Total No. of Printed Pages—11

HS/XII/A. Sc. Com/M/24

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MATHEMATICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 36 questions divided into four Sections—A, B, C and D. Section—A comprises of 20 questions of *1* mark each, Section—B comprises of 6 questions of *2* marks each, Section—C comprises of 6 questions of *4* marks each and Section—D comprises of 4 questions of *6* marks each.

(iii) There is no overall choice. However, internal choice has been provided in 7 questions of Section—A, 3 questions of Section—B, 5 questions of Section—C and 2 questions of Section—D. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculator is not permitted.

SECTION—A

1. Find the principal value of $\sin^{1}\ \frac{1}{2}$. 1

2. Differentiate w.r.t. x : 1

$$y \quad \log(\log x), \quad x \quad 1$$

3. Construct a 2 2 matrix $A \quad [a_{ij}]$, where $a_{ij} \quad \frac{(i \quad 2j)^2}{2}$. 1

Or

Find the values of x and y, if

$$\begin{matrix} 3x & y & y \\ 2x & y & 3 \end{matrix} \qquad \begin{matrix} 1 & 2 \\ 4 & 3 \end{matrix}$$ 1

4. Find the unit vector in the direction of the sum of the vectors $\vec{a} \quad 2\hat{i} \quad \hat{j} \quad 2\hat{k}$ and $\vec{b} \quad \hat{i} \quad \hat{j} \quad 3\hat{k}$. 1

5. Find the Cartesian equation of the line which passes through the point (2, 4, 5) and parallel to the line given by

$$\frac{x \quad 3}{3} \quad \frac{y \quad 4}{5} \quad \frac{z \quad 8}{6}$$ 1

6. Integrate the function $\sin x \quad \sin(\cos x)$. 1

Or

Evaluate the definite integral ${}_{0}^{1} \frac{dx}{\sqrt{1 \quad x^2}}$. 1

7. Find $\frac{dy}{dx}$ for the following : 1

$$2x \quad 3y \quad \sin y$$

8. Evaluate : 1

$${}_{/2}^{/2} \sin^7 x \, dx$$

9. Find the direction cosines of the vector $\vec{a} \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k}$. 1

Or

Find the scalar and vector components of the vector with initial point $(2, 1)$ and terminal point $(\quad 5, 7)$. 1

10. Find the value of k, if the function given by

$$f(x) \quad \begin{matrix} kx^2, & x & 1 \\ 4\ , & x & 1 \end{matrix}$$

is continuous at $x \quad 1$. 1

11. Differentiate $\log(\cos e^{x})$ w.r.t. x. 1

Or

Find $\frac{dy}{dx}$, if $x \quad a\cos \quad$ and $y \quad a\sin \quad$. 1

12. The radius of a circle is increasing at the rate of 0·7 cm/s. What is the rate of increase in its circumference? 1

13. Find the second-order derivative of $\log x$. 1

14. Examine the continuity of the function $f(x) \quad 5x \quad 3$ at $x \quad 5$. 1

15. Evaluate : 1

$$\frac{dx}{\sqrt{9 \quad 25x^2}}$$

Or

Integrate w.r.t. x : 1

$$\frac{1}{x \quad x\log x}$$

16. Verify that $y \quad e^{x} \quad 1$ is a solution of the differential equation $y \quad y \quad 0$. 1

Or

Find the general solution of the differential equation $\frac{dy}{dx} \quad \frac{1 \quad y^{2}}{1 \quad x^{2}}$. 1

Choose the correct answer :

17. Let R be the relation in the set $\mathbb{N}$ given by $R \quad \{(a, b) : a \quad b \quad 2, \ b \quad 6\}$. Then

(a) (2, 4) $\quad R$

(b) (3, 8) $\quad R$

(c) (6, 8) $\quad R$

(d) (8, 7) $\quad R$ 1

18. The corner points of the feasible region determined by a system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20) and (60, 0). The maximum value of $Z \quad 4x \quad 3y$ is

(a) 120

(b) 240

(c) 300

(d) 200 1

19. The matrix $\begin{matrix} 5 & 0 \\ 0 & 5 \end{matrix}$ is a

(a) diagonal matrix

(b) row matrix

(c) column matrix

(d) unit matrix 1

Or

The square matrices A and B will be inverse of each other, if

(a) $AB \quad O, \ BA \quad I$

(b) $AB \quad BA \quad O$

(c) $AB \quad BA \quad I$

(d) $AB \quad I, BA \quad O$ 1

20. The value of $(\hat{i} \quad \hat{j}) \ \hat{k} \quad (\hat{j} \quad \hat{k}) \ \hat{i}$ is

(a) 1

(b) 2

(c) 0

(d) 1 1

SECTION—B

21. Show that the function $f : \mathbb{R} \quad \mathbb{R}$ defined by $f(x) \quad \frac{1}{2x}$ is onto. Here, $\mathbb{R}$ is the set of non-zero real numbers. 2

