JEE Main 2026 January 22 Shift 1 Question Paper with Solutions

Time Allowed : 3 Hours	Maximum Marks : 300	Total Questions : 75

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Multiple choice questions (MCQs)
2. Questions with numerical values as answers.
3. There are three sections: **Mathematics, Physics, Chemistry**.
4. **Mathematics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
5. **Physics:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory..
6. **Chemistry:** 25 (20+5) 10 Questions with answers as a numerical value. Out of 10 questions, 5 questions are compulsory.
7. Total: 75 Questions (25 questions each).
8. 300 Marks (100 marks for each section).
9. **MCQs:** Four marks will be awarded for each correct answer and there will be a negative marking of one mark on each wrong answer.
10. **Questions with numerical value answers:** Candidates will be given four marks for each correct answer and there will be a negative marking of 1 mark for each wrong answer.

Mathematics

Section - A

1. The number of real solution of equation $x|x+4|+3|x+2|+10=0$ is/are :

(A) 1
(B) 2
(C) 3
(D) 4

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Question:
We need to find the number of real solutions for the equation involving absolute value functions: $x|x+4|+3|x+2|+10=0$. To solve this, we must analyze the equation in different intervals based on the points where the expressions inside the absolute value signs become zero.

Step 2: Identifying Critical Points and Intervals:
The critical points are where the arguments of the absolute value functions are zero.
These are $x+4=0 \Rightarrow x=-4$ and $x+2=0 \Rightarrow x=-2$.
These points divide the number line into three intervals:

- Case 1: $x < -4$
- Case 2: $-4 \leq x < -2$
- Case 3: $x \geq -2$

Step 3: Solving the Equation in Each Interval:
Case 1: $x < -4$
In this interval, $x+4 < 0$ and $x+2 < 0$.
So, $|x+4| = -(x+4)$ and $|x+2| = -(x+2)$.
The equation becomes:

$$x(-(x+4)) + 3(-(x+2)) + 10 = 0$$
$$-x^2 - 4x - 3x - 6 + 10 = 0$$
$$-x^2 - 7x + 4 = 0$$
$$x^2 + 7x - 4 = 0$$

Using the quadratic formula, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$:

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-4)}}{2(1)} = \frac{-7 \pm \sqrt{49+16}}{2} = \frac{-7 \pm \sqrt{65}}{2}$$

The two possible values for x are $x_1 = \frac{-7+\sqrt{65}}{2}$ and $x_2 = \frac{-7-\sqrt{65}}{2}$.
Since $\sqrt{64} = 8$, $\sqrt{65} \approx 8.06$.
$x_1 \approx \frac{-7+8.06}{2} = \frac{1.06}{2} = 0.53$. This is not in the interval $x < -4$.
$x_2 \approx \frac{-7-8.06}{2} = \frac{-15.06}{2} = -7.53$. This is in the interval $x < -4$.
So, we have one solution from this case: $x = \frac{-7-\sqrt{65}}{2}$.

Case 2: $-4 \leq x < -2$
In this interval, $x+4 \geq 0$ and $x+2 < 0$.
So, $|x+4| = x+4$ and $|x+2| = -(x+2)$.
The equation becomes:

$$x(x+4) + 3(-(x+2)) + 10 = 0$$
$$x^2 + 4x - 3x - 6 + 10 = 0$$
$$x^2 + x + 4 = 0$$

The discriminant is $D = b^2 - 4ac = 1^2 - 4(1)(4) = 1 - 16 = -15$.
Since $D < 0$, there are no real solutions in this interval.

Case 3: $x \geq -2$
In this interval, $x + 4 > 0$ and $x + 2 \geq 0$.
So, $|x + 4| = x + 4$ and $|x + 2| = x + 2$.
The equation becomes:

$$x(x + 4) + 3(x + 2) + 10 = 0$$
$$x^2 + 4x + 3x + 6 + 10 = 0$$
$$x^2 + 7x + 16 = 0$$

The discriminant is $D = b^2 - 4ac = 7^2 - 4(1)(16) = 49 - 64 = -15$.
Since $D < 0$, there are no real solutions in this interval.

Step 4: Final Answer:
Combining the results from all three cases, we find only one real solution, $x = \frac{-7-\sqrt{65}}{2}$.
Therefore, the number of real solutions is 1.

Quick Tip

When solving equations with multiple absolute value terms, always identify the critical points (where the arguments become zero) and analyze the equation in the intervals defined by these points.

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2. If xdy - ydx = $\sqrt{x^2 + y^2}$ dx. If y = y(x) & y(1) = 0 then y(3) is :

(A) 1
(B) 2
(C) 3
(D) 4

Correct Answer: (D) 4

Solution:

Step 1: Understanding the Question:
We are given a first-order differential equation and an initial condition. We need to find the value of the function y at x=3. The given equation is a homogeneous differential equation.

Step 2: Key Formula or Approach:
First, we rearrange the equation into the standard form $\frac{dy}{dx} = f(x, y)$.

$$xdy = (y + \sqrt{x^2 + y^2})dx$$
$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}$$

This is a homogeneous differential equation. We use the substitution $y = vx$, which implies $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Step 3: Detailed Explanation:
Substitute $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ into the equation:

$$v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$$

$$x\frac{dv}{dx} = \sqrt{1+v^2}$$

Now, we separate the variables:

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

Integrate both sides:

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

The standard integral $\int \frac{1}{\sqrt{a^2+x^2}}dx = \ln|x + \sqrt{a^2+x^2}|$. Here $a = 1$.

$$\ln|v + \sqrt{1+v^2}| = \ln|x| + C$$

where C is the constant of integration. We can write $C = \ln|A|$ for some constant A.

$$\ln|v + \sqrt{1+v^2}| = \ln|Ax|$$

$$v + \sqrt{1+v^2} = Ax$$

Substitute back $v = \frac{y}{x}$:

$$\frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2} = Ax$$

$$\frac{y}{x} + \frac{\sqrt{x^2+y^2}}{|x|} = Ax$$

Assuming $x > 0$ (since the initial condition is at x=1), we have:

$$\frac{y + \sqrt{x^2+y^2}}{x} = Ax$$

$$y + \sqrt{x^2+y^2} = Ax^2$$

Now, apply the initial condition $y(1) = 0$:

$$0 + \sqrt{1^2+0^2} = A(1)^2$$

$$1 = A$$

The particular solution is:

$$y + \sqrt{x^2+y^2} = x^2$$

We need to find $y(3)$. Substitute $x = 3$:

$$y(3) + \sqrt{3^2 + (y(3))^2} = 3^2$$

Let $y(3) = y$:

$$y + \sqrt{9 + y^2} = 9$$
$$\sqrt{9 + y^2} = 9 - y$$

Square both sides:

$$9 + y^2 = (9 - y)^2 = 81 - 18y + y^2$$
$$9 = 81 - 18y$$
$$18y = 81 - 9 = 72$$
$$y = \frac{72}{18} = 4$$

Step 4: Final Answer:
The value of y(3) is 4.

Quick Tip

Recognize homogeneous differential equations of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$. The standard substitution $y = vx$ simplifies them to a separable form, making them easier to solve.

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3. If $\int (\sin x)^{-11/2} (\cos x)^{-5/2} dx = \frac{p_1}{q_1}(\cot x)^{9/2} - \frac{p_2}{q_2}(\cot x)^{5/2} - \frac{p_3}{q_3}(\cot x)^{1/2} + \frac{p_4}{q_4}(\cot x)^{-3/2} + C$ where H.C.F. $\{p_i, q_i\} = 1$ & $i \in \{1, 2, 3, 4\}$. Then value of $\sum_{i=1}^{4}(p_i + q_i)$ is:

(A) 30
(B) 25
(C) 24
(D) 34

Correct Answer: (D) 34

Solution:

Step 1: Understanding the Question:
We are asked to evaluate a complex trigonometric integral and then compare the resulting coefficients with a given form to find the sum of certain values.

Step 2: Key Formula or Approach:
The integral is $I = \int (\sin x)^{-11/2} (\cos x)^{-5/2} dx$.
The sum of the powers is $-11/2 - 5/2 = -16/2 = -8$, which is a negative even integer. In such cases, substitution with $t = \tan x$ or $t = \cot x$ is effective. Let's use $t = \cot x$.
If $t = \cot x$, then $dt = -\csc^2 x \, dx$, which means $dx = -\sin^2 x \, dt$.
We also need expressions for $\sin x$ and $\cos x$ in terms of t: From $1 + \cot^2 x = \csc^2 x$, we get $\sin^2 x = \frac{1}{1+\cot^2 x} = \frac{1}{1+t^2}$.

From $\tan^2 x = \frac{1}{\cot^2 x} = \frac{1}{t^2}$, we get $\sec^2 x = 1 + \tan^2 x = 1 + \frac{1}{t^2} = \frac{t^2+1}{t^2}$, so $\cos^2 x = \frac{t^2}{t^2+1}$.

