DAY —— 09 SEAT NUMBER

2024	III	02

1100

J-866

(E)

MATHEMATICS & STATISTICS (88)
(COMMERCE)

Time : 3 Hrs. **(12 Pages)** **Max. Marks : 80**

General Instructions :

(i) *All questions are compulsory.*

(ii) *There are 6 questions divided into two sections.*

(iii) *Write answers of Section-I and Section-II in the same answer book.*

(iv) *Use of logarithmic tables is allowed. Use of calculator is not allowed.*

(v) *For L.P.P. graph paper is not necessary. Only rough sketch of graph is expected.*

(vi) *Start answer to each question on a new page.*

(vii) *For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabetical letter eg. (a) ..../ (b) ..../ (c) ..../ (d) .... No mark(s) shall be given if "ONLY" the correct answer or the alphabet of the correct answer is written. Only the first attempt will be considered for evaluation.*

SECTION - I

Q. 1. **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) : **[12]** (6)

(i) Which of the following is not a statement?

(a) Smoking is injurious to health

(b) $2 + 2 = 4$

(c) 2 is the only even prime number

(d) Come here

(ii) If $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$ then $(y, z) =$ _____.

(a) $(-1, 0)$ (b) $(1, 0)$

(c) $(1, -1)$ (d) $(-1, 1)$

(iii) If $y = \log\left(\frac{e^x}{x^2}\right)$, then $\frac{dy}{dx} = ?$

(a) $\frac{2-x}{x}$ (b) $\frac{x-2}{x}$

(c) $\frac{e-x}{ex}$ (d) $\frac{x-e}{ex}$

(iv) The value of $\int \frac{dx}{\sqrt{1-x}}$ is _____.

(a) $2\sqrt{1-x}+c$ (b) $-2\sqrt{1-x}+c$

(c) $\sqrt{x}+c$ (d) $x+c$

(v) $\int \frac{dx}{(x-8)(x+7)} =$ _____.

(a) $\frac{1}{15}\log\left|\frac{x+2}{x+1}\right|+c$ (b) $\frac{1}{15}\log\left|\frac{x+8}{x+7}\right|+c$

(c) $\frac{1}{15}\log\left|\frac{x-8}{x+7}\right|+c$ (d) $(x-8)(x+7)+c$

(vi) The differential equation of $y = k_1 e^x + k_2 e^{-x}$ is :

(a) $\frac{d^2y}{dx^2} - y = 0$ (b) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

(c) $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$ (d) $\frac{d^2y}{dx^2} + y = 0$

(B) State whether the following statements are true or false (1 mark each) : (3)

(i) $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(ii) For $\int \frac{x-1}{(x+1)^3} e^x dx = e^x f(x) + c,\ f(x) = (x+1)^2$

(iii) Order and degree of a differential equation are always positive integers.

(C) Fill in the following blanks (1 mark each) : (3)

(i) The slope of tangent at any point (a, b) is called as _______.

(ii) If $f'(x) = \frac{1}{x} + x$ and $f(1) = \frac{5}{2}$ then $f(x) = \log x + \frac{x^2}{2} +$ ______.

(iii) A solution of differential equation which can be obtained from the general solution by giving particular values to the arbitrary constants is called ______ solution.

Q. 2. **(A)** Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) Examine whether the following statement pattern is tautology, a contradiction or a contingency.

$\sim p \rightarrow (p \rightarrow \sim q)$

(ii) Find $\frac{dy}{dx}$ if, $x = e^{3t}$, $y = e^{(4t+5)}$

(iii) If $A = \begin{bmatrix} 7 & 3 & 0 \\ 0 & 4 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 1 & -4 \end{bmatrix}$ then find $A^T + 4B^T$

(B) Attempt any TWO of the following questions (4 marks each) : (8)

(i) Consider the following statements

(a) If D is dog, then D is very good.

(b) If D is very good, then D is dog.

(c) If D is not very good, then D is not a dog.

(d) If D is not a dog, then D is not very good.

Identify the pairs of statements having the same meaning. Justify.

(ii) Determine the minimum value of the function :

$f(x) = 2x^3 - 21x^2 + 36x - 20$.

(iii) Find the area of the regions bounded by the line $y = -2x$, the X-axis and the lines $x = -1$ and $x = 2$.

Q. 3. **(A)** Attempt any TWO of the following questions (3 marks each): (6) **[14]**

(i) Find $\frac{dy}{dx}$ if $y = x^{e^x}$

(ii) If $f'(x) = 4x^3 - 3x^2 + 2x + k$, $f(0) = 1$ and $f(1) = 4$, find $f(x)$.

(iii) Obtain the differential equation whose general solution is $x^3 + y^3 = 35ax$

(B) Attempt any ONE of the following questions (4 marks each): (4)

(i) Find the inverse of $\begin{bmatrix} 3 & 1 & 5 \\ 2 & 7 & 8 \\ 1 & 2 & 5 \end{bmatrix}$ by adjoint method.

