JEE Main 2026 Sequence, Series and Binomial Theorem Questions & Solutions PDF

Q1. In the expansion of $(x + 1/x)^{10}$, find the term independent of x.

Solution

General term: T_{r+1} = (10 r)x^{10-r} · $(1/ x)^r$ = (10 r)x^{10-2r}. The exponent of x is zero ⇒ 10 − 2r =0 ⇒ r = 5. So the term independent of x is (10 5) = 252.

Answer: (10 5) = 252

Q2. Find the sum of the coefficients of the even-power terms in the expansion of $(1 + x)^{12}$.

Solution

Sum of all coefficients = 2^{12} = 4096. Also from known result: sum of coefficients of even-power terms = sum of coefficients of odd-power terms = 2^{11} = 20482. (This is because set x = 1 gives the total sum and x = −1 gives the difference between even and odd sums.)

Answer: 20482

Q3. In a GP the 5th term is 48 and the 8th term is 384. Find the first term and the common ratio.

Solution

Let the first term = a, ratio = r. Then ar^4 = 48; ar^7 = 384. Divide: ar^7/ ar^4 = r^3 = 384/ 48 = 8 ⇒ r^3 = 8 ⇒ r = 2. Then $a2^4$ = 48 ⇒ a = 48/ 16 = 3. So, first term = 3, ratio = 2.

Answer: first term = 3, ratio = 2.

Q4. If a GP has first term 8 and common ratio r = 1/ 2, find its sum to infinity. Then find its 7th term.

Solution

Sum to infinity: since $|r| < 1$, $S_\infty = 8/ 1 - ½ = 8/ (1 / 2) = 16$. The 7th term = $ar^6 = 8 \times (1/ 2)^6 = 8 \times 1/ 64 = 8 / 64 = 1/ 8$.

Answer: 7th term = 1/ 8

Q5. In an AP, the 12th term is 50 and the 20th term is 82. Find the first term and common difference.

Solution

Let first term = a, difference = d.
12th term: $a + 11d = 50$
20th term: $a + 19d = 82$
Subtract: $8d = 32 \Rightarrow d = 4$. Then $a + 11 \times 4 = 50 \Rightarrow a = 50 - 44 = 6$

Answer: a = 6

Q6. Evaluate $\sum^{n}_{k=0} (n\ k)\ 2^k$

Solution

Use binomial theorem: $(1 + 2)^n = 3^n = \sum^{n}_{k=0} (n\ k)\ 1^{n-k}\ 2^{k}$
Thus the sum = 3^n

Answer: 3^n

Q7. In the expansion of $(x - y)^{10}$, find the coefficient of the term containing x^7y^3.

Solution

General term: $(10\ 7)x^{10-r}\ (-y)^r$
We want exponent of $x = 7 \Rightarrow 10 - r \Rightarrow r = 3$
Coefficient = $(10\ 3)\ (-1)^3 = 120 \times (-1) = -120$

Answer: -120

Q8. In the expansion of $(3 + x)^8$, the sum of the coefficients of all terms is _?

Solution

Set $x = 1 \Rightarrow (3 + 1)^8 = 4^8 = 65536$. That equals the sum of the coefficients of all terms.

Answer: 65536

Q9. Find the sum of the infinite GP: 81, 27, 9, …

Solution

First term a = 81, ratio r=27/81=1/3. |r| < 1 so sum to infinity = 81 / 1 - 1 / 3 = 81 / 2 / 3 = 81 X (3 / 2) = 121.5. Or as a fraction = 243/ 2.

Answer: 121.5

Q10. Expand $(1 + x)^5$ and find the coefficient of x^3.

Solution

By binomial theorem: coefficient = (5 3) = 10

Answer: 10

Q11. Evaluate the sum 1 + 1 / 2! + 1 / 3! + … + 1 / n!. (No closed simple form, but approximate as e).

Solution

Recognise the series approaches e - 1 as $n \to \infty$. For finite n, just state approximately.

Answer: $n \to \infty$

Q12. If the sum of an infinite GP is 16 and its first term is 12, find the common ratio.

Solution

S∞ = a / 1 - r = 16 ⇒ 12 / 1 - r = 16 ⇒ 1 - r = 12 / 16 = 3 / 4 ⇒ r = 1 / 4

Answer: r = 1 / 4

Q13. In the expansion of $(x + 2)^6$, what is the term independent of x?

Solution

General term: $(6\ r)x^{6-r}2^r$. Independent of x ⇒ exponent of x = 0 6-r =0 ⇒ r = 6.
Term = $(6\ 6)x^0 2^6$ = 1 X 64 = 64

Answer: 64

Q14. A series: S = 1 + 4 + 9 + 16 + ⋯ + n^2. Find the sum in closed form.

Solution

Famous formula: $\sum^n_{k=1} k^2 = n(n+1)\ (2n+1) / 6$

Answer: n(n+1) (2n+1) / 6

Q15. Expand $(1 - x)^7$ and find the coefficient of x^5.

Solution

Coefficient = $(7\ 5)\ (-1)^5$ = 21 X (-1) = -21

Answer: -21