JEE-Main-21-01-2026 (Memory Based)
[EVENING SHIFT]
MATHS

Question: Let $f(x)=x^3+x^2 f'(1)+2xf''(2)+f'''(3)$, $x \in R$. Then the value of $f'(5)$ is

Options:

(a) $\frac{117}{5}$

(b) $\frac{62}{5}$

(c) $\frac{657}{5}$

(d) $\frac{2}{5}$

Answer: (a)

$f'(1)=A \quad f''(2)=B \quad f'''(3)=C$

$f(x)=x^3+Ax^2+2xB+C$

$f'(x)=3x^2+2Ax+2B$

$f''(x)=6x+2A$

$f'''(x)=6 \Rightarrow f'''(3)=6=C$

$C=6$

$f''(2)=12+2A$

$B=12+2A \ldots\ldots(i)$

$f'(1)=A \Rightarrow A=3+2A+2B$

$\Rightarrow -A-2B=3 \ldots\ldots(ii)$

From (i) & (ii)

$A=-\frac{27}{5}$ & $B=\frac{6}{5}$

$f'(5)=3(5)^2+2\times-\frac{27}{5}\times 5+2\times\frac{6}{5}$

$=75-54+\frac{12}{5}=21+\frac{12}{5}$

$=\frac{117}{5}$

Question: The largest $n \in N$, for which 7^n divides 101!

Options:

(a) 16

(b) 18

(c) 19

(d) 15

Answer: (a)

Vedantu

$$\left[\frac{101}{7}\right] + \left[\frac{101}{7^2}\right]$$
$$= 14 + 2$$
$$= 16$$

Question: If three vectors are given as shown.

If angle between vectors $\vec{p}$ and $\bar{q}$ **is q where** $\cos\theta = \frac{1}{\sqrt{3}}$ and $\left|\vec{p}\right| = 2\sqrt{3}, \left|\vec{q}\right| = 2.$

$\left|\vec{p} \times \left(\vec{q} - 3\vec{r}\right)\right|^2 - 3\left|\vec{r}\right|^2$ **Then the value of**

Answer: (92)

$\bar{p} + \bar{r} = \bar{q}$

$\bar{r} = \bar{q} - \bar{p}$

$|\bar{p} \times (\bar{q} - 3\bar{q} + 3\bar{p})|^2 - 3|\bar{q} - \bar{p}|^2$

$|-2\bar{p} \times \bar{q}|^2 - 3|\bar{q} - \bar{p}|^2$

$4|\bar{p}|^2|\bar{q}|^2 \sin^2\theta - 3\left(|\bar{q}|^2 + |\bar{p}|^2 - 2|\bar{p}||\bar{q}|\cos\theta\right)$

$= 4 \times 12\,4\,(1 - \frac{1}{3}) - 3\left(4 + 12 - 2 \times 2\sqrt{3} \times \frac{1}{\sqrt{3}}\right)$

$= 16 \times 12 \times \frac{2}{3} - 3(16 - 4)$

$= 16 \times 8 - 3 \times 12$

$= 128 - 36 = 92$

Question: Let one end of a focal chord of the parabola y2 = 16x be (16,16). If P(α, β) divides this focal chord internally in the ratio 5 : 2; then the minimum value of $\alpha + \beta$ is equal to:

Options:

(a) 7

(b) 22

(c) 5

(d) 16

Answer: (a)

$4a = 16 \Rightarrow a = 4$

$2at_1 = 16 \Rightarrow t_1 = 2$

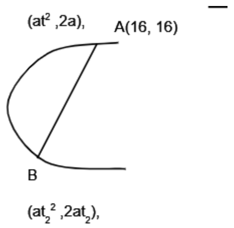
$t_1 t_2 = -1 \Rightarrow t_2 = -\frac{1}{2}$

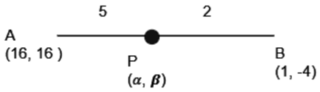
$B = (1, -4)$

$AP : PB = 5 : 2$

$\alpha = \frac{5+32}{7}, \beta = \frac{-20+32}{7}$

$\alpha + \beta = \frac{37+12}{7} = 7$





Question: If area bounded by curves y ≤ 1-2x and y ≤4-x2, x ≥0, y ≥0 is equal to $\frac{\alpha}{\beta}$ where G.C.D. $(\alpha, \beta) = 1$ then $\alpha + \beta$ is

Answer: $\left(\frac{20\sqrt{5}}{3}\right)$

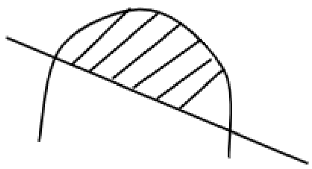
Solving $y = -2x,\ y = 4 - x^2$

$x^2 - 2x - 4 = 0$

$x = \frac{2 \pm \sqrt{4+16}}{2} = 1 + \sqrt{5}$

Area $= \frac{d^3}{6\,l} = \frac{\left(2\sqrt{5}\right)^3}{6 \times 1} = \frac{2 \times 5\sqrt{5}}{6}$

$= \frac{20\sqrt{5}}{3}$



Question: If $\frac{dy}{dx}$ **sec x - 2y = 2 + 3 sin x and** $y(0) = -\frac{7}{4}$ **then find** $y\left(\frac{\pi}{6}\right)$

