

**SECTION 1 (Maximum Marks: 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:

*Full Marks* : +3 If **ONLY** the correct option is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

Q.1 Let  $\mathbb{R}$  denote the set of all real numbers. Let  $a_i, b_i \in \mathbb{R}$  for  $i \in \{1, 2, 3\}$ . Define the functions  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $g: \mathbb{R} \rightarrow \mathbb{R}$ , and  $h: \mathbb{R} \rightarrow \mathbb{R}$  by

$$\begin{aligned}f(x) &= a_1 + 10x + a_2x^2 + a_3x^3 + x^4, \\g(x) &= b_1 + 3x + b_2x^2 + b_3x^3 + x^4, \\h(x) &= f(x+1) - g(x+2).\end{aligned}$$

If  $f(x) \neq g(x)$  for every  $x \in \mathbb{R}$ , then the coefficient of  $x^3$  in  $h(x)$  is

(A)	8
(B)	2
(C)	-4
(D)	-6

**Answer: C**

Q.2 Three students  $S_1, S_2$ , and  $S_3$  are given a problem to solve. Consider the following events:

$U$ : At least one of  $S_1, S_2$ , and  $S_3$  can solve the problem,  
 $V$ :  $S_1$  can solve the problem, given that neither  $S_2$  nor  $S_3$  can solve the problem,  
 $W$ :  $S_2$  can solve the problem and  $S_3$  cannot solve the problem,  
 $T$ :  $S_3$  can solve the problem.

For any event  $E$ , let  $P(E)$  denote the probability of  $E$ . If

$$P(U) = \frac{1}{2}, \quad P(V) = \frac{1}{10}, \quad \text{and} \quad P(W) = \frac{1}{12},$$

then  $P(T)$  is equal to

(A)	$\frac{13}{36}$	(B)	$\frac{1}{3}$	(C)	$\frac{19}{60}$	(D)	$\frac{1}{4}$
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**Answer: A**

Q.3

Let  $\mathbb{R}$  denote the set of all real numbers. Define the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  by

$$f(x) = \begin{cases} 2 - 2x^2 - x^2 \sin \frac{1}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

Then which one of the following statements is TRUE?

(A)	The function $f$ is <b>NOT</b> differentiable at $x = 0$
(B)	There is a positive real number $\delta$ , such that $f$ is a decreasing function on the interval $(0, \delta)$
(C)	For any positive real number $\delta$ , the function $f$ is <b>NOT</b> an increasing function on the interval $(-\delta, 0)$
(D)	$x = 0$ is a point of local minima of $f$

**Answer: C**

Q.4

Consider the matrix

$$P = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Let the transpose of a matrix  $X$  be denoted by  $X^T$ . Then the number of  $3 \times 3$  invertible matrices  $Q$  with integer entries, such that

$$Q^{-1} = Q^T \text{ and } PQ = QP,$$

is

(A)	32	(B)	8	(C)	16	(D)	24
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**Answer: C**

## SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

*Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

*Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;

*Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

*Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

*Negative Marks* : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then

choosing **ONLY** (A), (B) and (D) will get +4 marks;

choosing **ONLY** (A) and (B) will get +2 marks;

choosing **ONLY** (A) and (D) will get +2 marks;

choosing **ONLY** (B) and (D) will get +2 marks;

choosing **ONLY** (A) will get +1 mark;

choosing **ONLY** (B) will get +1 mark;

choosing **ONLY** (D) will get +1 mark;

choosing no option (i.e. the question is unanswered) will get 0 marks; and

choosing any other combination of options will get -2 marks.

Q.5

Let  $L_1$  be the line of intersection of the planes given by the equations

$$2x + 3y + z = 4 \text{ and } x + 2y + z = 5.$$

Let  $L_2$  be the line passing through the point  $P(2, -1, 3)$  and parallel to  $L_1$ . Let  $M$  denote the plane given by the equation

$$2x + y - 2z = 6.$$

Suppose that the line  $L_2$  meets the plane  $M$  at the point  $Q$ . Let  $R$  be the foot of the perpendicular drawn from  $P$  to the plane  $M$ .

Then which of the following statements is (are) TRUE?

(A)	The length of the line segment $PQ$ is $9\sqrt{3}$
(B)	The length of the line segment $QR$ is 15
(C)	The area of $\Delta PQR$ is $\frac{3}{2}\sqrt{234}$
(D)	The acute angle between the line segments $PQ$ and $PR$ is $\cos^{-1}\left(\frac{1}{2\sqrt{3}}\right)$

**Answer: A, C**

Q.6

Let  $\mathbb{N}$  denote the set of all natural numbers, and  $\mathbb{Z}$  denote the set of all integers. Consider the functions  $f: \mathbb{N} \rightarrow \mathbb{Z}$  and  $g: \mathbb{Z} \rightarrow \mathbb{N}$  defined by

$$f(n) = \begin{cases} (n+1)/2 & \text{if } n \text{ is odd,} \\ (4-n)/2 & \text{if } n \text{ is even,} \end{cases}$$

and

$$g(n) = \begin{cases} 3+2n & \text{if } n \geq 0, \\ -2n & \text{if } n < 0. \end{cases}$$

Define  $(g \circ f)(n) = g(f(n))$  for all  $n \in \mathbb{N}$ , and  $(f \circ g)(n) = f(g(n))$  for all  $n \in \mathbb{Z}$ .

Then which of the following statements is (are) TRUE?

(A)	$g \circ f$ is <b>NOT</b> one-one and $g \circ f$ is <b>NOT</b> onto
(B)	$f \circ g$ is <b>NOT</b> one-one but $f \circ g$ is onto
(C)	$g$ is one-one and $g$ is onto
(D)	$f$ is <b>NOT</b> one-one but $f$ is onto

**Answer: A, D**

Q.7

Let  $\mathbb{R}$  denote the set of all real numbers. Let  $z_1 = 1 + 2i$  and  $z_2 = 3i$  be two complex numbers, where  $i = \sqrt{-1}$ . Let

$$S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : |x + iy - z_1| = 2|x + iy - z_2|\}.$$

Then which of the following statements is (are) TRUE?

(A)	$S$ is a circle with centre $\left(-\frac{1}{3}, \frac{10}{3}\right)$
(B)	$S$ is a circle with centre $\left(\frac{1}{3}, \frac{8}{3}\right)$
(C)	$S$ is a circle with radius $\frac{\sqrt{2}}{3}$
(D)	$S$ is a circle with radius $\frac{2\sqrt{2}}{3}$

**Answer: A, D**

**SECTION 3 (Maximum Marks: 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:
 

<i>Full Marks</i>	<i>: +4</i>	If ONLY the correct numerical value is entered in the designated place;
<i>Zero Marks</i>	<i>: 0</i>	In all other cases.