Step 3: Detailed Explanation:
The integral is $I = \int \sin^{-11/2} x \cos^{-5/2} x \, dx$. Substitute $dx = -\sin^2 x \, dt$:

$$I = \int \sin^{-11/2} x \cos^{-5/2} x (-\sin^2 x) \, dt = -\int \sin^{-7/2} x \cos^{-5/2} x \, dt$$

Now substitute for $\sin x$ and $\cos x$ in terms of t: $\sin x = (1+t^2)^{-1/2}$ and $\cos x = t(1+t^2)^{-1/2}$.

$$I = -\int ((1+t^2)^{-1/2})^{-7/2} (t(1+t^2)^{-1/2})^{-5/2} \, dt$$

$$I = -\int (1+t^2)^{7/4} \cdot t^{-5/2} (1+t^2)^{5/4} \, dt$$

$$I = -\int t^{-5/2} (1+t^2)^{7/4+5/4} \, dt = -\int t^{-5/2} (1+t^2)^{12/4} \, dt$$

$$I = -\int t^{-5/2} (1+t^2)^3 \, dt$$

Expand $(1+t^2)^3 = 1 + 3t^2 + 3t^4 + t^6$.

$$I = -\int t^{-5/2} (1 + 3t^2 + 3t^4 + t^6) \, dt$$

$$I = -\int (t^{-5/2} + 3t^{-1/2} + 3t^{3/2} + t^{7/2}) \, dt$$

Now, integrate term by term using $\int t^n \, dt = \frac{t^{n+1}}{n+1}$:

$$I = -\left[\frac{t^{-3/2}}{-3/2} + \frac{3t^{1/2}}{1/2} + \frac{3t^{5/2}}{5/2} + \frac{t^{9/2}}{9/2} \right] + C$$

$$I = -\left[-\frac{2}{3} t^{-3/2} + 6t^{1/2} + \frac{6}{5} t^{5/2} + \frac{2}{9} t^{9/2} \right] + C$$

$$I = -\frac{2}{9} t^{9/2} - \frac{6}{5} t^{5/2} - 6t^{1/2} + \frac{2}{3} t^{-3/2} + C$$

Substitute back $t = \cot x$:

$$I = -\frac{2}{9} (\cot x)^{9/2} - \frac{6}{5} (\cot x)^{5/2} - 6(\cot x)^{1/2} + \frac{2}{3} (\cot x)^{-3/2} + C$$

Now, compare this with the given form: $\frac{p_1}{q_1}(\cot x)^{9/2} - \frac{p_2}{q_2}(\cot x)^{5/2} - \frac{p_3}{q_3}(\cot x)^{1/2} + \frac{p_4}{q_4}(\cot x)^{-3/2} + C$.
Assuming the first term in the problem statement should have a negative sign, making it $-\frac{p_1}{q_1}(\cot x)^{9/2}$, we compare coefficients:

- $-\frac{p_1}{q_1} = -\frac{2}{9} \Rightarrow p_1 = 2, q_1 = 9$ (HCF(2,9)=1)
- $-\frac{p_2}{q_2} = -\frac{6}{5} \Rightarrow p_2 = 6, q_2 = 5$ (HCF(6,5)=1)
- $-\frac{p_3}{q_3} = -6 \Rightarrow p_3 = 6, q_3 = 1$ (HCF(6,1)=1)

- $\frac{p_4}{q_4} = \frac{2}{3} \Rightarrow p_4 = 2, q_4 = 3$ (HCF(2,3)=1)

Now we calculate the required sum:

$$\sum_{i=1}^{4}(p_i + q_i) = (p_1 + q_1) + (p_2 + q_2) + (p_3 + q_3) + (p_4 + q_4)$$

$$= (2 + 9) + (6 + 5) + (6 + 1) + (2 + 3)$$

$$= 11 + 11 + 7 + 5 = 34$$

Step 4: Final Answer:
The value of the sum is 34.

Quick Tip

For integrals of the form $\int \sin^m x \cos^n x \, dx$, if $m + n$ is a negative even integer, the substitution $t = \tan x$ or $t = \cot x$ is a very effective strategy to simplify the integral.

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4. Let M = {1, 2, 3, ..., 16}, if a relation R defined on set M such that R = {(x, y) : 4y = 5x − 3, x, y ∈ M}. How many elements should be added to R to make it symmetric.

(A) 2
(B) 3
(C) 4
(D) 5

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Question:
We are given a set M and a relation R on M. We first need to find all the elements (ordered pairs) in R. Then, we need to determine how many more ordered pairs must be added to R to satisfy the property of symmetry.

Step 2: Finding the Elements of Relation R:
The relation is defined by the equation $4y = 5x - 3$, where $x, y \in \{1, 2, ..., 16\}$.
We can write this as $y = \frac{5x-3}{4}$. For y to be an integer, $5x - 3$ must be divisible by 4.
We can check this condition using modular arithmetic: $5x - 3 \equiv 0 \pmod 4$.
Since $5 \equiv 1 \pmod 4$ and $-3 \equiv 1 \pmod 4$, the condition becomes $x + 1 \equiv 0 \pmod 4$, or $x \equiv -1 \pmod 4$, which is $x \equiv 3 \pmod 4$.
This means x must be of the form $4k + 3$ for some integer k.
We test values of x in M = {1, 2, ..., 16} that satisfy this condition:

- If $x = 3$: $y = \frac{5(3)-3}{4} = \frac{12}{4} = 3$. Since $y = 3 \in M$, the pair (3, 3) is in R.
- If $x = 7$: $y = \frac{5(7)-3}{4} = \frac{32}{4} = 8$. Since $y = 8 \in M$, the pair (7, 8) is in R.
- If $x = 11$: $y = \frac{5(11)-3}{4} = \frac{52}{4} = 13$. Since $y = 13 \in M$, the pair (11, 13) is in R.
- If $x = 15$: $y = \frac{5(15)-3}{4} = \frac{72}{4} = 18$. Since $y = 18 \notin M$, this pair is not in R.

So, the relation R is $R = \{(3,3), (7,8), (11,13)\}$.

Step 3: Making the Relation Symmetric:
A relation R is symmetric if, for every ordered pair $(a, b) \in R$, the pair (b, a) must also be in R. Let's check the elements of R:

- For (3, 3): The reverse pair is (3, 3), which is already in R. This element satisfies the symmetric property.
- For (7, 8): The reverse pair is (8, 7). This pair is not in R, so it must be added.
- For (11, 13): The reverse pair is (13, 11). This pair is not in R, so it must be added.

Step 4: Final Answer:
To make the relation R symmetric, we need to add the pairs (8, 7) and (13, 11).
The number of elements to be added is 2.

> **Quick Tip**
>
> To make a relation R symmetric, for every pair (a, b) in R where $a \neq b$, you must ensure the pair (b, a) is also in R. Count how many such (b, a) pairs are missing.

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5. Image of point P (1, 2, a) with respect to line mirror $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is point Q (5, b, c), then value of $(a^2 + b^2 + c^2)$ is :

(A) 293
(B) 298
(C) 283
(D) 264

Correct Answer: (B) 298

Solution:

Step 1: Understanding the Question:
We are given a point P, a line, and the image of P in that line, Q. We need to find the values of the unknown coordinates a, b, and c, and then calculate $a^2 + b^2 + c^2$. There are two key properties of an image with respect to a line mirror.

Step 2: Applying Geometric Properties:
Property 1: The midpoint of PQ lies on the line.
The midpoint M of the segment PQ is given by:

$$M = \left(\frac{1+5}{2}, \frac{2+b}{2}, \frac{a+c}{2}\right) = \left(3, \frac{2+b}{2}, \frac{a+c}{2}\right)$$

Since M lies on the given line, its coordinates must satisfy the line's equation: $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.
Substituting the coordinates of M:

$$\frac{3-6}{3} = \frac{\frac{2+b}{2}-7}{2} = \frac{\frac{a+c}{2}-7}{-2}$$

$$\frac{-3}{3} = \frac{\frac{2+b-14}{2}}{2} = \frac{\frac{a+c-14}{2}}{-2}$$

$$-1 = \frac{b-12}{4} = \frac{a+c-14}{-4}$$

From $-1 = \frac{b-12}{4}$, we get $-4 = b - 12$, which gives $\mathbf{b = 8}$.
From $-1 = \frac{a+c-14}{-4}$, we get $4 = a + c - 14$, which gives $\mathbf{a + c = 18}$ (Equation I).

Property 2: The line segment PQ is perpendicular to the mirror line.
The direction ratios of the line segment PQ are $(5-1, b-2, c-a) = (4, b-2, c-a)$.
Since we found $b = 8$, the direction ratios of PQ are $(4, 6, c-a)$.
The direction ratios of the mirror line are given by the denominators in its equation: $(3, 2, -2)$.
For two lines to be perpendicular, the dot product of their direction ratios must be zero: $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

$$(4)(3) + (6)(2) + (c-a)(-2) = 0$$

$$12 + 12 - 2(c-a) = 0$$

$$24 = 2(c-a)$$

$\mathbf{c - a = 12}$ (Equation II).

Step 3: Solving for a, b, and c:
We have a system of two linear equations for a and c:

1. $a + c = 18$
2. $-a + c = 12$

Adding the two equations gives: $2c = 30 \Rightarrow \mathbf{c = 15}$.
Substituting $c = 15$ into the first equation: $a + 15 = 18 \Rightarrow \mathbf{a = 3}$.
So we have found $a = 3$, $b = 8$, and $c = 15$.

Step 4: Final Answer:
We need to calculate the value of $a^2 + b^2 + c^2$.

$$a^2 + b^2 + c^2 = 3^2 + 8^2 + 15^2$$

$$= 9 + 64 + 225$$

$$= 73 + 225 = 298$$

The final value is 298.

Quick Tip

Finding the image of a point in a line involves two key geometric conditions: the midpoint of the point and its image lies on the line, and the line connecting the point and its image is perpendicular to the mirror line.