Answer: $\left(\frac{-5}{2}\right)$

$\frac{dy}{dx} - (2\cos x)y = (2 + 3\sin x)\cos x$

$I.F = e^{-2\sin x}$

$ye^{-2\sin x} = \int e^{-2\sin x}(2+3\sin x)\cos x\,dx$

$= \int e^{-2}(2+3t)\,dt\,,\ t = \sin x$

$= (2+3t)\,.\,\frac{e^{-2t}}{-2} - \int 3.\,\frac{e^{-2t}}{-2}\,dt$

$= -\frac{1}{2}(2+3t)\,e^{-2t} - \frac{3}{4}\,e^{-2t} + c$

$ye^{-2\sin x} = -\frac{1}{2}(2+3\sin x)\,e^{-2\sin x} - \frac{3}{4}e^{-2\sin x} + c$

If $x = 0,\ y = \frac{-7}{4},\ \frac{-7}{4} = -1 - \frac{3}{4} + c \Rightarrow c = 0$

If $x = \frac{\pi}{6},\ ye^{-1} = -\frac{1}{2}\,(2+\frac{3}{2})e^{-1} - \frac{3}{4}e^{-1}$

$y = \frac{-7}{4} - \frac{3}{4} = \frac{-5}{2}$

Question: If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 29 & 49 \\ 1 & 2 \end{bmatrix}$ **satisfy** $(A^{15}+B)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ **then which of the following values of (x, y) satisfies above equation.**

Options:

(a) x = 11, y = 2

(b) x = 5, y = 7

(c) x = 18, y = 11

(d) x = 16, y = 3

Answer:

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}$

$p^2 = \begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow p^n = 0\,\forall n \geq 2$

$A^{15} = (I+p)^{15}$

$= I + {}^{15}C_1\,p + {}^{15}C_2\,p^2 + \ldots\ldots + {}^{15}C_{15}\,p^{15}$

$A^{15} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + 15\begin{pmatrix} 2 & -4 \\ 1 & -2 \end{pmatrix}$

$A^{15} = \begin{pmatrix} 31 & -60 \\ 15 & -29 \end{pmatrix}$

$(A^{15}+B)\begin{pmatrix} r \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 60 & -11 \\ 16 & -27 \end{pmatrix}\begin{pmatrix} r \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$60x - 11y = 0$

$16x - 27y = 0$

$x = 0\,\&\,y = 0$

Question: $a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \ldots\ldots, \frac{a_{10}}{2^9}$ **are in GP having common ratio** $\frac{1}{\sqrt{2}}$ **and** $a_1 + a_2 + a_3 + \ldots + a_{10} = 62$ **then find** a_1

Answer: $(2(\sqrt{2} - 1))$

$$\frac{\frac{a_2}{2}}{a_1} = \frac{1}{\sqrt{2}} \Rightarrow \frac{a_2}{a_1} = \sqrt{2}$$

$$\frac{a_1\left[\left(\sqrt{2}\right)^{10} - 1\right]}{\sqrt{2} - 1} = 62 \Rightarrow a_1 = 2\left(\sqrt{2} - 1\right)$$

Question: **Let** $f(x) = \lim_{n\to\infty}\left(\frac{1}{n^3}\sum_{k=1}^{n}\left[\frac{k^2}{3^x}\right]\right)$, **where [.] denotes the greatest integer function, then** $12\sum_{j=1}^{\infty} f(j)$ **is equal to**

Options:
(a) 2
(b) 3
(c) 4
(d) 1

Answer: (a)

$[A] = A - \{A\}$

$$f(x) = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n^3}\cdot\frac{k^2}{3^x} - \sum_{k=1}^{n}\frac{1}{n^3}\cdot\left\{\frac{k^2}{3^x}\right\}$$

$$\because \frac{1}{n^3} \to 0$$

$$\text{and } \left\{\frac{k^2}{3^x}\right\} \in (0,1)$$

$$f(x) = \frac{1}{3^x}\lim_{n\to\infty}\frac{1^2+2^2+\ldots\ldots+x^2}{n^3}$$

$$= \frac{1}{3^x}\lim_{n\to\infty}\frac{n(n+1)(3n+1)}{6\,n^3}$$

$$= \frac{1}{3^x} \times \frac{2}{6} = \left(\frac{1}{3}\right)^{x+1}$$

$$12\sum_{i=1}^{\infty} f(j) = 12\sum_{i=1}^{\infty}\left(\frac{1}{3}\right)^{j+1}$$

$$= 12\left[\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^3 + \ldots\ldots + \infty\right]$$

$$= 12\left[\frac{\frac{1}{9}}{1-\frac{1}{3}}\right] = 12 \times \frac{1}{9} \times \frac{3}{2}$$

$$= 12 \times \frac{1}{3\times 2} = 2$$

Question: **The number of solutions of equation tan3x = cot x in** $x \in [0,2\pi]$ **is**

Options:
(a) 4
(b) 6
(c) 2
(d) 8

Answer: (d)

$\tan 3x = \tan\left(\frac{\pi}{2} - x\right)$

$3x = n\pi + \frac{\pi}{2} - x$

$\Rightarrow x = (2n+1)\frac{\pi}{8}, 0 \leq n \leq 7$

$= 8$