**Q.8** Let the set of all relations  $R$  on the set  $\{a, b, c, d, e, f\}$ , such that  $R$  is reflexive and symmetric, and  $R$  contains exactly 10 elements, be denoted by  $\mathcal{S}$ .

Then the number of elements in  $\mathcal{S}$  is \_\_\_\_\_.

**Answer: 105**

**Q.9** For any two points  $M$  and  $N$  in the  $XY$ -plane, let  $\overrightarrow{MN}$  denote the vector from  $M$  to  $N$ , and  $\vec{0}$  denote the zero vector. Let  $P, Q$  and  $R$  be three distinct points in the  $XY$ -plane. Let  $S$  be a point inside the triangle  $\Delta PQR$  such that

$$\overrightarrow{SP} + 5 \overrightarrow{SQ} + 6 \overrightarrow{SR} = \vec{0}.$$

Let  $E$  and  $F$  be the mid-points of the sides  $PR$  and  $QR$ , respectively. Then the value of

$$\frac{\text{length of the line segment } EF}{\text{length of the line segment } ES}$$

is \_\_\_\_\_.

**Answer: [1.15 to 1.25]**

**Q.10** Let  $S$  be the set of all seven-digit numbers that can be formed using the digits 0, 1 and 2. For example, 2210222 is in  $S$ , but 0210222 is **NOT** in  $S$ .

Then the number of elements  $x$  in  $S$  such that at least one of the digits 0 and 1 appears exactly twice in  $x$ , is equal to \_\_\_\_\_.

**Answer: 762**

Q.11 Let  $\alpha$  and  $\beta$  be the real numbers such that

$$\lim_{x \rightarrow 0} \frac{1}{x^3} \left( \frac{\alpha}{2} \int_0^x \frac{1}{1-t^2} dt + \beta x \cos x \right) = 2.$$

Then the value of  $\alpha + \beta$  is \_\_\_\_\_.

**Answer: [2.35 to 2.45]**

Q.12 Let  $\mathbb{R}$  denote the set of all real numbers. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a function such that  $f(x) > 0$  for all  $x \in \mathbb{R}$ , and  $f(x+y) = f(x)f(y)$  for all  $x, y \in \mathbb{R}$ .

Let the real numbers  $a_1, a_2, \dots, a_{50}$  be in an arithmetic progression. If  $f(a_{31}) = 64f(a_{25})$ , and

$$\sum_{i=1}^{50} f(a_i) = 3(2^{25} + 1),$$

then the value of

$$\sum_{i=6}^{30} f(a_i)$$

is \_\_\_\_\_.

**Answer: 96**

Q.13 For all  $x > 0$ , let  $y_1(x)$ ,  $y_2(x)$ , and  $y_3(x)$  be the functions satisfying

$$\begin{aligned} \frac{dy_1}{dx} - (\sin x)^2 y_1 &= 0, \quad y_1(1) = 5, \\ \frac{dy_2}{dx} - (\cos x)^2 y_2 &= 0, \quad y_2(1) = \frac{1}{3}, \\ \frac{dy_3}{dx} - \left(\frac{2-x^3}{x^3}\right) y_3 &= 0, \quad y_3(1) = \frac{3}{5e}, \end{aligned}$$

respectively. Then

$$\lim_{x \rightarrow 0^+} \frac{y_1(x)y_2(x)y_3(x) + 2x}{e^{3x} \sin x}$$

is equal to \_\_\_\_\_.

**Answer: 2**

**SECTION 4 (Maximum Marks: 12)**

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme**:
 

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Q.14 Consider the following frequency distribution:

Value	4	5	8	9	6	12	11
Frequency	5	$f_1$	$f_2$	2	1	1	3

Suppose that the sum of the frequencies is 19 and the median of this frequency distribution is 6.

For the given frequency distribution, let  $\alpha$  denote the mean deviation about the mean,  $\beta$  denote the mean deviation about the median, and  $\sigma^2$  denote the variance.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

**List-I**

(P)  $7f_1 + 9f_2$  is equal to

(1) 146

(Q)  $19\alpha$  is equal to

(2) 47

(R)  $19\beta$  is equal to

(3) 48

(S)  $19\sigma^2$  is equal to

(4) 145

(5) 55

**List-II**

(A)	(P) $\rightarrow$ (5)	(Q) $\rightarrow$ (3)	(R) $\rightarrow$ (2)	(S) $\rightarrow$ (4)
(B)	(P) $\rightarrow$ (5)	(Q) $\rightarrow$ (2)	(R) $\rightarrow$ (3)	(S) $\rightarrow$ (1)
(C)	(P) $\rightarrow$ (5)	(Q) $\rightarrow$ (3)	(R) $\rightarrow$ (2)	(S) $\rightarrow$ (1)
(D)	(P) $\rightarrow$ (3)	(Q) $\rightarrow$ (2)	(R) $\rightarrow$ (5)	(S) $\rightarrow$ (4)

**Answer: C**

Q.15

Let  $\mathbb{R}$  denote the set of all real numbers. For a real number  $x$ , let  $[x]$  denote the greatest integer less than or equal to  $x$ . Let  $n$  denote a natural number.

Match each entry in List-I to the correct entry in List-II and choose the correct option.

**List-I****List-II**

(P) The minimum value of  $n$  for which the function

$$f(x) = \left[ \frac{10x^3 - 45x^2 + 60x + 35}{n} \right]$$

is continuous on the interval  $[1, 2]$ , is

(1) 8

(Q) The minimum value of  $n$  for which

$$g(x) = (2n^2 - 13n - 15)(x^3 + 3x), \\ x \in \mathbb{R},$$

(2) 9

(R) The smallest natural number  $n$  which is greater than 5, such that  $x = 3$  is a point of local minima of

$$h(x) = (x^2 - 9)^n(x^2 + 2x + 3), \\ \text{is}$$

(3) 5

(S) Number of  $x_0 \in \mathbb{R}$  such that

$$l(x) = \sum_{k=0}^4 \left( \sin|x - k| + \cos \left| x - k + \frac{1}{2} \right| \right),$$

$x \in \mathbb{R}$ , is NOT differentiable at  $x_0$ , is

(4) 6

(5) 10

(A)	(P) $\rightarrow$ (1) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (2) (S) $\rightarrow$ (5)
(B)	(P) $\rightarrow$ (2) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (3)
(C)	(P) $\rightarrow$ (5) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (3)
(D)	(P) $\rightarrow$ (2) (Q) $\rightarrow$ (3) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)

**Answer: B**

Q.16

Let  $\vec{w} = \hat{i} + \hat{j} - 2\hat{k}$ , and  $\vec{u}$  and  $\vec{v}$  be two vectors, such that  $\vec{u} \times \vec{v} = \vec{w}$  and  $\vec{v} \times \vec{w} = \vec{u}$ . Let  $\alpha, \beta, \gamma$ , and  $t$  be real numbers such that

$$\vec{u} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}, \quad -t\alpha + \beta + \gamma = 0, \quad \alpha - t\beta + \gamma = 0, \quad \text{and } \alpha + \beta - t\gamma = 0.$$

Match each entry in List-I to the correct entry in List-II and choose the correct option.

