Total No. of Printed Pages—12

HS/XII/A. Sc. Com/M/25

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MATHEMATICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 36 questions divided into four Sections—A, B, C and D. Section—A comprises of 20 questions of *1* mark each, Section—B comprises of 6 questions of *2* marks each, Section—C comprises of 6 questions of *4* marks each and Section—D comprises of 4 questions of *6* marks each.

(iii) There is no overall choice. However, internal choice has been provided in 9 questions of Section—A, 5 questions of Section—B, 5 questions of Section—C and 3 questions of Section—D. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculator is not permitted.

SECTION—A

1. Find the principal value of $\cot\ ^{1}(\ \sqrt{3})$. 1

Or

Evaluate : $\sin \frac{}{3} \quad \sin\ ^{1} \frac{1}{2}$ 1

2. Write the elements a_{23} and a_{32} of a $3 \quad 3$ matrix $A \quad [a_{ij}]$, whose elements a_{ij} are given by $a_{ij} \quad \frac{|i \quad j|}{2}$. 1

3. Check the continuity of the function f given by $f(x) \quad 2x \quad 3$ at $x \quad 1$. 1

Or

Find the value of k, so that the function

$$f(x) \quad \begin{matrix} kx^2, & \text{if } x \quad 2 \\ 3\ , & \text{if } x \quad 2 \end{matrix}$$

is continuous at $x \quad 2$. 1

4. Define an equivalence relation. 1

Or

If $R \quad \{(1, \quad 1), (2, \quad 2), (3, \quad 1)\}$ is a relation, then find the domain and the range of R. 1

5. Evaluate : $\quad x \log 2x\, dx$ 1

6. Find the equation of the line joining (1, 2) and (3, 6) using determinants. 1

Or

For what values of $\quad$ the matrix $\begin{matrix} 5 & & 1 \\ & 2 \quad 4 & \end{matrix}$ is invertible? 1

7. The random variable X has a probability distribution $P(X)$ of the following form, where k is some real number :

$$P(X) \quad \begin{matrix} k, & \text{if } X \quad 0 \\ 2k, & \text{if } X \quad 1 \\ 3k, & \text{if } X \quad 2 \\ 0, & \text{otherwise} \end{matrix}$$

Determine the value of k. 1

8. Find an anti-derivative of $(ax \quad b)^2$ by the method of inspection. 1

Or

Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} \quad 5x \quad \frac{dy}{dx}^{2} \quad 6y \quad \log x$$ 1

9. If   is the angle between any two non-zero vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a} \quad \vec{b}| \quad \vec{a} \quad \vec{b}$, then find . 1

Or

If a line makes angles 90°, 135° and 45° with the x-, y- and z-axes respectively, then find its direction cosines. 1

10. Given two independent events A and B such that $P(A) \quad 0 \; 3$ and $P(B) \quad 0 \; 6$. Find P (neither A nor B). 1

11. Find the second-order derivative of $x \cos x$ w.r.t. x. 1

12. If $f : \mathbb{R} \quad \mathbb{R}$ is a function defined by $f(x) \quad x^2, \quad x \quad \mathbb{R}$, then show that f is not one-one. 1

13. If $\,_0^a 3x^2 dx \quad 8$, then write the value of a. 1

Or

Differentiate $\sin^2 x$ w.r.t. $e^{\cos x}$. 1

14. Show that the function $f(x) \quad 3x \quad 17$ is strictly increasing on $\mathbb{R}$. 1

15. Show that the relation R in $\mathbb{R}$ defined as $R \quad \{(a, b) : a \quad b\}$ is transitive. 1

16. Find $\dfrac{dy}{dx}$, if $ax \quad by^2 \quad \cos y$. 1

Or

Find the integral $\quad \dfrac{\log x}{x}\, dx$. 1

Choose the correct answer :

17. The integrating factor of the differential equation $x\dfrac{dy}{dx} \quad y \quad 2x^2$ is

(a) $e^{\ x}$

(b) $e^{\ y}$

(c) $\dfrac{1}{x}$

(d) x 1

18. The maximum value of the objective function $Z \quad 3x \quad 4y$ subject to the constraints $x \quad 0$, $y \quad 0$ and $x \quad y \quad 1$ is

(a) 7

(b) 4

(c) 3

(d) 10 1

19. If $\vec{a}$ is a non-zero vector of magnitude 'a' and ' ' is a non-zero scalar, then $\vec{a}$ is a unit vector if

(a) $\quad 1$

(b) $\quad\;\; 1$

(c) $a \quad | \; |$

(d) $a \quad \dfrac{1}{| \; |}$ 1

Or

The value of $\hat{i} \quad (\hat{j} \quad \hat{k}) \quad \hat{j} \; (\hat{i} \quad \hat{k}) \quad \hat{k} \; (\hat{i} \quad \hat{j})$ is

(a) 0

(b) $\;\; 1$

(c) 1

(d) 3 1

20. $\quad e^x \sec x(1 \quad \tan x)\,dx$ is equal to

(a) $e^x \cos x \quad c$

(b) $e^x \sec x \quad c$

(c) $e^x \sin x \quad c$

(d) $e^x \tan x \quad c$ 1

SECTION—B

21. Find the vector and the Cartesian equations of the line through the point (5, 2, $\quad$ 4) and which is parallel to the vector $3\hat{i} \quad 2\hat{j} \quad 8\hat{k}$. 2