Or

Let $A \quad \mathbb{R} \quad \{3\}$ and $B \quad \mathbb{R} \quad \{1\}$. Consider the function $f : A \quad B$ defined by $f(x) \quad \frac{x \quad 2}{x \quad 3}$. Show that f is one-one. 2

22. Prove that the function $f(x) \quad \log x$ is strictly increasing on the interval $(0, \quad)$. 2

23. Evaluate : 2

$$\frac{dx}{(x \quad 1)(x \quad 2)}$$

Or

Find $\frac{dy}{dx}$, if $xy \quad y^2 \quad \tan x \quad y$. 2

24. If $y \quad 5\cos x \quad 3\sin x$, then prove that

$$\frac{d^2y}{dx^2} \quad y \quad 0$$ 2

Or

Find $\frac{dy}{dx}$, if $x \quad a(\quad \sin \quad)$ and $y \quad a(1 \quad \cos \quad)$. 2

25. Given that the events A and B are such that $P(A) \quad \frac{1}{2}$, $P(B) \quad p$ and $P(A \quad B) \quad \frac{3}{5}$. Find p, if A and B are independent. 2

26. Using determinants, find the value of k so that the points (3, 2), (k, 2) and (8, 8) lie on the same line. 2

SECTION—C

27. Using integrals, find the area enclosed by the circle $x^2 \quad y^2 \quad 4$. 4

Or

Evaluate : 4

$$\frac{2x \quad 1}{\sqrt{x^2 \quad 4x \quad 3}}\, dx$$

28. Differentiate $(\log x)^{\cos x}$ w.r.t. x. 4

Or

Using the properties of definite integral, evaluate

$$_0^{\;/2} (2 \log \sin x \quad \log \sin 2x)\, dx$$ 4

29. Show that the relation R on $\mathbb{Z}$ given by $R \ \{(x, y) : x \quad y$ is divisible by $2\}$ is an equivalence relation. 4

Or

Let A and B be two sets. Show that $f : A \quad B \quad B \quad A$ such that $f(a, b) \quad (b, a)$ is one-one and onto. 4

30. Solve the following LPP graphically : 4

$$\text{Minimize} \quad Z \quad 200x \quad 500y$$

subject to

$$x \quad 2y \quad 10$$
$$3x \quad 4y \quad 24$$
$$x, y \quad 0$$

31. Find the angle between the two lines whose equations are

$$\frac{x \quad 3}{3} \quad \frac{y \quad 1}{5} \quad \frac{z \quad 3}{4} \quad \text{and} \quad \frac{x \quad 1}{1} \quad \frac{y \quad 4}{1} \quad \frac{z \quad 5}{2}$$ 4

Or

Find and , if $(2\hat{i} \quad 6\hat{j} \quad 27\hat{k}) \quad (\hat{i} \quad \hat{j} \quad \hat{k}) \quad \vec{0}$. 4

32. Find the intervals in which the function f given by $f(x) \quad 2x^3 \quad 3x^2 \quad 36x \quad 7$ is—

(a) strictly increasing;

(b) strictly decreasing. 4

Or

If $y \quad e^{a\cos\ ^{1} x}, \quad 1 \quad x \quad 1$, then show that

$$(1 \quad x^2)y_2 \quad xy_1 \quad a^2 y \quad 0$$ 4

SECTION—D

33. Solve the following system of linear equations : 6

$$\begin{array}{cccc} x & y & z & 6 \\ x & y & z & 2 \\ 2x & y & z & 1 \end{array}$$

34. Find the shortest distance between the lines l_1 and l_2, whose vector equations are

$$\vec{r} \quad (\hat{i} \quad \hat{j}) \qquad (2\hat{i} \quad \hat{j} \quad \hat{k})$$

$$\vec{r} \quad (2\hat{i} \quad \hat{j} \quad \hat{k}) \qquad (3\hat{i} \quad 5\hat{j} \quad 2\hat{k})$$ 6

35. Show that a closed right circular cylinder of given total surface area and maximum volume is such that its height is equal to its diameter. 6

Or

Solve the linear differential equation

$$\cos^2 x \frac{dy}{dx} \quad y \quad \tan x \quad 0 \quad x \quad \frac{}{4}$$

given that $y \quad 1$, when $x \quad 0$. 6

36. State Bayes' theorem and use it to solve the following : 2+4=6

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3 / 4 be the probability that he knows the answer and 1 / 4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1 / 4, what is the probability that the student knows the answer given that he answered it correctly?

Or

A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 \quad k$

Determine the following : 6

(i) k

(ii) $P(X \quad 3)$

(iii) $P(X \quad 6)$

(iv) $P(0 \quad X \quad 3)$

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