**List-I**

(P)  $|\vec{v}|^2$  is equal to  
 (Q) If  $\alpha = \sqrt{3}$ , then  $\gamma^2$  is equal to  
 (R) If  $\alpha = \sqrt{3}$ , then  $(\beta + \gamma)^2$  is equal to  
 (S) If  $\alpha = \sqrt{2}$ , then  $t + 3$  is equal to

**List-II**

(1) 0  
 (2) 1  
 (3) 2  
 (4) 3  
 (5) 5

(A)	(P) $\rightarrow$ (2)	(Q) $\rightarrow$ (1)	(R) $\rightarrow$ (4)	(S) $\rightarrow$ (5)
(B)	(P) $\rightarrow$ (2)	(Q) $\rightarrow$ (4)	(R) $\rightarrow$ (3)	(S) $\rightarrow$ (5)
(C)	(P) $\rightarrow$ (2)	(Q) $\rightarrow$ (1)	(R) $\rightarrow$ (4)	(S) $\rightarrow$ (3)
(D)	(P) $\rightarrow$ (5)	(Q) $\rightarrow$ (4)	(R) $\rightarrow$ (1)	(S) $\rightarrow$ (3)

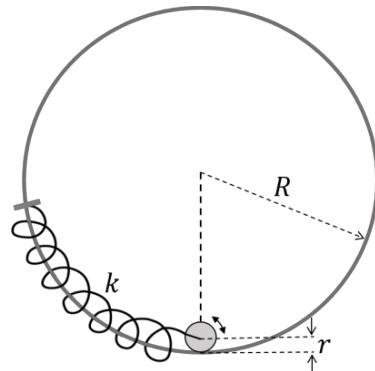
**Answer: A**

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Q.1

The center of a disk of radius  $r$  and mass  $m$  is attached to a spring of spring constant  $k$ , inside a ring of radius  $R > r$  as shown in the figure. The other end of the spring is attached on the periphery of the ring. Both the ring and the disk are in the same vertical plane. The disk can only roll along the inside periphery of the ring, without slipping. The spring can only be stretched or compressed along the periphery of the ring, following the Hooke's law. In equilibrium, the disk is at the bottom of the ring. Assuming small displacement of the disc, the time period of oscillation of center of mass of the disk is written as  $T = \frac{2\pi}{\omega}$ . The correct expression for  $\omega$  is ( $g$  is the acceleration due to gravity):

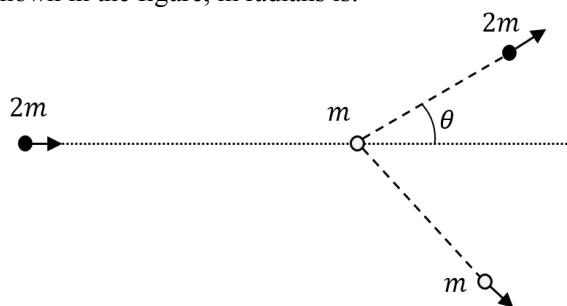


(A)	$\sqrt{\frac{2}{3}\left(\frac{g}{R-r} + \frac{k}{m}\right)}$	(B)	$\sqrt{\frac{2g}{3(R-r)} + \frac{k}{m}}$
(C)	$\sqrt{\frac{1}{6}\left(\frac{g}{R-r} + \frac{k}{m}\right)}$	(D)	$\sqrt{\frac{1}{4}\left(\frac{g}{R-r} + \frac{k}{m}\right)}$

**Answer: A**

Q.2

In a scattering experiment, a particle of mass  $2m$  collides with another particle of mass  $m$ , which is initially at rest. Assuming the collision to be perfectly elastic, the maximum angular deviation  $\theta$  of the heavier particle, as shown in the figure, in radians is:

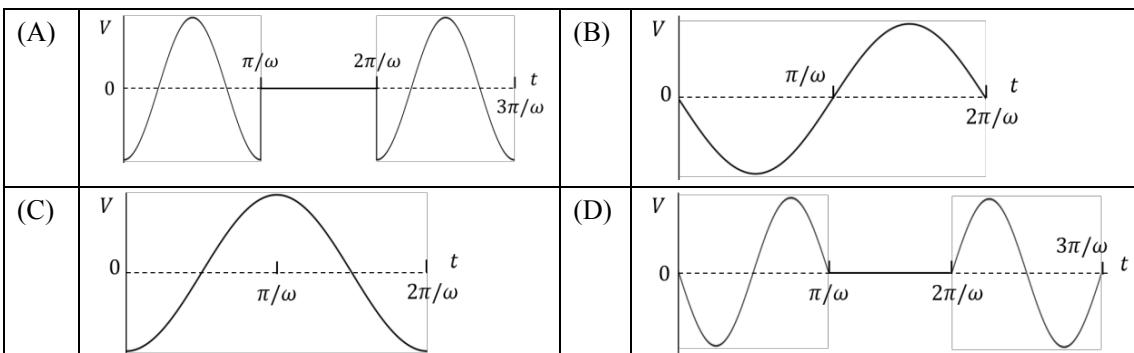
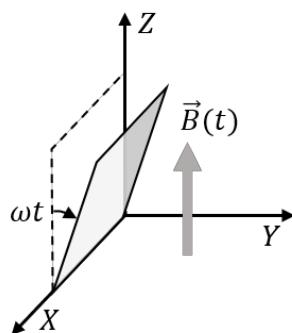


(A)	$\pi$	(B)	$\tan^{-1}\left(\frac{1}{2}\right)$	(C)	$\frac{\pi}{3}$	(D)	$\frac{\pi}{6}$
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**Answer: D**