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6. The no. of solution in $x \in \left(-\frac{1}{2\sqrt{6}}, \frac{1}{2\sqrt{6}}\right)$ of equation $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{6}$ is :

(A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Question:
We are asked to find the number of solutions to a trigonometric equation involving inverse tangent functions within a specific interval.

Step 2: Key Formula or Approach:
We use the formula for the sum of two inverse tangent functions:

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left(\frac{A+B}{1-AB}\right), \quad \text{provided } AB < 1$$

Here, $A = 4x$ and $B = 6x$. The condition is $(4x)(6x) < 1 \Rightarrow 24x^2 < 1 \Rightarrow x^2 < \frac{1}{24}$.
This means $|x| < \frac{1}{\sqrt{24}} = \frac{1}{2\sqrt{6}}$. The given interval for x is $\left(-\frac{1}{2\sqrt{6}}, \frac{1}{2\sqrt{6}}\right)$, which satisfies the condition $AB < 1$.

Step 3: Detailed Explanation:
Applying the formula to the given equation:

$$\tan^{-1}(4x) + \tan^{-1}(6x) = \tan^{-1}\left(\frac{4x+6x}{1-(4x)(6x)}\right) = \tan^{-1}\left(\frac{10x}{1-24x^2}\right)$$

We are given that this equals $\frac{\pi}{6}$.

$$\tan^{-1}\left(\frac{10x}{1-24x^2}\right) = \frac{\pi}{6}$$

Taking the tangent of both sides:

$$\frac{10x}{1-24x^2} = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

Now, we solve for x:

$$10\sqrt{3}x = 1 - 24x^2$$
$$24x^2 + 10\sqrt{3}x - 1 = 0$$

This is a quadratic equation in x. Using the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$:

$$x = \frac{-10\sqrt{3} \pm \sqrt{(10\sqrt{3})^2 - 4(24)(-1)}}{2(24)}$$

$$x = \frac{-10\sqrt{3} \pm \sqrt{300 + 96}}{48} = \frac{-10\sqrt{3} \pm \sqrt{396}}{48}$$

$$x = \frac{-10\sqrt{3} \pm \sqrt{36 \times 11}}{48} = \frac{-10\sqrt{3} \pm 6\sqrt{11}}{48} = \frac{-5\sqrt{3} \pm 3\sqrt{11}}{24}$$

The two possible solutions are $x_1 = \frac{-5\sqrt{3}+3\sqrt{11}}{24}$ and $x_2 = \frac{-5\sqrt{3}-3\sqrt{11}}{24}$.

Step 4: Checking the Solutions against the Interval:

The interval is $\left(-\frac{1}{2\sqrt{6}}, \frac{1}{2\sqrt{6}}\right)$.

$\frac{1}{2\sqrt{6}} = \frac{\sqrt{6}}{12} \approx \frac{2.449}{12} \approx 0.204$. So the interval is approx $(-0.204, 0.204)$.

Approximate values of the solutions: $\sqrt{3} \approx 1.732$, $\sqrt{11} \approx 3.317$.

$x_1 \approx \frac{-5(1.732)+3(3.317)}{24} = \frac{-8.66+9.951}{24} = \frac{1.291}{24} \approx 0.0538.$

This value lies inside the interval $(-0.204, 0.204)$.

$x_2 \approx \frac{-5(1.732)-3(3.317)}{24} = \frac{-8.66-9.951}{24} = \frac{-18.611}{24} \approx -0.775.$

This value lies outside the interval $(-0.204, 0.204)$.

Therefore, there is only one solution in the given interval.

Quick Tip

When using the formula for $\tan^{-1} A + \tan^{-1} B$, always check the condition $AB < 1$. The given interval often provides a hint about which form of the formula to use.

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7. If the value of $\frac{\cos^2 48° - \sin^2 12°}{\sin^2 24° - \sin^2 6°}$ is $\frac{\alpha+\beta\sqrt{5}}{\gamma}$ then value of $(\alpha + \beta + \gamma)$ (where $\alpha, \beta, \gamma \in \mathbb{N}$ and are in lowest form) :

(A) 3
(B) 4
(C) 5

(D) 6

Correct Answer: (D) 6

Solution:

Step 1: Understanding the Question:
We need to simplify a trigonometric expression and express it in the form $\frac{\alpha+\beta\sqrt{5}}{\gamma}$. Then we have to find the sum of the integers α, β, γ.

Step 2: Key Formula or Approach:
We will use the following trigonometric identities:

1. $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$
2. $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

We will also need the values of some standard angles: $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$ and $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.

Step 3: Detailed Explanation:
Let's simplify the numerator and the denominator separately.
Numerator: $\cos^2 48^\circ - \sin^2 12^\circ$ Using the identity $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$:

$$\cos(48^\circ + 12^\circ)\cos(48^\circ - 12^\circ) = \cos(60^\circ)\cos(36^\circ)$$

We know $\cos 60^\circ = \frac{1}{2}$ and $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$.

$$\text{Numerator} = \frac{1}{2} \cdot \frac{\sqrt{5}+1}{4} = \frac{\sqrt{5}+1}{8}$$

Denominator: $\sin^2 24^\circ - \sin^2 6^\circ$ Using the identity $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$:

$$\sin(24^\circ + 6^\circ)\sin(24^\circ - 6^\circ) = \sin(30^\circ)\sin(18^\circ)$$

We know $\sin 30^\circ = \frac{1}{2}$ and $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$.

$$\text{Denominator} = \frac{1}{2} \cdot \frac{\sqrt{5}-1}{4} = \frac{\sqrt{5}-1}{8}$$

Now, let's find the value of the given expression:

$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{(\sqrt{5}+1)/8}{(\sqrt{5}-1)/8} = \frac{\sqrt{5}+1}{\sqrt{5}-1}$$

To rationalize the denominator, we multiply the numerator and denominator by $(\sqrt{5}+1)$:

$$\frac{(\sqrt{5}+1)}{(\sqrt{5}-1)} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)} = \frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2 - 1^2} = \frac{5+1+2\sqrt{5}}{5-1} = \frac{6+2\sqrt{5}}{4}$$

Simplifying the expression by dividing by 2:

$$\frac{3+\sqrt{5}}{2}$$

Step 4: Final Answer:
We are given that the value is $\frac{\alpha+\beta\sqrt{5}}{\gamma}$. Comparing this with our result $\frac{3+1\sqrt{5}}{2}$, we get: $\alpha = 3$, $\beta = 1$, $\gamma = 2$.
These are natural numbers and in the lowest form.
The required value is $\alpha + \beta + \gamma = 3 + 1 + 2 = 6$.

Quick Tip

Memorizing product-to-sum formulas and specific values like $\sin 18°$ and $\cos 36°$ is crucial for solving such trigonometry problems quickly in competitive exams.

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8. If a line $ax + y = 1$ does not intersect the hyperbola $x^2 - 9y^2 = 9$ then a possible value of α is :

(A) 0.2
(B) 0.3
(C) 0.4
(D) 0.5

Correct Answer: (D) 0.5

Solution:

Step 1: Understanding the Question:
We are given the equation of a line and a hyperbola. We need to find the condition on the parameter 'a' such that the line does not intersect the hyperbola, and then identify a possible value for 'a' from the options.

Step 2: Key Formula or Approach:
First, we write both equations in their standard forms.
The equation of the hyperbola is $x^2 - 9y^2 = 9$. Dividing by 9, we get:

$$\frac{x^2}{9} - \frac{y^2}{1} = 1$$

This is a standard hyperbola with $a_h^2 = 9$ and $b_h^2 = 1$. (Using a_h, b_h to avoid confusion with parameter 'a' in the line).
The equation of the line is $ax + y = 1$, which can be written as $y = -ax + 1$.
This is in the slope-intercept form $y = mx + c$, with slope $m = -a$ and y-intercept $c = 1$.
For a line $y = mx + c$ and a hyperbola $\frac{x^2}{a_h^2} - \frac{y^2}{b_h^2} = 1$, the condition for the line to not intersect the hyperbola is $c^2 < a_h^2 m^2 - b_h^2$.
(The condition for tangency is $c^2 = a_h^2 m^2 - b_h^2$, and for intersection at two points is $c^2 > a_h^2 m^2 - b_h^2$).

Step 3: Detailed Explanation:
Substitute the values from our problem into the condition for no intersection: $c = 1$, $m = -a$, $a_h^2 = 9$, $b_h^2 = 1$.

$$1^2 < 9(-a)^2 - 1$$
$$1 < 9a^2 - 1$$
$$2 < 9a^2$$
$$a^2 > \frac{2}{9}$$

Taking the square root of both sides:

$$|a| > \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$$

Now, we need to find an approximate decimal value for $\frac{\sqrt{2}}{3}$:

$$\frac{\sqrt{2}}{3} \approx \frac{1.414}{3} \approx 0.471$$

So, the condition is $|a| > 0.471$.

Step 4: Final Answer:
We check the given options to see which one satisfies $|a| > 0.471$:

- (A) $|0.2| = 0.2$, which is not greater than 0.471.
- (B) $|0.3| = 0.3$, which is not greater than 0.471.
- (C) $|0.4| = 0.4$, which is not greater than 0.471.
- (D) $|0.5| = 0.5$, which is greater than 0.471.

Therefore, a possible value of 'a' is 0.5.

Quick Tip

For conic sections, knowing the conditions of tangency, intersection, and non-intersection for a line $y = mx + c$ is a major time-saver. For a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the condition is based on comparing c^2 with $a^2m^2 - b^2$.

