

SET-A

Roll No.

Total No. of Printed Pages—16

607 R

(Regular)

M

(Science/Arts)

(For Students registered in 2021)

2 0 2 3 (A)

MATHEMATICS

SCIENCE/ARTS

Full Marks : 100

Time : 3 hours

The figures in the right-hand margin indicate marks

ଦକ୍ଷିଣ ପାର୍ଶ୍ଵରେ ଥୁବା ସଂଖ୍ୟା ପ୍ରଶ୍ନର ମୂଲ୍ୟାଙ୍କ ସୂଚାଭାବିତ

Answer the questions as per directions given in each

ପ୍ରଶ୍ନେକ ପ୍ରଶ୍ନରେ ଦିଆଯାଇଥିବା ନିର୍ଦ୍ଦେଶ ଅନୁଯାୟୀ ସମସ୍ତ ଉତ୍ତର ଦିଆ

*Electronic gadgets are not allowed in the
Examination Hall*

ପରୀକ୍ଷା ହଲ୍ଦରେ ଇଲେକ୍ଟ୍ରୋନିକ ଯନ୍ତ୍ରର ବ୍ୟବହାର ନିଷେଧ ଆଚାର

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(Turn Over)

GROUP—A

ক—বিভাগ

(Marks : 10)

(নম্বর : 10)

1. Answer all questions : $1 \times 10 = 10$

সমস্ত প্রশ্নের উত্তর দিঅ :

(a) If R is an equivalence relation on A , then link the domain of R and the range of R .

যদি R , A উপরিষ্ঠ এক সমতুল্য সমষ্টি হুৰ্বা, তেবে R র পরিসর ও বিস্তৃতি মধ্যে সংপর্ক লেখ।

(b) Write the value of $\cos^{-1} \cos(3\pi/2)$.

$\cos^{-1} \cos(3\pi/2)$ ৰ মান লেখ।

(c) Write the number of entries in a square matrix of order 4.

গোটী৏ 4 অৰ্দ্ধের ম্যাট্রিক্সৰ উপাদান সংখ্যা লেখ।

(d) Write the value of

$$\begin{vmatrix} 0 & 7 & 0 \\ 22 & 390 & 22 \\ 1 & 208 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 7 & 0 \\ 22 & 390 & 22 \\ 1 & 208 & 0 \end{vmatrix} \text{ র মূল্য লেখ।}$$

(e) A random variable X has the probability distribution as given below :

X	1	2	3	4	5
$P(X)$	0.1	k	0.3	$2k$	0.2

Then write the value of k .

নিম্নৰে এক যাদুলু ঢিল X ৰ সমাব্যটা বেঁচে দিআয়াকৰিঃ

X	1	2	3	4	5
$P(X)$	0.1	k	0.3	$2k$	0.2

k ৰ মান নির্ণয় কৰ।

(f) Write the function $f(x)$, where $f'(x) = f(x)$.

$f(x)$ ফলনটি লেখ, যেৱেৱে $f'(x) = f(x)$.

(g) Write the set of values of x for which the function $f(x) = \sin x - x$ is increasing.

x মূল্যৰ যেৱে ষেৱ পাৰ্শ্ব ফলন $f(x) = \sin x - x$ বৰ্দ্ধমান, তাৰা লেখ।

(4)

(h) If $\int_1^2 f(x) dx = 7$, then write the value of

$$\int_1^2 f(\phi(x)) d(\phi(x)).$$

যদি $\int_1^2 f(x) dx = 7$, তবে $\int_1^2 f(\phi(x)) d(\phi(x))$ র
মান লেখ।

(i) Write the degree of the differential

$$\text{equation } \frac{d^2y}{dx^2} = \sqrt{1 + (dy/dx)}.$$

$\frac{d^2y}{dx^2} = \sqrt{1 + (dy/dx)}$ অবকল সমাকরণের ঘাত
লেখ।

(j) Write the equation of the plane meeting in the coordinate axes in A , B and C in the order, given that (a, b, c) is the centroid of the triangle ABC .

গোটি এ সমতল অক্ষগুଡ়িকু যথাক্রমে A , B ও C দিয়ে রেখ করো। যদি (a, b, c) ত্রিভুজ ABC র ভরকেন্দ্র হুয়ে, তবে সমতলের
সমাকরণ লেখ।

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GROUP—B

শ্রেণী

(Marks : 60)

(নম্বর : 60)

2. Answer any three questions :

 $4 \times 3 = 12$

যেকোণী তিনোটি প্রশ্নের উত্তর দিঅ :

(a) If $f(x) = \sin x$, $g(x) = x^5$, then find $g \circ f$ and $f \circ g$.