Q.3

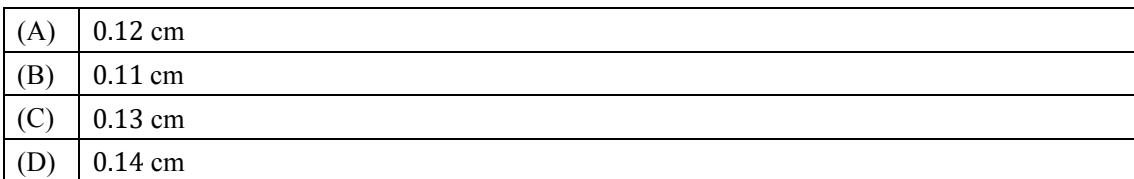
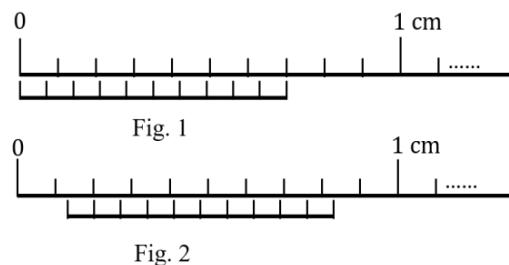
A conducting square loop initially lies in the  $XZ$  plane with its lower edge hinged along the  $X$ -axis. Only in the region  $y \geq 0$ , there is a time dependent magnetic field pointing along the  $Z$ -direction,  $\vec{B}(t) = B_0(\cos \omega t)\hat{k}$ , where  $B_0$  is a constant. The magnetic field is zero everywhere else. At time  $t = 0$ , the loop starts rotating with constant angular speed  $\omega$  about the  $X$  axis in the clockwise direction as viewed from the  $+X$  axis (as shown in the figure). Ignoring self-inductance of the loop and gravity, which of the following plots correctly represents the induced e.m.f. ( $V$ ) in the loop as a function of time:



**Answer: A**

Q.4

Figure 1 shows the configuration of main scale and Vernier scale before measurement. Fig. 2 shows the configuration corresponding to the measurement of diameter  $D$  of a tube. The measured value of  $D$  is:



**Answer: C**

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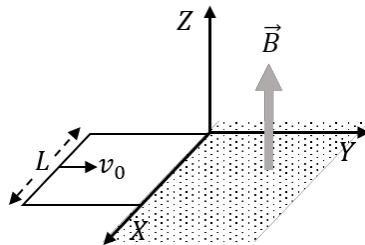
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

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- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  - choosing **ONLY** (A), (B) and (D) will get +4 marks;
  - choosing **ONLY** (A) and (B) will get +2 marks;
  - choosing **ONLY** (A) and (D) will get +2 marks;
  - choosing **ONLY** (B) and (D) will get +2 marks;
  - choosing **ONLY** (A) will get +1 mark;
  - choosing **ONLY** (B) will get +1 mark;
  - choosing **ONLY** (D) will get +1 mark;
  - choosing no option (i.e. the question is unanswered) will get 0 marks; and
  - choosing any other combination of options will get -2 marks.

Q.5

A conducting square loop of side  $L$ , mass  $M$  and resistance  $R$  is moving in the  $XY$  plane with its edges parallel to the  $X$  and  $Y$  axes. The region  $y \geq 0$  has a uniform magnetic field,  $\vec{B} = B_0 \hat{k}$ . The magnetic field is zero everywhere else. At time  $t = 0$ , the loop starts to enter the magnetic field with an initial velocity  $v_0 \hat{j}$  m/s, as shown in the figure. Considering the quantity  $K = \frac{B_0^2 L^2}{RM}$  in appropriate units, ignoring self-inductance of the loop and gravity, which of the following statements is/are correct:



(A)	If $v_0 = 1.5KL$ , the loop will stop before it enters completely inside the region of magnetic field.
(B)	When the complete loop is inside the region of magnetic field, the net force acting on the loop is zero.
(C)	If $v_0 = \frac{KL}{10}$ , the loop comes to rest at $t = \left(\frac{1}{K}\right) \ln\left(\frac{5}{2}\right)$ .
(D)	If $v_0 = 3KL$ , the complete loop enters inside the region of magnetic field at time $t = \left(\frac{1}{K}\right) \ln\left(\frac{3}{2}\right)$ .

**Answer: B, D**

Q.6

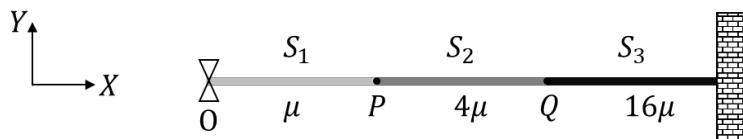
Length, breadth and thickness of a strip having a uniform cross section are measured to be 10.5 cm, 0.05 mm, and  $6.0 \mu\text{m}$ , respectively. Which of the following option(s) give(s) the volume of the strip in  $\text{cm}^3$  with correct significant figures:

(A)	$3.2 \times 10^{-5}$	(B)	$32.0 \times 10^{-6}$	(C)	$3.0 \times 10^{-5}$	(D)	$3 \times 10^{-5}$
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**Answer: D**

Q.7

Consider a system of three connected strings,  $S_1$ ,  $S_2$  and  $S_3$  with uniform linear mass densities  $\mu$  kg/m,  $4\mu$  kg/m and  $16\mu$  kg/m, respectively, as shown in the figure.  $S_1$  and  $S_2$  are connected at the point  $P$ , whereas  $S_2$  and  $S_3$  are connected at the point  $Q$ , and the other end of  $S_3$  is connected to a wall. A wave generator  $O$  is connected to the free end of  $S_1$ . The wave from the generator is represented by  $y = y_0 \cos(\omega t - kx)$  cm, where  $y_0$ ,  $\omega$  and  $k$  are constants of appropriate dimensions. Which of the following statements is/are correct:



(A)	When the wave reflects from $P$ for the first time, the reflected wave is represented by $y = \alpha_1 y_0 \cos(\omega t + kx + \pi)$ cm, where $\alpha_1$ is a positive constant.
(B)	When the wave transmits through $P$ for the first time, the transmitted wave is represented by $y = \alpha_2 y_0 \cos(\omega t - kx)$ cm, where $\alpha_2$ is a positive constant.
(C)	When the wave reflects from $Q$ for the first time, the reflected wave is represented by $y = \alpha_3 y_0 \cos(\omega t - kx + \pi)$ cm, where $\alpha_3$ is a positive constant.
(D)	When the wave transmits through $Q$ for the first time, the transmitted wave is represented by $y = \alpha_4 y_0 \cos(\omega t - 4kx)$ cm, where $\alpha_4$ is a positive constant.

**Answer: A, D**

**SECTION 3 (Maximum Marks: 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:
 

<i>Full Marks</i>	: +4	If ONLY the correct numerical value is entered in the designated place;
<i>Zero Marks</i>	: 0	In all other cases.