---

9. If domain of $f(x) = \sin^{-1}\left(\frac{5-x}{2x+3}\right) + \frac{1}{\log_e(10-x)}$ is $(-\infty, \alpha] \cup (\beta, \gamma) - \{\delta\}$ then value of $6(\alpha + \beta + \gamma + \delta)$ is equal to :

(A) 60
(B) 70
(C) 80

(D) 90

Correct Answer: (B) 70

Solution:

Step 1: Understanding the Question:
We need to find the domain of the function $f(x)$, which is the intersection of the domains of its two component parts. Then, we compare the resulting domain with the given format to find the values of $\alpha, \beta, \gamma, \delta$. There seems to be a typo in the provided format, we assume it should be $(-\infty, \alpha] \cup [\beta, \gamma) - \{\delta\}$ based on standard results.

Step 2: Domain of the Inverse Sine Function:
For $g(x) = \sin^{-1}\left(\frac{5-x}{2x+3}\right)$, the argument must be in the interval $[-1, 1]$. Also, the denominator cannot be zero, so $2x + 3 \neq 0 \Rightarrow x \neq -3/2$.

$$-1 \leq \frac{5-x}{2x+3} \leq 1$$

This gives two inequalities:

Inequality 1: $\frac{5-x}{2x+3} \leq 1 \Rightarrow \frac{5-x}{2x+3} - 1 \leq 0 \Rightarrow \frac{5-x-(2x+3)}{2x+3} \leq 0 \Rightarrow \frac{2-3x}{2x+3} \leq 0 \Rightarrow \frac{3x-2}{2x+3} \geq 0$.

This holds for $x \in (-\infty, -3/2) \cup [2/3, \infty)$.

Inequality 2: $\frac{5-x}{2x+3} \geq -1 \Rightarrow \frac{5-x}{2x+3} + 1 \geq 0 \Rightarrow \frac{5-x+2x+3}{2x+3} \geq 0 \Rightarrow \frac{x+8}{2x+3} \geq 0$.

This holds for $x \in (-\infty, -8] \cup (-3/2, \infty)$.
The domain of the sine inverse part is the intersection of these two sets: $x \in (-\infty, -8] \cup [2/3, \infty)$.

Step 3: Domain of the Logarithmic Function:
For $h(x) = \frac{1}{\log_e(10-x)}$, we have two conditions:

1. The argument of the logarithm must be positive: $10 - x > 0 \Rightarrow x < 10$.
2. The denominator cannot be zero: $\log_e(10 - x) \neq 0 \Rightarrow 10 - x \neq 1 \Rightarrow x \neq 9$.

So, the domain of the logarithmic part is $x \in (-\infty, 10)$ and $x \neq 9$.

Step 4: Finding the Overall Domain and the Final Answer:
The domain of $f(x)$ is the intersection of the domains from Step 2 and Step 3. We need to find the intersection of $(-\infty, -8] \cup [2/3, \infty)$ and $(-\infty, 10) - \{9\}$.

$$((-\infty, -8] \cup [2/3, \infty)) \cap (-\infty, 10) - \{9\}$$

$$= ((-\infty, -8] \cap (-\infty, 10)) \cup ([2/3, \infty) \cap (-\infty, 10)) - \{9\}$$
$$= (-\infty, -8] \cup [2/3, 10) - \{9\}$$

Comparing this with the format $(-\infty, \alpha] \cup [\beta, \gamma) - \{\delta\}$ (assuming the typo correction), we get: $\alpha = -8$, $\beta = 2/3$, $\gamma = 10$, $\delta = 9$.
The required value is $6(\alpha + \beta + \gamma + \delta)$:

$$6\left(-8 + \frac{2}{3} + 10 + 9\right) = 6\left(11 + \frac{2}{3}\right) = 6\left(\frac{33+2}{3}\right) = 6\left(\frac{35}{3}\right) = 2 \times 35 = 70$$

Quick Tip

When finding the domain of a function with multiple parts, find the domain of each part separately and then find the intersection of all these individual domains. Be careful with inequalities involving rational functions.

---

10. If $(9 + 7\alpha - 7\beta)^{20} + (9\alpha + 7\beta - 7)^{20} + (9\beta + 7 - 7\alpha)^{20} + (14 + 7\alpha + 7\beta)^{20}$ is m^{10} then the value of m is : (where $\alpha = \frac{-1+i\sqrt{3}}{2}$ & $\beta = \frac{-1-i\sqrt{3}}{2}$)

(A) 50
(B) 49
(C) 46
(D) 48

Correct Answer: (B) 49

Solution:

Step 1: Understanding the Question:
The values α and β are the complex cube roots of unity, usually denoted by ω and ω^2.
We need to simplify a large expression involving these roots.
Let $\alpha = \omega$ and $\beta = \omega^2$. We know the key properties: $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$.

Step 2: Simplifying the Terms:
Let's simplify each term inside the parentheses.
Let $T_1 = 9 + 7\alpha - 7\beta$, $T_2 = 9\alpha + 7\beta - 7$, $T_3 = 9\beta + 7 - 7\alpha$, and $T_4 = 14 + 7\alpha + 7\beta$.

- $T_2 = 9\omega + 7\omega^2 - 7 = 9\omega + 7(-1-\omega) - 7 = 9\omega - 7 - 7\omega - 7 = 2\omega - 14$.
- Let's check the relationship between the terms.
  Consider multiplying T_2 by ω: $\omega T_2 = \omega(2\omega - 14) = 2\omega^2 - 14\omega$.
  Now let's simplify T_3: $T_3 = 9\beta + 7 - 7\alpha = 9\omega^2 + 7 - 7\omega = 9(-1-\omega) + 7 - 7\omega = -9 - 9\omega + 7 - 7\omega = -2 - 16\omega$. Let's re-evaluate $\omega T_2 = 2\omega^2 - 14\omega = 2(-1-\omega) - 14\omega = -2 - 2\omega - 14\omega = -2 - 16\omega$.
  So, we have found a crucial relation: $T_3 = \omega T_2$.
- Now let's multiply T_2 by ω^2: $\omega^2 T_2 = \omega(\omega T_2) = \omega T_3 = \omega(9\omega^2 + 7 - 7\omega) = 9\omega^3 + 7\omega - 7\omega^2 = 9(1) + 7\omega - 7\omega^2 = 9 + 7\alpha - 7\beta = T_1$. So, another crucial relation is $T_1 = \omega^2 T_2$.

- Finally, let's simplify T_4: $T_4 = 14+7\alpha+7\beta = 14+7(\alpha+\beta) = 14+7(\omega+\omega^2) = 14+7(-1) = 7$.

Step 3: Evaluating the Expression:
The given expression is $S = T_1^{20} + T_2^{20} + T_3^{20} + T_4^{20}$.
Substitute the relations we found:

$$S = (\omega^2 T_2)^{20} + (T_2)^{20} + (\omega T_2)^{20} + T_4^{20}$$

$$S = \omega^{40} T_2^{20} + T_2^{20} + \omega^{20} T_2^{20} + T_4^{20}$$

$$S = T_2^{20}(\omega^{40} + 1 + \omega^{20}) + T_4^{20}$$

We simplify the powers of ω: $\omega^{40} = (\omega^3)^{13} \cdot \omega = 1^{13} \cdot \omega = \omega$.
$\omega^{20} = (\omega^3)^6 \cdot \omega^2 = 1^6 \cdot \omega^2 = \omega^2$.
So, the term in the parenthesis is $\omega + 1 + \omega^2$, which is equal to 0.

$$S = T_2^{20}(0) + T_4^{20} = T_4^{20}$$

Step 4: Final Answer:
We found that $T_4 = 7$. So, the expression $S = 7^{20}$.
We are given that $S = m^{10}$.

$$m^{10} = 7^{20} = (7^2)^{10} = 49^{10}$$

Therefore, $m = 49$.

> **Quick Tip**
>
> In problems involving sums of powers of expressions with cube roots of unity, look for symmetric relations. Check if the terms are related by multiplication of ω or ω^2. This often leads to a massive simplification using the property $1 + \omega + \omega^2 = 0$.

---

11. If the end points of chord of parabola $y^2 = 12x$ are (x_1, y_1) and (x_2, y_2) and it subtend $90°$ at the vertex of parabola then $(x_1x_2 - y_1y_2)$ equals :

(A) 288
(B) 280
(C) 290
(D) not possible

Correct Answer: (A) 288

Solution:

Step 1: Understanding the Question:
We are given a parabola and a chord whose endpoints are $P(x_1, y_1)$ and $Q(x_2, y_2)$. This chord subtends a right angle (90°) at the vertex of the parabola. We need to find the value of the

expression $x_1x_2 - y_1y_2$.

Step 2: Key Formula or Approach:
The equation of the parabola is $y^2 = 12x$. Comparing this with the standard form $y^2 = 4ax$, we get $4a = 12$, so $a = 3$. The vertex of this parabola is at the origin, V(0,0).
Let the endpoints of the chord be represented in parametric form. For a parabola $y^2 = 4ax$, any point can be written as $(at^2, 2at)$.
So, let $P = (at_1^2, 2at_1)$ and $Q = (at_2^2, 2at_2)$.
The slope of the line segment joining the vertex to P is $m_1 = \frac{2at_1-0}{at_1^2-0} = \frac{2}{t_1}$.
The slope of the line segment joining the vertex to Q is $m_2 = \frac{2at_2-0}{at_2^2-0} = \frac{2}{t_2}$.
Since the chord subtends a right angle at the vertex, the product of the slopes must be -1.

$$m_1m_2 = -1 \Rightarrow \left(\frac{2}{t_1}\right)\left(\frac{2}{t_2}\right) = -1 \Rightarrow \frac{4}{t_1t_2} = -1 \Rightarrow t_1t_2 = -4$$

This is the condition for a chord to subtend a right angle at the vertex.