(ii) The consumption expenditure E_c of a person with income x, is given by $E_c = 0.0006x^2 + 0.003x$. Find average propensity to consume (APC), marginal propensity to consume (MPC) when his income is ₹ 200. Also find his marginal propensity to save (MPS).

(C) Attempt any ONE of the following questions (Activity) (4 marks each): (4)

(i) Complete the following activity :

$$\int_0^2 \frac{dx}{4+x-x^2} = \int_0^2 \frac{dx}{-x^2 + \square + \square}$$

$$= \int_0^2 \frac{dx}{-x^2 + x + \frac{1}{4} - \square + 4}$$

$$= -\int_0^2 \frac{dx}{\left(x - \frac{1}{2}\right)^2 - \left(\square\right)^2}$$

$$= \frac{1}{\sqrt{17}} \log\left(\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right)$$

(ii) The rate of growth of population is proportional to the number of inhabitants. If the population doubles in 25 years and the present population is 1,00,000, when will the city have population 4,00,000?

Solution : Let 'P' be the population at time 't'.
Since rate of growth of population is proportional to the no. of inhabitants :

$$\frac{dP}{dt} \propto P$$

$\therefore$ Differential equation can be written as $\frac{dP}{dt} = kP$

where k is constant of proportionality.

$\therefore$ $\frac{dP}{P} = k \cdot dt$

On integrating we get

$\boxed{\quad\quad} = kt + c$ .....(i)

(i) When $t = 0,\ P = 1{,}00{,}000$

$\therefore$ from (i)

$\log 1{,}00{,}000 = k(0) + c$

$\therefore$ $c = \boxed{\quad\quad}$

$\therefore$ $\log\left(\frac{P}{1{,}00{,}000}\right) = kt$ .....(ii)

(ii) When $t = 25,\ P = 2{,}00{,}000$

as population doubles in 25 years

$\therefore$ from (ii) $\log 2 = 25k$

$\therefore$ $k = \boxed{\quad\quad}$

$\therefore$ $\log\left(\frac{P}{1{,}00{,}000}\right) = \left(\frac{1}{25}\log 2\right) \cdot t$

(iii) $\therefore$ when $P = 4{,}00{,}000$

$$\log\left(\frac{4{,}00{,}000}{1{,}00{,}000}\right) = \left(\frac{1}{25}\log 2\right) \cdot t$$

$\therefore \quad \log 4 = \left(\frac{1}{25}\log 2\right)\cdot t$

$\therefore \quad t = \boxed{}\ years$

SECTION - II

Q. 4. **(A)** Select and write the correct answer of the following multiple choice type of questions (1 mark each) : **[12]** (6)

(i) The difference between face value and present worth is called _____.

(a) Banker's discount

(b) True discount

(c) Banker's gain

(d) Cash value

(ii) In an ordinary annuity, payments or receipts occur at _____.

(a) beginning of each period

(b) end of each period

(c) mid of each period

(d) quarterly basis

(iii) b_{xy} and b_{yx} are _____.

(a) Independent of change of origin and scale

(b) Independent of change of origin but not of scale

(c) Independent of change of scale but not of origin

(d) Affected by change of origin and scale

(iv) Dorbish-Bowley's Price Index Number is given by _____.

(a) $\dfrac{\dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_1} + \dfrac{\Sigma p_0 q_1}{\Sigma p_1 q_0}}{2} \times 100$

(b) $\dfrac{\dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_0} + \dfrac{\Sigma p_0 q_0}{\Sigma p_1 q_1}}{2} \times 100$

(c) $\dfrac{\dfrac{\Sigma p_1 q_0}{\Sigma p_0 q_0} + \dfrac{\Sigma p_1 q_1}{\Sigma p_0 q_1}}{2} \times 100$

(d) $\dfrac{\dfrac{\Sigma p_0 q_0}{\Sigma p_1 q_0} + \dfrac{\Sigma p_0 q_1}{\Sigma p_1 q_1}}{2} \times 100$

(v) Objective function of L.P.P. is _____.

(a) a constraint

(b) a function to be maximised or minimised

(c) a relation between the decision variables

(d) a feasible region

(vi) To use the Hungarian method, a profit maximization assignment problem requires _____.

(a) Converting all profits to opportunity losses

(b) A dummy person or job

(c) Matrix expansion

(d) Finding the maximum number of lines to cover all the zeros in the reduced matrix.

(B) State whether the following statements are true or false (1 mark each) : (3)

(i) Broker is an agent who gives a guarantee to seller that the buyer will pay the selling price of goods.