**Q.8** A person sitting inside an elevator performs a weighing experiment with an object of mass 50 kg. Suppose that the variation of the height  $y$  (in m) of the elevator, from the ground, with time  $t$  (in s) is given by  $y = 8 \left[ 1 + \sin \left( \frac{2\pi t}{T} \right) \right]$ , where  $T = 40\pi$  s. Taking acceleration due to gravity,  $g = 10$  m/s<sup>2</sup>, the maximum variation of the object's weight (in N) as observed in the experiment is \_\_\_\_\_

**Answer: 2**

**Q.9** A cube of unit volume contains  $35 \times 10^7$  photons of frequency  $10^{15}$  Hz. If the energy of all the photons is viewed as the average energy being contained in the electromagnetic waves within the same volume, then the amplitude of the magnetic field is  $\alpha \times 10^{-9}$  T. Taking permeability of free space  $\mu_0 = 4\pi \times 10^{-7}$  Tm/A, Planck's constant  $h = 6 \times 10^{-34}$  Js and  $\pi = \frac{22}{7}$ , the value of  $\alpha$  is \_\_\_\_\_

**Answer: [21 to 25]**

Q.10 Two identical plates P and Q, radiating as perfect black bodies, are kept in vacuum at constant absolute temperatures  $T_P$  and  $T_Q$ , respectively, with  $T_Q < T_P$ , as shown in Fig. 1. The radiated power transferred per unit area from P to Q is  $W_0$ . Subsequently, two more plates, identical to P and Q, are introduced between P and Q, as shown in Fig. 2. Assume that heat transfer takes place only between adjacent plates. If the power transferred per unit area in the direction from P to Q (Fig. 2) in the steady state is  $W_S$ , then the ratio  $\frac{W_0}{W_S}$  is \_\_\_\_

Fig. 1

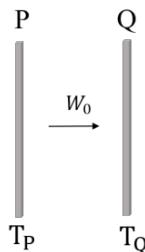
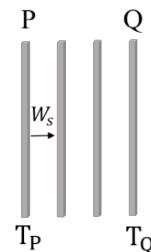
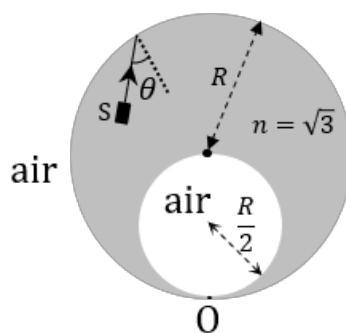


Fig. 2



**Answer: 3**

Q.11 A solid glass sphere of refractive index  $n = \sqrt{3}$  and radius  $R$  contains a spherical air cavity of radius  $\frac{R}{2}$ , as shown in the figure. A very thin glass layer is present at the point O so that the air cavity (refractive index  $n = 1$ ) remains inside the glass sphere. An unpolarized, unidirectional and monochromatic light source S emits a light ray from a point inside the glass sphere towards the periphery of the glass sphere. If the light is reflected from the point O and is fully polarized, then the angle of incidence at the inner surface of the glass sphere is  $\theta$ . The value of  $\sin \theta$  is \_\_\_\_



**Answer: 0.5 OR 0.75**

Q.12

A single slit diffraction experiment is performed to determine the slit width using the equation,  $\frac{bd}{D} = m\lambda$ , where  $b$  is the slit width,  $D$  the shortest distance between the slit and the screen,  $d$  the distance between the  $m^{\text{th}}$  diffraction maximum and the central maximum, and  $\lambda$  is the wavelength.  $D$  and  $d$  are measured with scales of least count of 1 cm and 1 mm, respectively. The values of  $\lambda$  and  $m$  are known precisely to be 600 nm and 3, respectively. The absolute error (in  $\mu\text{m}$ ) in the value of  $b$  estimated using the diffraction maximum that occurs for  $m = 3$  with  $d = 5 \text{ mm}$  and  $D = 1 \text{ m}$  is \_\_\_\_

**Answer: [75 to 79]**

Q.13

Consider an electron in the  $n = 3$  orbit of a hydrogen-like atom with atomic number  $Z$ . At absolute temperature  $T$ , a neutron having thermal energy  $k_B T$  has the same de Broglie wavelength as that of this electron. If this temperature is given by  $T = \frac{Z^2 h^2}{\alpha \pi^2 a_0^2 m_N k_B}$ , (where  $h$  is the Planck's constant,  $k_B$  is the Boltzmann constant,  $m_N$  is the mass of the neutron and  $a_0$  is the first Bohr radius of hydrogen atom) then the value of  $\alpha$  is \_\_\_\_

**Answer: 72**

**SECTION 4 (Maximum Marks: 12)**

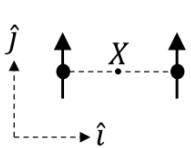
- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme**:  
*Full Marks : +4 ONLY if the option corresponding to the correct combination is chosen;*  
*Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);*  
*Negative Marks : -1 In all other cases.*

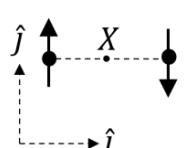
Q.14

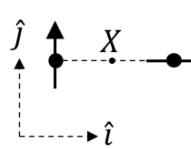
List-I shows four configurations, each consisting of a pair of ideal electric dipoles. Each dipole has a dipole moment of magnitude  $p$ , oriented as marked by arrows in the figures. In all the configurations the dipoles are fixed such that they are at a distance  $2r$  apart along the  $x$  direction. The midpoint of the line joining the two dipoles is  $X$ . The possible resultant electric fields  $\vec{E}$  at  $X$  are given in List-II.

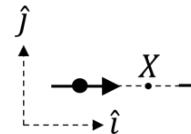
Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

**List-I**

(P) 

(Q) 

(R) 

(S) 

**List-II**

(1)  $\vec{E} = 0$

(2)  $\vec{E} = -\frac{p}{2\pi\epsilon_0 r^3} \hat{j}$

(3)  $\vec{E} = -\frac{p}{4\pi\epsilon_0 r^3} (\hat{i} - \hat{j})$

(4)  $\vec{E} = \frac{p}{4\pi\epsilon_0 r^3} (2\hat{i} - \hat{j})$

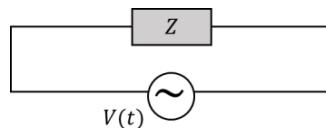
(5)  $\vec{E} = \frac{p}{\pi\epsilon_0 r^3} \hat{i}$