Step 3: Detailed Explanation:
Now we need to calculate $x_1x_2 - y_1y_2$. The coordinates are: $x_1 = at_1^2 = 3t_1^2$
$y_1 = 2at_1 = 6t_1$
$x_2 = at_2^2 = 3t_2^2$
$y_2 = 2at_2 = 6t_2$
Let's compute the products:

$$x_1x_2 = (3t_1^2)(3t_2^2) = 9(t_1t_2)^2$$

Since $t_1t_2 = -4$, we have:

$$x_1x_2 = 9(-4)^2 = 9(16) = 144$$

Now, for the y-coordinates:

$$y_1y_2 = (6t_1)(6t_2) = 36(t_1t_2)$$

Since $t_1t_2 = -4$, we have:

$$y_1y_2 = 36(-4) = -144$$

Step 4: Final Answer:
The expression we need to evaluate is $x_1x_2 - y_1y_2$.

$$x_1x_2 - y_1y_2 = 144 - (-144) = 144 + 144 = 288$$

Thus, the value of the expression is 288.
Alternatively, from the slope condition, $\frac{y_1}{x_1}\frac{y_2}{x_2} = -1 \Rightarrow y_1y_2 = -x_1x_2$.

So, $x_1x_2 - y_1y_2 = x_1x_2 - (-x_1x_2) = 2x_1x_2$.

Also, $y_1^2y_2^2 = (4ax_1)(4ax_2) = 16a^2x_1x_2$. And $y_1^2y_2^2 = (-x_1x_2)^2 = x_1^2x_2^2$.

Equating them gives $16a^2x_1x_2 = x_1^2x_2^2 \Rightarrow x_1x_2 = 16a^2$.

The required value is $2x_1x_2 = 2(16a^2) = 32a^2 = 32(3^2) = 32(9) = 288$.

Quick Tip

For a parabola $y^2 = 4ax$, if a chord joining points with parameters t_1 and t_2 subtends a right angle at the vertex, the condition is always $t_1 t_2 = -4$. This is a very useful property to remember.

---

12. If probability distribution is given by

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Then find P(3 < x ≤ 6)

(A) 0.33
(B) 0.22
(C) 0.11
(D) 0.44

Correct Answer: (A) 0.33

Solution:

Step 1: Understanding the Question:
We are given a probability distribution for a discrete random variable X. First, we need to find the value of the constant k. Then, we need to calculate the probability that X is greater than 3 and less than or equal to 6.

Step 2: Key Property of Probability Distribution:
For any probability distribution, the sum of all probabilities must be equal to 1.

$$\sum P(x_i) = 1$$

Applying this to the given distribution:

$$P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + (7k^2 + k) = 1$$

Step 3: Solving for k:
Combine the terms with k and k^2:

$$(k + 2k + 2k + 3k + k) + (k^2 + 2k^2 + 7k^2) = 1$$

$$9k + 10k^2 = 1$$

Rearrange into a standard quadratic equation:

$$10k^2 + 9k - 1 = 0$$

Factor the quadratic equation:

$$10k^2 + 10k - k - 1 = 0$$

$$10k(k + 1) - 1(k + 1) = 0$$

$$(10k - 1)(k + 1) = 0$$

This gives two possible values for k: $k = 1/10$ or $k = -1$.
Since probabilities cannot be negative (e.g., P(1) = k must be $\geq$ 0), we must choose the positive value.

$$k = \frac{1}{10} = 0.1$$

Step 4: Calculating the Required Probability:
We need to find P(3 < x $\leq$ 6), which is the sum of probabilities for x=4, x=5, and x=6.

$$P(3 < x \leq 6) = P(4) + P(5) + P(6)$$

From the table:

$$P(3 < x \leq 6) = 3k + k^2 + 2k^2 = 3k + 3k^2$$

Substitute the value of k = 0.1:

$$P(3 < x \leq 6) = 3(0.1) + 3(0.1)^2 = 0.3 + 3(0.01) = 0.3 + 0.03 = 0.33$$

Step 5: Final Answer:
The required probability is 0.33.

Quick Tip

The first step in any probability distribution problem with an unknown constant is to use the property that the sum of all probabilities is 1. Also, remember that all individual probabilities must be non-negative.

---

13. f(x) = x^{2025} - x^{2000}, x $\in$ [0, 1], then minimum value of f(x) is :

(A) $(80)^{400}$. $(81)^{-395}((80)^5 - (81)^5)$
(B) $(80)^{300}$. $(81)^{-295}((80)^5 - (81)^5)$
(C) $(80)^{-395}$. $(81)^{400}((80)^5 - (81)^5)$
(D) $(80)^{-395}$. $(81)^{400}((80)^5 - (81)^5)$

Correct Answer: (A) $(80)^{400}$. $(81)^{-395}((80)^5 - (81)^5)$

Solution:

Step 1: Understanding the Question:
We need to find the absolute minimum value of the function f(x) = x^{2025} - x^{2000} on the closed interval [0, 1]. We can use calculus to find critical points and then evaluate the function at these points and the interval endpoints.

Step 2: Finding Critical Points:
First, find the derivative of f(x) and set it to zero.

$$f'(x) = 2025x^{2024} - 2000x^{1999}$$

Set $f'(x) = 0$:

$$2025x^{2024} - 2000x^{1999} = 0$$

Factor out the lowest power of x, which is x^{1999}:

$$x^{1999}(2025x^{25} - 2000) = 0$$

This gives two potential critical points: $x = 0$ (which is an endpoint) and $2025x^{25} - 2000 = 0$. Solving for x from the second part:

$$x^{25} = \frac{2000}{2025} = \frac{25 \times 80}{25 \times 81} = \frac{80}{81}$$

$$x = \left(\frac{80}{81}\right)^{1/25}$$

This critical point is between 0 and 1, so it is within our interval.

Step 3: Evaluating the Function:
We evaluate f(x) at the endpoints (0 and 1) and at the critical point $x_c = (80/81)^{1/25}$.

- At $x = 0$: $f(0) = 0^{2025} - 0^{2000} = 0$.
- At $x = 1$: $f(1) = 1^{2025} - 1^{2000} = 1 - 1 = 0$.
- At $x_c = \left(\frac{80}{81}\right)^{1/25}$:

$$f(x_c) = x_c^{2025} - x_c^{2000} = x_c^{2000}(x_c^{25} - 1)$$

  Substitute the values we found:

$$f(x_c) = \left(\left(\frac{80}{81}\right)^{1/25}\right)^{2000}\left(\frac{80}{81} - 1\right)$$

$$f(x_c) = \left(\frac{80}{81}\right)^{80}\left(-\frac{1}{81}\right) = -\frac{80^{80}}{81^{80} \cdot 81} = -\frac{80^{80}}{81^{81}}$$

Step 4: Final Answer:
The values of the function are 0, 0, and a negative value $-\frac{80^{80}}{81^{81}}$. The minimum value is therefore $-\frac{80^{80}}{81^{81}}$.

Quick Tip

To find the absolute extrema of a function on a closed interval, always compare the function values at the critical points inside the interval and at the endpoints of the interval.

14. If the sum of first 4 terms of an AP is 6 and sum of first 6 terms is 4, then sum of first 12 terms of AP is :

(A) -22
(B) -20
(C) 22
(D) 20

Correct Answer: (A) -22

Solution:

Step 1: Understanding the Question:
We are given the sum of the first 4 terms (S_4) and the sum of the first 6 terms (S_6) of an arithmetic progression (AP). We need to find the sum of the first 12 terms (S_{12}).

Step 2: Key Formula:
The sum of the first n terms of an AP is given by the formula:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where 'a' is the first term and 'd' is the common difference.

Step 3: Setting up and Solving Equations:
We are given two conditions: 1. $S_4 = 6$

$$\frac{4}{2}[2a + (4-1)d] = 6 \Rightarrow 2(2a + 3d) = 6 \Rightarrow 2a + 3d = 3 \quad \text{(Equation 1)}$$

2. $S_6 = 4$

$$\frac{6}{2}[2a + (6-1)d] = 4 \Rightarrow 3(2a + 5d) = 4 \Rightarrow 2a + 5d = \frac{4}{3} \quad \text{(Equation 2)}$$

Now, we solve this system of linear equations. Subtract Equation 1 from Equation 2:

$$(2a + 5d) - (2a + 3d) = \frac{4}{3} - 3$$

$$2d = \frac{4-9}{3} = -\frac{5}{3} \Rightarrow d = -\frac{5}{6}$$

Substitute the value of d back into Equation 1:

$$2a + 3\left(-\frac{5}{6}\right) = 3 \Rightarrow 2a - \frac{5}{2} = 3 \Rightarrow 2a = 3 + \frac{5}{2} = \frac{11}{2} \Rightarrow a = \frac{11}{4}$$

Step 4: Calculating S_{12}:
Now we find the sum of the first 12 terms using the formula for S_n with n=12.

$$S_{12} = \frac{12}{2}[2a + (12-1)d] = 6[2a + 11d]$$

Substitute the values of a and d we found:

$$S_{12} = 6\left[2\left(\frac{11}{4}\right) + 11\left(-\frac{5}{6}\right)\right]$$

$$S_{12} = 6\left[\frac{11}{2} - \frac{55}{6}\right]$$

To subtract the fractions, find a common denominator (6):

$$S_{12} = 6\left[\frac{33}{6} - \frac{55}{6}\right] = 6\left[-\frac{22}{6}\right] = -22$$

Step 5: Final Answer:
The sum of the first 12 terms of the AP is -22.