(ii) $\sum \frac{p_0 q_0}{p_1 q_1} \times 100$ is the Value Index Number by simple aggregate method.

(iii) The optimum value of the objective function of L.P.P. occurs at the center of the feasible region.

(C) Fill in the blanks (1 mark each) : (3)

(i) The banker's discount is always _____ than the true discount.

(ii) The cost of living index number using Weighted Relative Method is given by _____.

(iii) The time interval between starting the first job and completing the last job including the idle time (if any) in a particular order by the given set of machines is called _____.

Q. 5. (A) Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) Deepak's salary was increased from ₹ 4,000 to ₹ 5,000. The sales being the same, due to reduction in the rate of commission from 3% to 2%, his income remains unchanged. Find his sales.

(ii) For a bivariate data, the regression co-efficient of Y on X is 0.4 and the regression co-efficient of X on Y is 0.9. Find the value of variance of Y if variance of X is 9.

(iii) The following table shows the index of industrial production for the period from 1976 to 1985, using the year 1976 as the base year. Obtain the trend values for the following data using 4 yearly centered moving averages :

Years	**1976**	**1977**	**1978**	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**
Index	0	2	3	3	2	4	5	6	7	10

(B) Attempt any TWO of the following questions (4 marks each) : (8)

(i) If for the following data, Walsh's Price Index Number is 150, find 'x' :

Commodity	Base Year		Current Year	
	Price p_0	Quantity q_0	Price p_1	Quantity q_1
A	5	3	10	3
B	x	4	16	9
C	15	5	23	5
D	10	2	26	8

(ii) A toy manufacturing company produces five types of toys. Each toy has to go through three machines A, B and C in the order ABC. The time required in hours for each process is given in the following table :

Type	1	2	3	4	5
Machine *A*	16	20	12	14	22
Machine *B*	10	12	4	6	8
Machine *C*	8	18	16	12	10

Find the total elapsed time and also find idle time for machine B.

(iii) A random variable X has the following probability distribution :

x	1	2	3	4	5	6	7
$P(x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (a) k (b) $P(X<3)$ (c) $P(X>6)$

Q. 6. (A) Attempt any TWO of the following questions (3 marks each) : (6) **[14]**

(i) The building is insured for 75% of its value. The annual premium at 0.70 percent amounts to ₹ 2,625. If the building is damaged to the extent of 60% due to fire, how much can be claimed under the policy?

(ii) Three new machines M_1, M_2, M_3 are to be installed in a machine shop. There are four vacant places A, B, C, D. Due to limited space, machine M_2 can not be placed at B. The cost matrix (in hundred ₹) is as follows :

Machine	**Places**			
	A	**B**	**C**	**D**
M_1	13	10	12	11
M_2	15	–	13	20
M_3	5	7	10	6

Determine the optimum assignment schedule and find the minimum cost.

(iii) The eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg in the lot of 10 eggs.

(B) Attempt any ONE of the following questions (4 marks each) : (4)

(i) Following table shows the all India infant mortality rates (per '000) for years 1980 to 2010 :

Year	**1980**	**1985**	**1990**	**1995**	**2000**	**2005**	**2010**
IMR	10	7	5	4	3	1	0

Fit the trend line to the above data by the method of least squares.

(ii) Minimize : $z = 6x + 2y$

Subject to : $x + 2y \geq 3$,

$x + 4y \geq 4$,

$3x + y \geq 3$,

$x \geq 0, \quad y \geq 0$

(C) Attempt any ONE of the following questions (Activity) (4 marks each) : (4)

(i) For a bivariate data $\bar{x} = 10,\ \bar{y} = 12,\ V(X) = 9,\ \sigma_y = 4$ and $r = 0.6$

Estimate y when $x = 5$

Solution : Line of regression of Y on X is

$$Y - \bar{y} = \boxed{}(X - \bar{x})$$

$$\therefore \quad Y - 12 = r \cdot \frac{\sigma_y}{\sigma_x}(X - 10)$$

$$\therefore \quad Y - 12 = 0.6 \times \frac{4}{\boxed{}}(X - 10)$$

$\therefore$ When $x = 5$

$$Y - 12 = \boxed{}(5 - 10)$$

$$\therefore \quad Y - 12 = -4$$

$$\therefore \quad Y = \boxed{}$$

(ii) If $X \sim P(m)$ with $P(X = 1) = P(X = 2)$ then find the mean and $P(X = 2)$.

Given $e^{-2} = 0.1353$

Solution : Since $P(X = 1) = P(X = 2)$

$$\therefore \quad \frac{e^{\boxed{}} m^1}{1!} = \frac{e^{-m} m^2}{\boxed{}}$$

$$\therefore \quad m = \boxed{}$$

$$\therefore \quad P(X = 2) = \frac{e^{-2} \cdot 2^2}{2!} = \boxed{}$$

◆◆◆