(A)	P→3, Q→1, R→2, S→4
(B)	P→4, Q→5, R→3, S→1
(C)	P→2, Q→1, R→4, S→5
(D)	P→2, Q→1, R→3, S→5

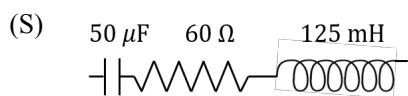
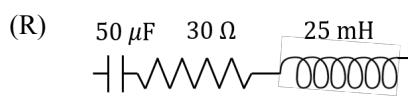
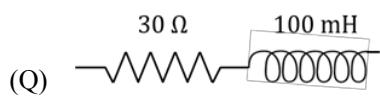
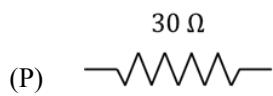
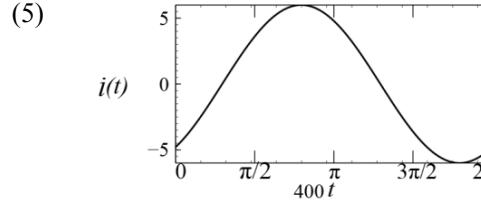
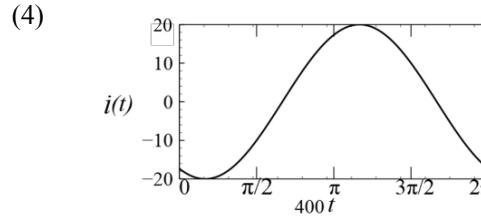
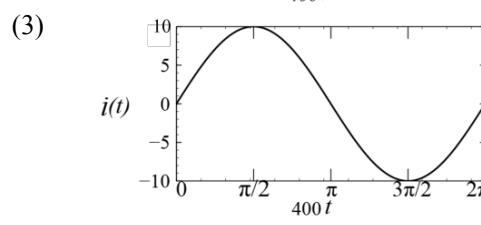
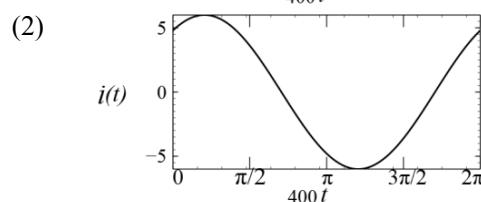
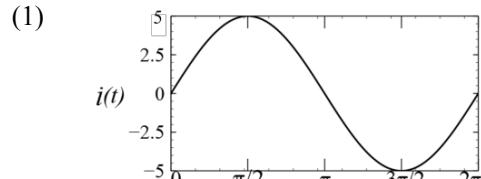
**Answer: C**

Q.15

A circuit with an electrical load having impedance  $Z$  is connected with an AC source as shown in the diagram. The source voltage varies in time as  $V(t) = 300 \sin(400t)$  V, where  $t$  is time in s. List-I shows various options for the load. The possible currents  $i(t)$  in the circuit as a function of time are given in List-II.



Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

**List-I****List-II**

(A)	P→3, Q→5, R→2, S→1
(B)	P→1, Q→5, R→2, S→3
(C)	P→3, Q→4, R→2, S→1
(D)	P→1, Q→4, R→2, S→5

**Answer: A**

Q.16

List-I shows various functional dependencies of energy ( $E$ ) on the atomic number ( $Z$ ). Energies associated with certain phenomena are given in List-II.

Choose the option that describes the correct match between the entries in **List-I** to those in **List-II**.

**List-I**

(P)  $E \propto Z^2$

(Q)  $E \propto (Z - 1)^2$

(R)  $E \propto Z(Z - 1)$

(S)  $E$  is practically independent of  $Z$ **List-II**

(1) energy of characteristic x-rays

(2) electrostatic part of the nuclear binding energy for stable nuclei with mass numbers in the range 30 to 170

(3) energy of continuous x-rays

(4) average nuclear binding energy per nucleon for stable nuclei with mass number in the range 30 to 170

(5) energy of radiation due to electronic transitions from hydrogen-like atoms

(A)	P→4, Q→3, R→1, S→2
(B)	P→5, Q→2, R→1, S→4
(C)	P→5, Q→1, R→2, S→4
(D)	P→3, Q→2, R→1, S→5

**Answer: C**

**SECTION 1 (Maximum Marks: 12)**

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated **according to the following marking scheme**:
 

*Full Marks* : +3 If **ONLY** the correct option is chosen;

*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

*Negative Marks* : -1 In all other cases.

**Q.1** The heating of  $\text{NH}_4\text{NO}_2$  at 60–70 °C and  $\text{NH}_4\text{NO}_3$  at 200–250 °C is associated with the formation of nitrogen containing compounds **X** and **Y**, respectively. **X** and **Y**, respectively, are

(A)	$\text{N}_2$ and $\text{N}_2\text{O}$
(B)	$\text{NH}_3$ and $\text{NO}_2$
(C)	$\text{NO}$ and $\text{N}_2\text{O}$
(D)	$\text{N}_2$ and $\text{NH}_3$

**Answer: A**

**Q.2** The correct order of the wavelength maxima of the absorption band in the ultraviolet-visible region for the given complexes is

(A)	$[\text{Co}(\text{CN})_6]^{3-} < [\text{Co}(\text{NH}_3)_6]^{3+} < [\text{Co}(\text{NH}_3)_5(\text{H}_2\text{O})]^{3+} < [\text{Co}(\text{NH}_3)_5(\text{Cl})]^{2+}$
(B)	$[\text{Co}(\text{NH}_3)_5(\text{Cl})]^{2+} < [\text{Co}(\text{NH}_3)_5(\text{H}_2\text{O})]^{3+} < [\text{Co}(\text{NH}_3)_6]^{3+} < [\text{Co}(\text{CN})_6]^{3-}$
(C)	$[\text{Co}(\text{CN})_6]^{3-} < [\text{Co}(\text{NH}_3)_5(\text{Cl})]^{2+} < [\text{Co}(\text{NH}_3)_5(\text{H}_2\text{O})]^{3+} < [\text{Co}(\text{NH}_3)_6]^{3+}$
(D)	$[\text{Co}(\text{NH}_3)_6]^{3+} < [\text{Co}(\text{CN})_6]^{3-} < [\text{Co}(\text{NH}_3)_5(\text{Cl})]^{2+} < [\text{Co}(\text{NH}_3)_5(\text{H}_2\text{O})]^{3+}$

