6x

$\therefore MR(x = 5) = 66$

**ANS : 66**

(v) Given $C(x) = 210x + 7000$ , $R(x) = 280x$

Minimum number must be sold daily when $R(x) = C(x)$

$\Rightarrow 210x + 7000 = 280x$

$\Rightarrow 70x = 7000$

$\Rightarrow x = 100.$

$\therefore$ Minimum number that must be sold is 100.

**ANS: 100**

**Question 20** **[2]**

(i) $$C(x) = 200 + 20x - \frac{1}{2}x^2$$

$\Rightarrow AC = \frac{200}{x} + 20 - \frac{1}{2}x$

To find AC falls:

$$\frac{d}{dx}(AC) = \frac{d}{dx}\left(\frac{200}{x} + 20 - \frac{1}{2}x\right)$$

$$= -\frac{200}{x^2} - \frac{1}{2} = -\left(\frac{200}{x^2} + \frac{1}{2}\right) < 0, \forall\, x > 0.$$

Hence AC falls continuously.

**OR**

(ii) Given, $x = 100 - 4p$

$\Rightarrow p = \frac{100 - x}{4}$

$\therefore R(x) = px = \frac{100x - x^2}{4}$

$\Rightarrow MR = \frac{d}{dx}\left(\frac{100x - x^2}{4}\right) = \frac{100 - 2x}{4}$

$\because$ MR = 0

$\therefore x = 50$

**Question 21** **[4]**

(i) $$\bar{x} = \frac{\sum x}{n} = 170, \qquad \bar{y} = \frac{\sum y}{n} = 192,$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = 0.6 \times \frac{20}{60} = 0.2$$

Regression equation y on x is

$y - 192 = 0.2(x - 170)$

$\Rightarrow y - 192 = 0.2x - 34$
$\Rightarrow y = 0.2x + 158$
Put $x = 200$
$\Rightarrow y = 0.2(200) + 158 = 198$
$\therefore$ Expenditure on food and entertainment = ₹ 198.

**OR**

(ii)

(a) Regression lines are $3x + 2y - 26 = 0$ ……………….(i)
$6x + y - 31 = 0$ …………… (ii)
$(\bar{x}, \bar{y})$ satisfies the equation (i) and (ii) ,
$\therefore$ Solving (i) and (ii) we get , $\bar{x} = 4$ and $\bar{y} = 7$.

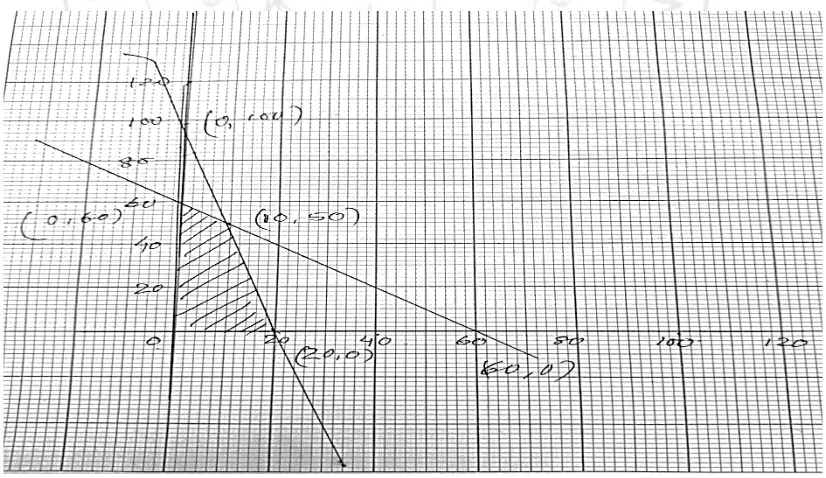
(b) On finding we get, $b_{yx} = -\frac{1}{6}$, $b_{xy} = -\frac{3}{2}$

$\therefore r^2 = b_{yx} \times b_{xy} = -\frac{1}{6} \times -\frac{3}{2} = \frac{1}{4}$

$\therefore r = -\frac{1}{2}$. [$\because b_{yx}, b_{xy}$ and $r$ must have same sign]

**Question 22** **[4]**

(i) From graph, corner points are: A (0, 60) , B ( 10, 50), C ( 20, 0) , D ( 0, 0)




(ii) $Z = px + qy$
Given , Z is maximum at (0, 60) and (10, 50)
$\therefore$ 0. p + 60. q = 10. p + 50.q
$\Rightarrow$10p = 10q
$\Rightarrow$p = q
So there can be infinite number of optimal solutions.