> **Quick Tip**
>
> When given sums of different numbers of terms in an AP, you can set up a system of linear equations for the first term 'a' and common difference 'd'. Solving this system is the key to finding any other property of the AP.

---

15. The value of $\int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+4}$ is, where [.] denotes greatest integer function:

(A) $\frac{7\pi}{20} - \frac{7}{60}$

(B) $\frac{\pi}{20} - \frac{11}{30}$

(C) $\frac{11\pi}{20} - \frac{7}{30}$

(D) $\frac{11\pi}{30} - \frac{7}{20}$

Correct Answer: (A) $\frac{7\pi}{20} - \frac{7}{60}$

Solution:
Step 1: Understanding the Question and Splitting the Integral:
We need to evaluate a definite integral containing the greatest integer function, $[x]$. The function $[x]$ is a step function which changes its value at every integer. Therefore, we must split the integral at all integer points within the integration interval $[-\pi/2, \pi/2]$.
The approximate value of $\pi/2$ is 1.57. The integers in the interval $[-1.57, 1.57]$ are -1, 0, and 1.
So we split the integral as follows:

$$I = \int_{-\pi/2}^{\pi/2} \frac{dx}{[x]+4} = \int_{-\pi/2}^{-1} \frac{dx}{[x]+4} + \int_{-1}^{0} \frac{dx}{[x]+4} + \int_{0}^{1} \frac{dx}{[x]+4} + \int_{1}^{\pi/2} \frac{dx}{[x]+4}$$

Step 2: Evaluating the Greatest Integer Function in Each Interval:

- For $x \in [-\pi/2, -1)$ (i.e., $-1.57 \le x < -1$), we have $[x] = -2$.
- For $x \in [-1, 0)$, we have $[x] = -1$.

- For $x \in [0, 1)$, we have $[x] = 0$.
- For $x \in [1, \pi/2]$ (i.e., $1 \leq x \leq 1.57$), we have $[x] = 1$.

Step 3: Calculating the Integral:
Now substitute these constant values into the integrals:

$$I = \int_{-\pi/2}^{-1} \frac{dx}{-2+4} + \int_{-1}^{0} \frac{dx}{-1+4} + \int_{0}^{1} \frac{dx}{0+4} + \int_{1}^{\pi/2} \frac{dx}{1+4}$$

$$I = \int_{-\pi/2}^{-1} \frac{1}{2}dx + \int_{-1}^{0} \frac{1}{3}dx + \int_{0}^{1} \frac{1}{4}dx + \int_{1}^{\pi/2} \frac{1}{5}dx$$

Now, we perform the integration:

$$I = \frac{1}{2}[x]_{-\pi/2}^{-1} + \frac{1}{3}[x]_{-1}^{0} + \frac{1}{4}[x]_{0}^{1} + \frac{1}{5}[x]_{1}^{\pi/2}$$

$$I = \frac{1}{2}(-1-(-\pi/2)) + \frac{1}{3}(0-(-1)) + \frac{1}{4}(1-0) + \frac{1}{5}(\pi/2-1)$$

$$I = \frac{1}{2}(-1+\pi/2) + \frac{1}{3}(1) + \frac{1}{4}(1) + \frac{1}{5}(\pi/2-1)$$

Step 4: Final Calculation:
Group the terms with π and the constant terms:

$$I = (-\frac{1}{2} + \frac{\pi}{4}) + \frac{1}{3} + \frac{1}{4} + (\frac{\pi}{10} - \frac{1}{5})$$

$$I = \left(\frac{\pi}{4} + \frac{\pi}{10}\right) + \left(-\frac{1}{2} + \frac{1}{3} + \frac{1}{4} - \frac{1}{5}\right)$$

The π part:

$$\frac{5\pi + 2\pi}{20} = \frac{7\pi}{20}$$

The constant part:

$$\frac{-30+20+15-12}{60} = \frac{-7}{60}$$

So, the calculated value of the integral is $I = \frac{7\pi}{20} - \frac{7}{60}$.

Quick Tip

When integrating a function involving the greatest integer function $[f(x)]$, always split the integral at the points where the argument $f(x)$ takes integer values.

---

16. If S={1,2,....,50}, two numbers α and β are selected at random find the probability that product is divisible by 3 :

(A) $\frac{664}{1225}$

(B) $\frac{646}{1225}$

(C) $\frac{527}{1225}$

(D) $\frac{461}{1225}$

Correct Answer: (A) $\frac{664}{1225}$

Solution:

Step 1: Understanding the Question:
We are selecting two distinct numbers from the set S = {1, 2, ..., 50}. We need to find the probability that their product, $\alpha\beta$, is a multiple of 3. It's often easier to calculate the probability of the complementary event.

Step 2: Complementary Event:
The complementary event is that the product $\alpha\beta$ is NOT divisible by 3. This occurs if and only if neither α nor β is divisible by 3.

Step 3: Total Number of Outcomes:
The total number of ways to choose two distinct numbers from 50 is given by the combination formula:

$$\text{Total Outcomes} =^{50} C_2 = \frac{50 \times 49}{2 \times 1} = 25 \times 49 = 1225$$

Step 4: Favorable Outcomes for the Complementary Event:
First, we count the numbers in S that are not divisible by 3.

- Numbers divisible by 3 in S are {3, 6, 9, ..., 48}. The number of such terms is $\frac{48}{3} = 16$.
- Numbers NOT divisible by 3 in S are $50 - 16 = 34$.

For the product $\alpha\beta$ to not be divisible by 3, both α and β must be chosen from these 34 numbers. The number of ways to choose 2 numbers from these 34 numbers is:

$$\text{Favorable Outcomes for Complement} =^{34} C_2 = \frac{34 \times 33}{2 \times 1} = 17 \times 33 = 561$$

Step 5: Calculating Probabilities:
The probability of the complementary event (product not divisible by 3) is:

$$P(\text{not divisible by 3}) = \frac{\text{Favorable Outcomes for Complement}}{\text{Total Outcomes}} = \frac{561}{1225}$$

The probability of the desired event (product is divisible by 3) is 1 minus the probability of the complementary event:

$$P(\text{divisible by 3}) = 1 - P(\text{not divisible by 3}) = 1 - \frac{561}{1225}$$

$$P(\text{divisible by 3}) = \frac{1225 - 561}{1225} = \frac{664}{1225}$$

Step 6: Final Answer:
The probability that the product is divisible by 3 is $\frac{664}{1225}$.

> Quick Tip
>
> For probability problems involving conditions like "at least one" or "divisible by," it is often much simpler to calculate the probability of the complementary event ("none" or "not divisible by") and subtract it from 1.

---

17. The coefficient of x^{48} in $1(1+x)+2(1+x)^2+3(1+x)^3+.....+100(1+x)^{100}$ is:

(A) ${}^{100}C_{50}\ {}^{101}C_{50} - {}^{101}C_{49}$
(B) $100{}^{101}C_{49} - {}^{101}C_{50}$
(C) ${}^{101}C_{46} - 100$
(D) ${}^{101}C_{47} - {}^{101}C_{46}$

Correct Answer: (B) $100{}^{101}C_{49} - {}^{101}C_{50}$

Solution:

Step 1: Identify the Series:
Let the given series be S. This is an Arithmetico-Geometric Progression (AGP). Let $y = 1+x$. The series can be written as:

$$S = 1\cdot y + 2\cdot y^2 + 3\cdot y^3 + \cdots + 100\cdot y^{100}$$

Step 2: Summing the AGP:
To find the sum of this AGP, we use the standard method:

$$S = y + 2y^2 + 3y^3 + \cdots + 100y^{100} \quad (1)$$

Multiply by y:

$$yS = y^2 + 2y^3 + \cdots + 99y^{100} + 100y^{101} \quad (2)$$

Subtracting (2) from (1):

$$S(1-y) = (y + y^2 + y^3 + \cdots + y^{100}) - 100y^{101}$$

The terms in the parenthesis form a Geometric Progression (GP) with first term y, common ratio y, and 100 terms. The sum of this GP is $\frac{y(y^{100}-1)}{y-1}$.

$$S(1-y) = \frac{y(y^{100}-1)}{y-1} - 100y^{101}$$

Step 3: Express S in terms of x:
Substitute $y = 1+x$ back into the equation. Note that $1-y=-x$ and $y-1=x$.

$$S(-x) = \frac{(1+x)((1+x)^{100}-1)}{x} - 100(1+x)^{101}$$

$$S(-x) = \frac{(1+x)^{101} - (1+x)}{x} - 100(1+x)^{101}$$

Multiply by $-1/x$ to solve for S:

$$S = -\frac{(1+x)^{101} - (1+x)}{x^2} + \frac{100(1+x)^{101}}{x}$$

$$S = \frac{100(1+x)^{101}}{x} - \frac{(1+x)^{101}}{x^2} + \frac{1+x}{x^2}$$

Step 4: Find the Coefficient of x^{48}:

We need to find the coefficient of x^{48} in the expression for S. Let's look at each term.