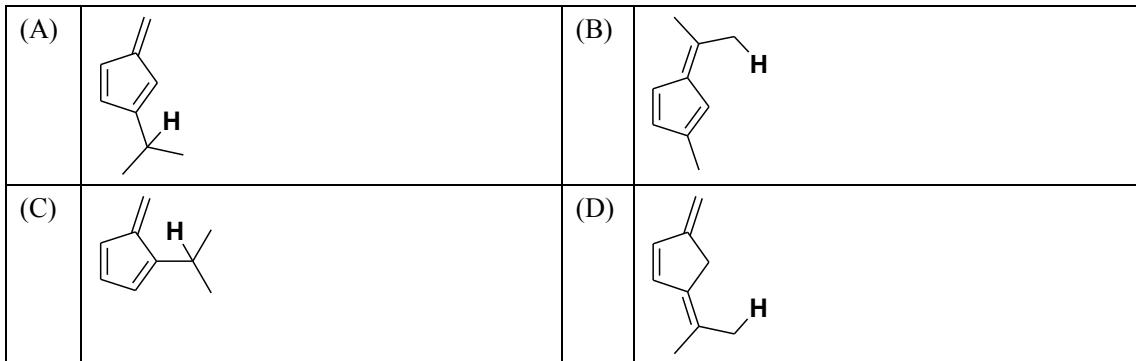
**Answer: A**

**Q.3** One of the products formed from the reaction of permanganate ion with iodide ion in neutral aqueous medium is

(A)	$\text{I}_2$	(B)	$\text{IO}_3^-$	(C)	$\text{IO}_4^-$	(D)	$\text{IO}_2^-$
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**Answer: B**

Q.4 Consider the depicted hydrogen (**H**) in the hydrocarbons given below. The most acidic hydrogen (**H**) is



**Answer: B**

## SECTION 2 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated **according to the following marking scheme**:

*Full Marks* : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

*Partial Marks* : +3 If all the four options are correct but **ONLY** three options are chosen;

*Partial Marks* : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

*Partial Marks* : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);

*Negative Marks* : -2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  - choosing ONLY (A), (B) and (D) will get +4 marks;
  - choosing ONLY (A) and (B) will get +2 marks;
  - choosing ONLY (A) and (D) will get +2 marks;
  - choosing ONLY (B) and (D) will get +2 marks;
  - choosing ONLY (A) will get +1 mark;
  - choosing ONLY (B) will get +1 mark;
  - choosing ONLY (D) will get +1 mark;
  - choosing no option (i.e. the question is unanswered) will get 0 marks; and
  - choosing any other combination of options will get -2 marks.

**Q.5** Regarding the molecular orbital (MO) energy levels for homonuclear diatomic molecules, the **INCORRECT** statement(s) is(are)

(A)	Bond order of $\text{Ne}_2$ is zero.
(B)	The highest occupied molecular orbital (HOMO) of $\text{F}_2$ is $\sigma$ -type.
(C)	Bond energy of $\text{O}_2^+$ is smaller than the bond energy of $\text{O}_2$ .
(D)	Bond length of $\text{Li}_2$ is larger than the bond length of $\text{B}_2$ .

**Answer: B, C**

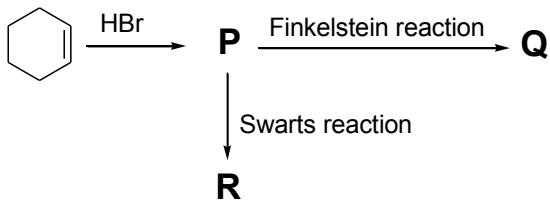
**Q.6** The pair(s) of diamagnetic ions is(are)

(A)	$\text{La}^{3+}$ , $\text{Ce}^{4+}$
(B)	$\text{Yb}^{2+}$ , $\text{Lu}^{3+}$
(C)	$\text{La}^{2+}$ , $\text{Ce}^{3+}$
(D)	$\text{Yb}^{3+}$ , $\text{Lu}^{2+}$

**Answer: A, B**

Q.7

For the reaction sequence given below, the correct statement(s) is(are)



(In the options, X is any atom other than carbon and hydrogen, and it is different in P, Q and R)

(A)	C–X bond length in P, Q and R follows the order Q > R > P.
(B)	C–X bond enthalpy in P, Q and R follows the order R > P > Q.
(C)	Relative reactivity toward S <sub>N</sub> 2 reaction in P, Q and R follows the order P > R > Q.
(D)	pK <sub>a</sub> value of the conjugate acids of the leaving groups in P, Q and R follows the order R > Q > P.

**Answer: B**

**SECTION 3 (Maximum Marks: 24)**

- This section contains **SIX (06)** questions.
- The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated **according to the following marking scheme**:

**Full Marks** : +4 If ONLY the correct numerical value is entered in the designated place;  
**Zero Marks** : 0 In all other cases.

Q.8 In an electrochemical cell, dichromate ions in aqueous acidic medium are reduced to  $\text{Cr}^{3+}$ . The current (in amperes) that flows through the cell for 48.25 minutes to produce 1 mole of  $\text{Cr}^{3+}$  is \_\_\_\_\_.

**Use:** 1 Faraday = 96500 C mol<sup>-1</sup>

**Answer: 100**

Q.9 At 25 °C, the concentration of  $\text{H}^+$  ions in  $1.00 \times 10^{-3}$  M aqueous solution of a weak monobasic acid having acid dissociation constant ( $K_a$ ) of  $4.00 \times 10^{-11}$  is  $X \times 10^{-7}$  M. The value of  $X$  is \_\_\_\_\_.

**Use:** Ionic product of water ( $K_w$ ) =  $1.00 \times 10^{-14}$  at 25 °C

**Answer: [2.2 to 2.3]**

Q.10 Molar volume ( $V_m$ ) of a van der Waals gas can be calculated by expressing the van der Waals equation as a cubic equation with  $V_m$  as the variable. The ratio (in mol dm<sup>-3</sup>) of the coefficient of  $V_m^2$  to the coefficient of  $V_m$  for a gas having van der Waals constants  $a = 6.0 \text{ dm}^6 \text{ atm mol}^{-2}$  and  $b = 0.060 \text{ dm}^3 \text{ mol}^{-1}$  at 300 K and 300 atm is \_\_\_\_\_.

**Use:** Universal gas constant (R) = 0.082 dm<sup>3</sup> atm mol<sup>-1</sup> K<sup>-1</sup>

**Answer: [-7.2 to -7]**

Q.11 Considering ideal gas behavior, the expansion work done (in kJ) when 144 g of water is electrolyzed completely under constant pressure at 300 K is \_\_\_\_\_.