যদি $f(x) = \sin x$, $g(x) = x^5$, তবে $g \circ f$
এবং $f \circ g$ নির্ণয় কর।

(b) Show that the relation $R = \{(m, n) : \frac{m}{n}$ is a power of 5} on the set $Z - \{0\}$ is an equivalence relation.

যেহেতু $Z - \{0\}$ রপরিষ্ঠি $R = \{(m, n) : \frac{m}{n}$ ৫র ঘাত
থাণে } যন্ত্রে এক সমতুল্য সমষ্টি, দর্শাও।

(c) Show that

দর্শাও যে

$$\sin^{-1} \frac{4}{5} + 2 \tan^{-1} \frac{1}{3} = \frac{\pi}{2}$$

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(d) Solve (ସମାଧାନ କର) :

$$\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} x = \frac{\pi}{2}$$

(e) Solve the following LPP graphically :

ଲେଖଚିତ୍ର ସାହାଯ୍ୟରେ ନିମ୍ନ ପ୍ରଦତ୍ତ LPP ର ସମାଧାନ

କର :

Maximize (ଗରିଷ୍ଠ ମାନ ନିର୍ଣ୍ଣୟ କର)

$$Z = 20x + 40y$$

subject to (ଯେପରିକି)

$$x + y \leq 1$$

$$6x + 2y \leq 3$$

$$x, y \geq 0$$

3. Answer any three questions :

4×3=12

ଯେକୋଣେ ତିନେଟି ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଆ :

(a) If (ଯଦି) $\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x & 2 \\ 1 & y \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -1 & 4 \end{bmatrix}$, then

(ଡେବେ) find x and y (x ଏବଂ y ନିର୍ଣ୍ଣୟ କର).

(b) Find the inverse of the following matrix :

ନିମ୍ନଲିଖିତ ମ୍ୟାଟ୍ରିଜ୍ବର ବିଲୋମ ନିର୍ଣ୍ଣୟ କର :

$$\begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

(c) Show that (ଦର୍ଶାନ ଯେ)

$$\begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$$

(d) If $P(A) = 0.6$ and $P(B|A) = 0.5$, then find $P(A \cup B)$, when A and B are independent.

ଯଦି $P(A) = 0.6$, $P(B|A) = 0.5$, $P(A \cup B)$ ନିର୍ଣ୍ଣୟ କର ଯେତେବେଳେ A ଏବଂ B ସ୍ବନ୍ଧୀ।

(e) A person takes 4 tests in succession. The probability of his passing the first test is p , that of his passing each succeeding test is p or $p/2$, depending on his passing or failing the preceding test. Find the probability of his passing just three tests.

ଜଣେ ବ୍ୟକ୍ତି ଗୋଟିଏ ପରେ ଗୋଟିଏ 4ଟି ପରୀକ୍ଷା ଦିଅଛି। ପ୍ରଥମ ପରୀକ୍ଷା ପାସ କରିବାର ସମ୍ଭାବ୍ୟତା p ଅଟେ, ତା' ପରର ପ୍ରତ୍ୟେକ ପରୀକ୍ଷା ପାସ କରିବାର ସମ୍ଭାବ୍ୟତା ପୂର୍ବ ପରୀକ୍ଷାର ପାସ କିମ୍ବା ଫେଲ୍ ଉପରେ ନିର୍ଭର କରି p କିମ୍ବା $p/2$ ହୁଏ। ମାତ୍ର 3ଟି ପରୀକ୍ଷା ପାସ କରିବାର ସମ୍ଭାବ୍ୟତା ନିର୍ଣ୍ଣୟ କର।

(8)

4. Answer any three questions :

4×3=12

ଯେକୋଣସି ତିନୋଟି ପ୍ରଶ୍ନର ଭାବର ଦିଅ :

(a) If $y = x^{x^x}$, then find $\frac{dy}{dx}$.ଯଦି $y = x^{x^x}$, ତେବେ $\frac{dy}{dx}$ ନିର୍ଣ୍ଣୟ କରା(b) If $x = a \sin \theta$ and $y = b \cos \theta$, then find

$$\frac{d^2y}{dx^2}.$$

ଯଦି $x = a \sin \theta$ ଓ $y = b \cos \theta$, $\frac{d^2y}{dx^2}$ ନିର୍ଣ୍ଣୟ

କରା

(c) Test the continuity and differentiability of the following function at $x = 0$:ନିମ୍ନ ପ୍ରଦତ୍ତ ଫଳନର $x = 0$ ଠାରେ ଅବିହିନିତା ଓ
ଅବକଳନୀୟତା ପରୀକ୍ଷା କରା :

$$f(x) = \begin{cases} \frac{1-e^{-x}}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

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(Continued)

(9)