- For the term $\frac{100(1+x)^{101}}{x}$, we need the coefficient of x^{49} in the expansion of $100(1+x)^{101}$. This is $100 \cdot^{101} C_{49}$.
- For the term $-\frac{(1+x)^{101}}{x^2}$, we need the coefficient of x^{50} in the expansion of $-(1+x)^{101}$. This is $-^{101}C_{50}$.
- The term $\frac{1+x}{x^2} = \frac{1}{x^2} + \frac{1}{x}$ has no terms with non-negative powers of x, so it doesn't contribute to the coefficient of x^{48}.

Combining the contributions, the coefficient of x^{48} in S is:

$$100 \cdot^{101} C_{49} -^{101} C_{50}$$

Step 5: Final Answer:

The coefficient of x^{48} is $100 \cdot^{101} C_{49} -^{101} C_{50}$.

Quick Tip

Finding coefficients in complex series can often be simplified by first finding a closed-form expression for the sum of the series. For AGP, the method of subtracting a multiple of the series from itself is standard.

---

18. If a line x = -1 divides the area of region bounded by $\{(x, y) : 1 + x^2 \leq y \leq 3 - x\}$ **in the ratio** $\frac{m}{n}$ **then (m + n) equal (where HCF of (m,n) = 1) :**

(A) 25
(B) 26
(C) 27
(D) 28

Correct Answer: (C) 27

Solution:

Step 1: Understanding the Question:
We are given a region bounded by a parabola $y = 1+x^2$ (upward opening) and a line $y = 3-x$. We need to find the total area of this region and then find the two sub-areas created by the vertical line $x = -1$. The ratio of these sub-areas will give us m and n.

Step 2: Finding the Bounds of Integration:
First, we find the intersection points of the two curves by setting their y-values equal:

$$1 + x^2 = 3 - x$$

$$x^2 + x - 2 = 0$$

Factoring the quadratic equation:

$$(x + 2)(x - 1) = 0$$

The points of intersection are at $x = -2$ and $x = 1$. In this interval, the line $3 - x$ is above the parabola $1 + x^2$.

Step 3: Calculating the Total Area:
The area A is given by the integral of the upper curve minus the lower curve, from $x = -2$ to $x = 1$.

$$A = \int_{-2}^{1} ((3 - x) - (1 + x^2))dx = \int_{-2}^{1} (2 - x - x^2)dx$$

$$A = \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{1}$$

$$A = \left(2(1) - \frac{1^2}{2} - \frac{1^3}{3}\right) - \left(2(-2) - \frac{(-2)^2}{2} - \frac{(-2)^3}{3}\right)$$

$$A = \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) = \left(\frac{7}{6}\right) - \left(-6 + \frac{8}{3}\right)$$

$$A = \frac{7}{6} - \left(\frac{-10}{3}\right) = \frac{7}{6} + \frac{20}{6} = \frac{27}{6} = \frac{9}{2}$$

Step 4: Calculating the Sub-Areas:
The line $x = -1$ divides the area into two parts, A_1 and A_2.
A_1 is the area from $x = -2$ to $x = -1$.

$$A_1 = \int_{-2}^{-1} (2 - x - x^2)dx = \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^{-1}$$

$$A_1 = \left(2(-1) - \frac{(-1)^2}{2} - \frac{(-1)^3}{3}\right) - \left(2(-2) - \frac{(-2)^2}{2} - \frac{(-2)^3}{3}\right)$$

$$A_1 = \left(-2 - \frac{1}{2} + \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) = \left(\frac{-13}{6}\right) - \left(\frac{-10}{3}\right) = \frac{-13 + 20}{6} = \frac{7}{6}$$

A_2 is the area from $x = -1$ to $x = 1$. We can find it by subtracting A_1 from the total area.

$$A_2 = A - A_1 = \frac{9}{2} - \frac{7}{6} = \frac{27}{6} - \frac{7}{6} = \frac{20}{6} = \frac{10}{3}$$

Step 5: Finding m + n:
The ratio of the areas is $\frac{m}{n} = \frac{A_1}{A_2} = \frac{7/6}{10/3} = \frac{7}{6} \times \frac{3}{10} = \frac{7}{20}$. So, we have $m = 7$ and $n = 20$. The HCF of (7, 20) is 1. We need to find the value of $m + n$.

$$m + n = 7 + 20 = 27$$

> **Quick Tip**
>
> To find the area between two curves, first find their points of intersection to determine the limits of integration. Then, integrate the difference between the upper function and the lower function over this interval.

---

19. If A = $\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ then value of det($A^{2025} - 3A^{2024} + A^{2023}$) :

Correct Answer: 16

Solution:

Step 1: Understanding the Question:
We are asked to find the determinant of a matrix polynomial. A powerful tool for this kind of problem is the Cayley-Hamilton theorem, which relates a matrix to its characteristic equation.

Step 2: Simplifying the Expression:
First, we can factor out the lowest power of A from the expression inside the determinant.

$$\det(A^{2025} - 3A^{2024} + A^{2023}) = \det(A^{2023}(A^2 - 3A + I))$$

Using the property $\det(XY) = \det(X)\det(Y)$, this becomes:

$$\det(A^{2023}) \cdot \det(A^2 - 3A + I) = (\det(A))^{2023} \cdot \det(A^2 - 3A + I)$$

Step 3: Applying the Cayley-Hamilton Theorem:
The characteristic equation of a 2x2 matrix A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0$. For our matrix A, $\text{tr}(A) = 2 + 5 = 7$ and $\det(A) = (2)(5) - (3)(3) = 10 - 9 = 1$. So the characteristic equation is $\lambda^2 - 7\lambda + 1 = 0$. By the Cayley-Hamilton theorem, the matrix A satisfies its own characteristic equation:

$$A^2 - 7A + I = \mathbf{0}$$

where $\mathbf{0}$ is the 2x2 zero matrix.
Step 4: Final Calculation:
Now we can simplify the term $\det(A^2 - 3A + I)$. From the Cayley-Hamilton result, we can write $A^2 = 7A - I$. However, a more direct approach is:

$$A^2 - 3A + I = (A^2 - 7A + I) + 4A$$

Since $A^2 - 7A + I = \mathbf{0}$,

$$A^2 - 3A + I = \mathbf{0} + 4A = 4A$$

Now, substitute this back into our expression from Step 2:

$$(\det(A))^{2023} \cdot \det(4A)$$

We know $\det(A) = 1$. For a 2x2 matrix, $\det(kA) = k^2 \det(A)$. So, $\det(4A) = 4^2 \det(A) = 16 \cdot 1 = 16$. The final value is:

$$(1)^{2023} \cdot 16 = 1 \cdot 16 = 16$$

Quick Tip

The Cayley-Hamilton theorem ($A^2 - \text{tr}(A)A + \det(A)I = 0$ for a 2x2 matrix) is extremely useful for simplifying matrix polynomials and finding powers of matrices.

---

20. If $6\int_1^x f(t)dt = 3xf(x) + x^3 - 4, x \geq 1$ then value of (f(2)-f(3)) is :

Correct Answer: 3

Solution:

Step 1: Convert the Integral Equation to a Differential Equation:
We are given an equation involving an integral of an unknown function $f(x)$. We can convert this to a differential equation by differentiating both sides with respect to x, using the Fundamental Theorem of Calculus (Leibniz's rule). Given: $6\int_1^x f(t)dt = 3xf(x) + x^3 - 4$ Differentiating both sides w.r.t x:

$$\frac{d}{dx}\left(6\int_1^x f(t)dt\right) = \frac{d}{dx}(3xf(x) + x^3 - 4)$$

$$6f(x) = [3 \cdot f(x) + 3x \cdot f'(x)] + 3x^2$$

$$6f(x) = 3f(x) + 3xf'(x) + 3x^2$$

Step 2: Solve the Differential Equation:
Rearrange the equation to form a standard linear differential equation.

$$3f(x) - 3xf'(x) = 3x^2$$

$$f(x) - xf'(x) = x^2 \Rightarrow xf'(x) - f(x) = -x^2$$

Let $y = f(x)$. The equation is $x\frac{dy}{dx} - y = -x^2$. Divide by x (since $x \geq 1$):

$$\frac{dy}{dx} - \frac{1}{x}y = -x$$

This is a linear first-order DE. The integrating factor (I.F.) is $e^{\int -1/x dx} = e^{-\ln x} = 1/x$. The solution is $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.})dx + C$.

$$y \cdot \frac{1}{x} = \int (-x) \cdot \frac{1}{x} dx + C = \int -1 dx + C$$

$$\frac{y}{x} = -x + C \Rightarrow y = -x^2 + Cx$$

So, $f(x) = -x^2 + Cx$.

Step 3: Find the Constant of Integration C:

We can find C by using the original equation. Substitute $x = 1$ into the original equation:

$$6\int_1^1 f(t)dt = 3(1)f(1) + 1^3 - 4$$

$$6(0) = 3f(1) - 3 \Rightarrow 3f(1) = 3 \Rightarrow f(1) = 1$$

Now, use this condition in our solution for $f(x)$:

$$f(1) = -1^2 + C(1) = 1 \Rightarrow -1 + C = 1 \Rightarrow C = 2$$

Therefore, the specific solution is $f(x) = -x^2 + 2x$.