**Use:** Universal gas constant (R) = 8.3 J K<sup>-1</sup> mol<sup>-1</sup>; Atomic mass (in amu): H = 1, O = 16

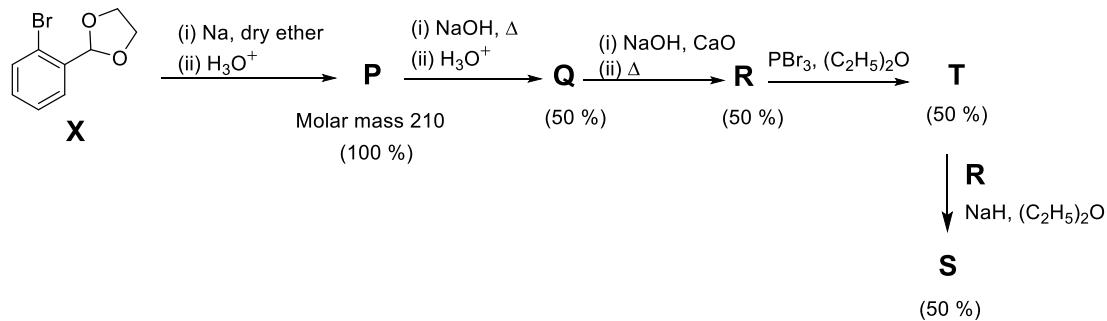
**Answer: [-29.95 to -29.8] OR [29.8 to 29.95]**

Q.12 The monomer (X) involved in the synthesis of Nylon 6,6 gives positive carbylamine test. If 10 moles of X are analyzed using Dumas method, the amount (in grams) of nitrogen gas evolved is \_\_\_\_\_.

**Use:** Atomic mass of N (in amu) = 14

**Answer: 280**

Q.13 The reaction sequence given below is carried out with 16 moles of **X**. The yield of the major product in each step is given below the product in parentheses. The amount (in grams) of **S** produced is \_\_\_\_\_.



Use: Atomic mass (in amu): H = 1, C = 12, O = 16, Br = 80

**Answer: 175**

**SECTION 4 (Maximum Marks: 12)**

- This section contains **THREE (03)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated **according to the following marking scheme**:
 

*Full Marks* : +4 **ONLY** if the option corresponding to the correct combination is chosen;  
*Zero Marks* : 0 If none of the options is chosen (i.e. the question is unanswered);  
*Negative Marks* : -1 In all other cases.

Q.14

The correct match of the group reagents in **List-I** for precipitating the metal ion given in **List-II** from solutions, is

**List-I**

(P) Passing H<sub>2</sub>S in the presence of NH<sub>4</sub>OH  
 (Q) (NH<sub>4</sub>)<sub>2</sub>CO<sub>3</sub> in the presence of NH<sub>4</sub>OH  
 (R) NH<sub>4</sub>OH in the presence of NH<sub>4</sub>Cl  
 (S) Passing H<sub>2</sub>S in the presence of dilute HCl

**List-II**

(1) Cu<sup>2+</sup>  
 (2) Al<sup>3+</sup>  
 (3) Mn<sup>2+</sup>  
 (4) Ba<sup>2+</sup>  
 (5) Mg<sup>2+</sup>

(A)	P → 3; Q → 4; R → 2; S → 1
(B)	P → 4; Q → 2; R → 3; S → 1
(C)	P → 3; Q → 4; R → 1; S → 5
(D)	P → 5; Q → 3; R → 2; S → 4

**Answer: A**

Q.15

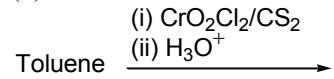
The major products obtained from the reactions in **List-II** are the reactants for the named reactions mentioned in **List-I**. Match each entry in **List-I** with the appropriate entry in **List-II** and choose the correct option.

**List-I**

(P) Stephen reaction

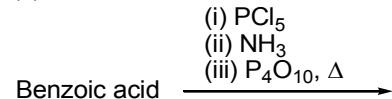
**List-II**

(1)



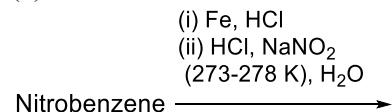
(Q) Sandmeyer reaction

(2)



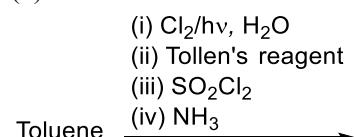
(R) Hoffmann bromamide degradation reaction

(3)

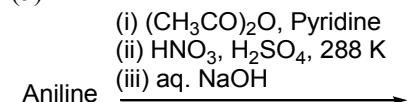


(S) Cannizzaro reaction

(4)



(5)



(A)	$\text{P} \rightarrow 2; \text{Q} \rightarrow 4; \text{R} \rightarrow 1; \text{S} \rightarrow 3$
(B)	$\text{P} \rightarrow 2; \text{Q} \rightarrow 3; \text{R} \rightarrow 4; \text{S} \rightarrow 1$
(C)	$\text{P} \rightarrow 5; \text{Q} \rightarrow 3; \text{R} \rightarrow 4; \text{S} \rightarrow 2$
(D)	$\text{P} \rightarrow 5; \text{Q} \rightarrow 4; \text{R} \rightarrow 2; \text{S} \rightarrow 1$

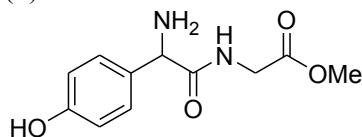
**Answer: B**

Q.16

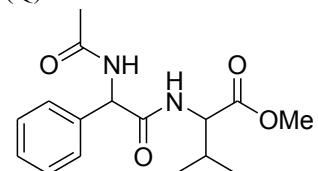
Match the compounds in **List-I** with the appropriate observations in **List-II** and choose the correct option.

**List-I**

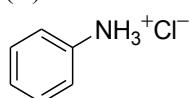
(P)



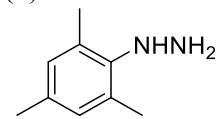
(Q)



(R)



(S)

**List-II**

(1) Reaction with phenyl diazonium salt gives yellow dye.

(2) Reaction with ninhydrin gives purple color and it also reacts with  $\text{FeCl}_3$  to give violet color.

(3) Reaction with glucose will give corresponding hydrazone.

(4) Lassaigne extract of the compound treated with dilute  $\text{HCl}$  followed by addition of aqueous  $\text{FeCl}_3$  gives blood red color.

(5) After complete hydrolysis, it will give ninhydrin test and it **DOES NOT** give positive phthalein dye test.

(A)	$P \rightarrow 1; Q \rightarrow 5; R \rightarrow 4; S \rightarrow 2$
(B)	$P \rightarrow 2; Q \rightarrow 5; R \rightarrow 1; S \rightarrow 3$
(C)	$P \rightarrow 5; Q \rightarrow 2; R \rightarrow 1; S \rightarrow 4$
(D)	$P \rightarrow 2; Q \rightarrow 1; R \rightarrow 5; S \rightarrow 3$

**Answer: B****END OF THE QUESTION PAPER**