(d) Find the equation of the normal to the

curve $y = (\log x)^2$ at $x = \frac{1}{e}$.ବାକ୍ୟ $y = (\log x)^2$ ପ୍ରତି $x = \frac{1}{e}$ ଠାରେ ଲମ୍ବର
ସମୀକରଣ ନିର୍ଣ୍ଣୟ କରା(e) Find two numbers x and y , whose sum is 15 such that xy^2 is maximum.ଯୋଗପାଳ 15 ହେଉଥିବାର ଦୁଇଟି ସଂଖ୍ୟା x ଓ y
ନିର୍ଣ୍ଣୟ କର ଯେପରିକି xy^2 ସର୍ବାଧିକ ହେବା

5. Answer any three questions :

4×3=12

ଯେକୋଣସି ତିନୋଟି ପ୍ରଶ୍ନର ଭାବର ଦିଅ :

(a) Evaluate :

ମୂଲ୍ୟାଙ୍କନ କରା :

$$\int \frac{\sin x}{\sin(x+\alpha)} dx$$

(b) Evaluate :

ମୂଲ୍ୟାଙ୍କନ କରା :

$$\int x^2 \tan^{-1} x dx$$

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(Turn Over)

(10)

(c) Find the area of the region bounded by the curve $y^2 - x^2$ and the double ordinate through (2, 0).

$y^2 - x^2$ ഒരു ഓ (2, 0) ദേശ യാഭത്താ ദിക്ക് കോടി ഭാരാ പാശ ശ്രദ്ധിച്ച ശ്രദ്ധിച്ച നിർണ്ണയ കരാ.

(d) Solve

ഇനാധാര കരാ :

$$x \log x \frac{dy}{dx} + y = \log x$$

(e) Solve :

ഇനാധാര കരാ :

$$\frac{dy}{dx} = e^{3x+4y}$$

6. Answer any three questions : $4 \times 3 = 12$

മേരുക്കേണ്ടി പ്രശ്നര തിരഞ്ഞെടുക്കുക :

(a) Prove that two vectors \vec{a} and \vec{b} are perpendicular iff $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$.

പ്രമാണ കരാ യേ \vec{a} ഓ \vec{b} ദിശാക്കൂദ്ധ പരബ്രഹ്മ പ്രതി ലൈ അംഗീ യഥി ഓ കേബലം യഥി $|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$.

(11)

(b) Find the value of λ so that the three vectors $\vec{i} + 2\vec{j} + 3\vec{k}$, $4\vec{i} + \vec{j} + \lambda\vec{k}$ and $\lambda\vec{i} - 4\vec{j} + \vec{k}$ are coplanar

$\vec{i} + 2\vec{j} + 3\vec{k}$, $4\vec{i} + \vec{j} + \lambda\vec{k}$ ഓ $\lambda\vec{i} - 4\vec{j} + \vec{k}$ ദിശാക്കൂദ്ധ ഘമഭക്കായ ദേശാഘാർ λ ഓ മൂലം നിർണ്ണയ കരാ।

(c) Show that

ബശ്ശാം യേ

$$(\vec{a} \times \vec{b})^2 = a^2 b^2 - (\vec{a} \cdot \vec{b})^2$$

(d) Prove that the measure of the angle between two main diagonals of a cube is $\cos^{-1} \frac{1}{3}$.

പ്രമാണ കരാ യേ ഏക ഘനര ദുഃഖ പ്രധാന കണ്ണ മാറ്റു കോണര മാപക $\cos^{-1} \frac{1}{3}$ അംഗീ

(e) Find the point where the line $\frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{2}$ meets the plane $2x + y + z = 2$.

$\frac{x-2}{1} = \frac{y}{-1} = \frac{z-1}{2}$ രേഖാം യേജ് ബിഹുരേ $2x + y + z = 2$ ഘമഭക്ക ഘിച്ച മീറ്റിച ഹുഏ, താഴാ നിർണ്ണയ കരാ।

(12)

GROUP—C

ଗ—ବିଭାଗ

(Marks : 30)

(ନୟର : 30)

7. Answer any one question :

6

ଯେକୌଣସି ଗୋଟିଏ ପ୍ରଶ୍ନର ଭରର ଦିଆ :

(a) If (ସଦି) $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$,
show that (ଦର୍ଶାଅ ଯେ)

$$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$$

(b) Let f be a real function. Show that $h(x) = f(x) + f(-x)$ is always an even function and $g(x) = f(x) - f(-x)$ is always an odd function. Also express $e^x + \sin x$ as the sum of an even function and an odd function.