22. Evaluate the integral : 2

$$\int_0^{\ /2} \frac{\sin x}{1 \quad \cos^2 x}\, dx$$

Or

Find $\frac{dy}{dx}$, if $x \quad a(\cos \quad\quad \sin \ )$ and $y \quad a(\sin \quad\quad \cos \ )$. 2

23. Find the unit vector in the direction of vector $\overline{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6) respectively. 2

Or

Find $|\vec{a} \quad \vec{b}|$, if two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| \quad 2$, $|\vec{b}| \quad 3$ and $\vec{a} \quad \vec{b} \quad 4$. 2

24. Find the values of x, y and z from the following equation : 2

$$\begin{matrix} x & y & 2 \\ 5 & z & xy \end{matrix} \qquad \begin{matrix} 6 & 2 \\ 5 & 8 \end{matrix}$$

Or

If $A \quad \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}$, then show that $A^2 \quad 5A \quad 7I \quad 0$. 2

25. Find the interval in which the function given by $f(x) \quad x^2 \quad 2x \quad 5$ is strictly increasing. 2

Or

Evaluate : $\quad e^x (\sin x \quad \cos x)\, dx$ 2

26. Find the values of p so that the lines

$$\frac{1 \quad x}{3} \quad \frac{7y \quad 14}{2p} \quad \frac{z \quad 3}{2} \text{ and } \frac{7 \quad 7x}{3p} \quad \frac{y \quad 5}{1} \quad \frac{6 \quad z}{5}$$

are at right angles. 2

Or

If $\vec{a} \quad 2\hat{i} \quad 2\hat{j} \quad 3\hat{k}$, $\vec{b} \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$ and $\vec{c} \quad 3\hat{i} \quad \hat{j}$ are such that $\vec{a} \quad \vec{b}$ is perpendicular to $\vec{c}$, then find the value of . 2

Section—C

27. By using the properties of definite integrals, evaluate the integral

$$\int_0^{\pi/4} \log(1 + \tan x)\, dx$$ 4

Or

Find $\int \frac{3x - 2}{(x+1)^2 (x+3)}\, dx.$ 4

28. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate. 4

Or

Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is *(a)* increasing and *(b)* decreasing. 2+2

29. Let $f : X \to Y$ be a function. Define a relation R in X given by $R = \{(a, b) : f(a) = f(b)\}$. Show that R is an equivalence relation. 4

Or

Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3 - 4x$ is bijective. 4

30. Find $\frac{dy}{dx}$, if $y \quad x^{x\cos x} \quad \frac{x^2 \quad 1}{x^2 \quad 1}$. 4

Or

If $y \quad 3\cos(\log x) \quad 4\sin(\log x)$, then show that $x^2 y_2 \quad xy_1 \quad y \quad 0$. 4

31. Solve the following linear programming problem graphically : 4

Maximize $Z \quad 4x \quad y$

subject to the constraints

$$x \quad y \quad 50$$
$$3x \quad y \quad 90$$
$$x \quad 0, \ y \quad 0$$

32. Find the equation of the curve passing through the point $0, \ \frac{}{4}$ whose differential equation is

$$\sin x \cos y \, dx \quad \cos x \sin y \, dy \quad 0$$ 4

Or

Solve the homogeneous differential equation

$$(x^2 \quad xy)\,dy \quad (x^2 \quad y^2)\,dx$$ 4

SECTION—D

33. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume. 6

Or

Find the area of the region bounded by the ellipse $\frac{x^2}{16} \quad \frac{y^2}{9} \quad 1$ above x-axis. 6

34. If $A \quad \begin{matrix} 2 & 3 & 5 \\ 3 & 2 & 4 \\ 1 & 1 & 2 \end{matrix}$, then find $A^{\ 1}$. Using $A^{\ 1}$, solve the following system of equations : 6

$$\begin{matrix} 2x & 3y & 5z & 11 \\ 3x & 2y & 4z & 5 \\ x & y & 2z & 3 \end{matrix}$$

35. Find the Cartesian equation of the line passing through the point (1, 2, 4) and perpendicular to the two lines

$$\frac{x \quad 8}{3} \quad \frac{y \quad 19}{16} \quad \frac{z \quad 10}{7} \quad \text{and} \quad \frac{x \quad 15}{3} \quad \frac{y \quad 29}{8} \quad \frac{z \quad 5}{5}$$

Also write the vector form of the line so obtained. 5+1

Or

Find the shortest distance between the lines whose vector equations are

$$\vec{r} \quad (1 \quad t)\hat{i} \quad (t \quad 2)\hat{j} \quad (3 \quad 2t)\hat{k} \text{ and}$$
$$\vec{r} \quad (s \quad 1)\hat{i} \quad (2s \quad 1)\hat{j} \quad (2s \quad 1)\hat{k}$$

6

36. State Bayes' theorem on probability. Use this theorem to solve any *one* of the following : 1+5

(a) Bag *I* contains 3 red and 4 black balls and Bag *II* contains 4 red and 5 black balls. One ball is transferred from Bag *I* to Bag *II* and then one ball is drawn from Bag *II*. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

Or

(b) A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

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