ମନେକର f ଏକ ବାସ୍ତ୍ଵବ ଫଳନ,
 $h(x) = f(x) + f(-x)$ ସର୍ବଦା ଏକ ସମ-ଫଳନ ଓ
 $g(x) = f(x) - f(-x)$ ସର୍ବଦା ଏକ ବିଷମ ଫଳନ
ବୋଲି ଦର୍ଶାଅ। $e^x + \sin x$ ଏକ ସମ ଓ ଏକ
ବିଷମ ଫଳନର ଯୋଗଫଳ ଭାବେ ଦର୍ଶାଅ।

(c) Solve the following LPP by graphical method :

ଲେଖଣିତ୍ର ସାହାଯ୍ୟରେ ନିମ୍ନ LPP ର ସମାଧାନ କର :

Minimize (ଲାଗ୍ରିମ୍ ମାନ ନିର୍ଣ୍ଣୟ କର)

$$Z = x + 3y$$

subject to (ଯେପରିକି)

$$x + 2y \geq 2$$

$$3x + y \geq 3$$

$$4x + 3y \geq 6$$

$$x, y \geq 0$$

8. Answer any one question :

6

ଯେକୌଣସି ଗୋଟିଏ ପ୍ରଶ୍ନର ଭରର ଦିଆ :

(a) If (ସଦି) $A + B + C = \pi$, prove that (ପ୍ରମାଣ କର ଯେ)

$$\begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix} = 0$$

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(Continued)

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(b) Solve the following equations by matrix method :

ନିମ୍ନ ସମୀକରଣଗୁଡ଼ିକର ନ୍ୟାଟ୍ରିକ୍ ପଦ୍ଧତିରେ
ସମାଧାନ କର :

$$\begin{aligned}x + 2y - 3z &= 4 \\2x + 4y - 5z &= 12 \\3x - y + z &= 3\end{aligned}$$

(c) Four cards are drawn successively from a well-shuffled pack of 52 cards with replacement after each draw. Find the probability that—

(i) all four cards are spades;
(ii) only three cards are spades;
(iii) none is a spade.

ଉଲ୍ଲଙ୍ଘନ କରିବାରେ ଫେର୍ରା ଯାଇଥିବା ଏକ 52 ପଦା ବିଶିଷ୍ଟ ତାସମୂଠାରୁ ପ୍ରତ୍ୟେକ ଟଣା ପରେ ସଂସ୍କାର ସହିତ 4ଟି ପଦା ଟଣାଗଲା। ନିମ୍ନଲିଖିତ ଘଟଣାଗୁଡ଼ିକର ସମ୍ଭାବ୍ୟତା ନିର୍ଣ୍ଣୟ କର :

(i) ସମସ୍ତ 4ଟି କଳାପାନ ଅଟେ;
(ii) କେବଳ 3ଟି କଳାପାନ ଅଟେ;
(iii) କୌଣସିଟି କଳାପାନ ନୁହେଁ।

9. Answer any one question :

ଯେକୌଣସି ଗୋଟିଏ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

(a) Find (ନିର୍ଣ୍ଣୟ କର) $\frac{dy}{dx}$, if (ଯଦି)

$$x^m y^n = \left(\frac{x}{y} \right)^{m+n}.$$

(b) Find the points on the curve $y = x^2 + 1$, which are nearest to the point (0, 2).

ବକ୍ଷ $y = x^2 + 1$ ଉପରିଷ୍ଠ ବିନ୍ଦୁମାନ, ଯେଉଁମାନେ (0, 2) ବିନ୍ଦୁର ଅତି ନିକଟରେ, ସେବୁଡ଼ିକୁ ନିର୍ଣ୍ଣୟ କର।

10. Answer any one question :

ଯେକୌଣସି ଗୋଟିଏ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

(a) Evaluate :

ମୂଲ୍ୟାଙ୍କନ କର :

$$\int \frac{x+5}{x^2 + 6x + 13} dx$$

(b) Find the area of the circle $x^2 + y^2 = 2ax$.

$x^2 + y^2 = 2ax$ ବୃତ୍ତର କ୍ଷେତ୍ରଫଳ ନିର୍ଣ୍ଣୟ କର।

(c) Solve :

ସମାଧାନ କର :

$$(2x + y + 1)dx + (4x + 2y - 1)dy = 0$$

11. Answer any *one* question :

6

ଯେକୋଣସି ଗୋଟିଏ ପ୍ରଶ୍ନର ଉତ୍ତର ଦିଅ :

(a) Show that

ଦର୍ଶାଅ ଯେ

$$[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$$

(b) Find the equation of the line through the point (1, 2, 3) and parallel to the line $x - y + 2z - 5 = 0 = 3x + y + z - 6$.

(1, 2, 3) ବିନ୍ଦୁ ଦେଇ ଯାଉଥୁବା 3

$x - y + 2z - 5 = 0 = 3x + y + z - 6$ ରେଖା

ସହିତ ସମାନ୍ତର ରେଖାର ସମୀକରଣ ନିର୍ଣ୍ଣୟ କର।

★ ★ ★