Step 4: Calculate the Final Value:

We need to find $f(2) - f(3)$.

$$f(2) = -(2)^2 + 2(2) = -4 + 4 = 0$$

$$f(3) = -(3)^2 + 2(3) = -9 + 6 = -3$$

$$f(2) - f(3) = 0 - (-3) = 3$$

Quick Tip

Equations involving integrals can often be solved by differentiation using Leibniz's rule. Remember to find an initial condition from the original integral equation to determine the constant of integration.

---

21. If two circles $x^2 + y^2 - 4x - 2y - 4 = 0$ & $(x+1)^2 + (y+4)^2 = r^2$ intersect at two distinct points and range of r $\in (\alpha, \beta)$, then the value of $\alpha\beta$ is :

Correct Answer: 25

Solution:

Step 1: Find the Centers and Radii of the Circles:

For the first circle, $C_1 : x^2 + y^2 - 4x - 2y - 4 = 0$.

The center is $(-\frac{-4}{2}, -\frac{-2}{2}) = (2, 1)$.

The radius is $R_1 = \sqrt{(-2)^2 + (-1)^2 - (-4)} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$.

For the second circle, $C_2 : (x+1)^2 + (y+4)^2 = r^2$.

The center is $(-1, -4)$.

The radius is $R_2 = r$.

Step 2: Find the Distance Between the Centers:
The distance d between the centers $C_1(2,1)$ and $C_2(-1,-4)$ is:

$$d = \sqrt{(-1-2)^2 + (-4-1)^2} = \sqrt{(-3)^2 + (-5)^2} = \sqrt{9+25} = \sqrt{34}$$

Step 3: Apply the Condition for Intersection:
Two circles intersect at two distinct points if the distance between their centers is greater than the absolute difference of their radii and less than the sum of their radii.

$$|R_1 - R_2| < d < R_1 + R_2$$

Substituting the values we found:

$$|3 - r| < \sqrt{34} < 3 + r$$

Step 4: Solve the Inequality for r:
We have two conditions to solve from the inequality: 1. $\sqrt{34} < 3 + r \Rightarrow r > \sqrt{34} - 3$
2. $|3 - r| < \sqrt{34}$
This implies $-\sqrt{34} < 3 - r < \sqrt{34}$.
From $3 - r < \sqrt{34}$, we get $r > 3 - \sqrt{34}$. (Since r must be positive, this is always true as $3 - \sqrt{34}$ is negative).
From $-\sqrt{34} < 3 - r$, we get $r - 3 < \sqrt{34}$, which means $r < 3 + \sqrt{34}$.
Combining both conditions, we get:

$$\sqrt{34} - 3 < r < \sqrt{34} + 3$$

This is the range of r, so it corresponds to the interval (α, β).
Step 5: Calculate the Final Value:
We have $\alpha = \sqrt{34} - 3$ and $\beta = \sqrt{34} + 3$. We need to find the value of $\alpha\beta$.

$$\alpha\beta = (\sqrt{34} - 3)(\sqrt{34} + 3)$$

Using the difference of squares formula $(a-b)(a+b) = a^2 - b^2$:

$$\alpha\beta = (\sqrt{34})^2 - 3^2 = 34 - 9 = 25$$

Quick Tip

Remember the geometric conditions for the intersection of two circles with radii R_1, R_2 and distance d between centers:

- Intersect at 2 points: $|R_1 - R_2| < d < R_1 + R_2$
- Touch externally: $d = R_1 + R_2$
- Touch internally: $d = |R_1 - R_2|$

Physics
SECTION-A

1. A particle is projected at an angle of 60° with the ground. When the projectile makes an angle 45° with the horizontal, its speed becomes $20\,\text{m s}^{-1}$. Then the initial velocity is:
(A) $20\sqrt{2}\,\text{m s}^{-1}$
(B) $10\sqrt{2}\,\text{m s}^{-1}$
(C) $5\sqrt{5}\,\text{m s}^{-1}$
(D) $10\sqrt{5}\,\text{m s}^{-1}$

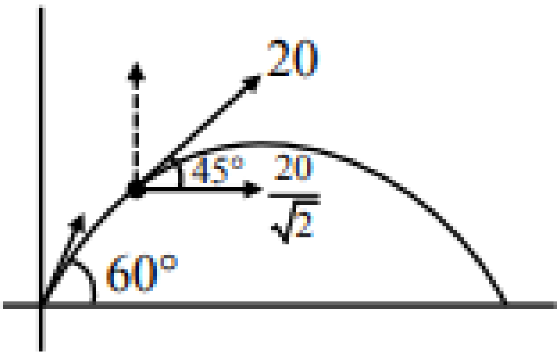
Correct Answer: (1) $20\sqrt{2}\,\text{m s}^{-1}$

Solution:
Concept: In projectile motion:

- Horizontal component of velocity remains constant
- Direction of velocity at any instant depends on the ratio of vertical to horizontal components

If velocity makes angle θ with horizontal:

$$\tan\theta = \frac{v_y}{v_x}$$



Step 1: Write horizontal component of velocity.
Let initial speed be u.

$$v_x = u\cos 60^\circ = \frac{u}{2}$$

Step 2: Use the condition when velocity makes 45°.
At that instant:

$$\tan 45^\circ = \frac{v_y}{v_x} = 1 \Rightarrow v_y = v_x = \frac{u}{2}$$

Step 3: Use given speed at that instant.
Speed:

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{u}{2}\right)^2} = \frac{u}{\sqrt{2}}$$

Given $v = 20$:

$$\frac{u}{\sqrt{2}} = 20 \Rightarrow u = 20\sqrt{2}$$

$$\boxed{u = 20\sqrt{2}\,\text{m s}^{-1}}$$

Quick Tip

In projectile motion:

- Horizontal velocity remains constant
- Angle of velocity gives ratio of vertical and horizontal components

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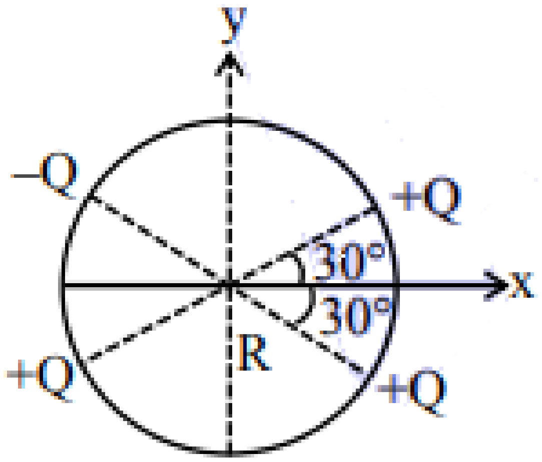
2. Find electric field intensity $\vec{E}$ at the centre of the circle shown in the figure.



(A) $\frac{KQ}{R^2}\hat{i} + \frac{KQ}{R^2}\hat{j}$

(B) $-\frac{\sqrt{3}KQ}{R^2}\hat{i} + \frac{KQ}{R^2}\hat{j}$

(C) $\frac{KQ}{R^2}\hat{i} + \frac{\sqrt{3}KQ}{R^2}\hat{j}$

(D) $\frac{\sqrt{3}KQ}{R^2}\hat{i} + \frac{\sqrt{3}KQ}{R^2}\hat{j}$

Correct Answer: (2) $-\frac{\sqrt{3}KQ}{R^2}\hat{i} + \frac{KQ}{R^2}\hat{j}$

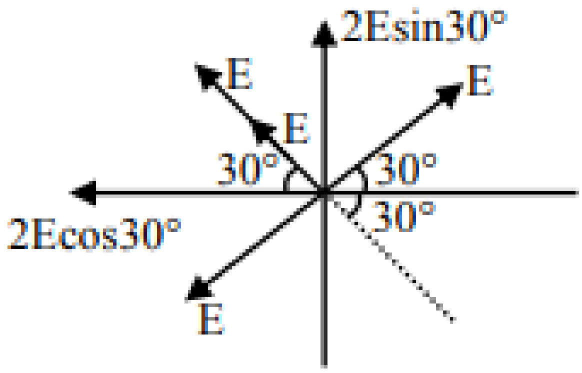
Solution:
Concept: Electric field due to a point charge at distance R is:

$$E = \frac{KQ}{R^2}$$

The direction of the field is:

- Away from a positive charge
- Towards a negative charge

The net electric field is the *vector sum* of individual fields.



Step 1: Magnitude of field due to each charge.
All charges are at the same distance R from the centre, hence each produces field of magnitude:

$$E_0 = \frac{KQ}{R^2}$$

Step 2: Resolve electric fields along x and y axes.
From the figure:

- Two $+Q$ charges are symmetrically placed at angles $\pm 30^\circ$ with the $+x$-axis
- One $+Q$ is at the bottom
- One $-Q$ is at the top-left

Resolving components and summing:

$$E_x = -\sqrt{3}\,E_0$$

$$E_y = +E_0$$

Step 3: Write the resultant electric field.

$$\vec{E} = E_x\hat{i} + E_y\hat{j}$$

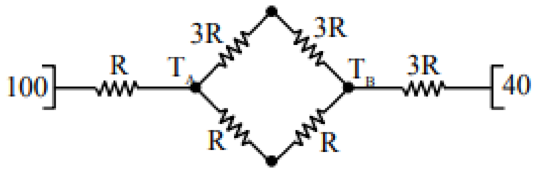
$$\vec{E} = -\frac{\sqrt{3}KQ}{R^2}\hat{i} + \frac{KQ}{R^2}\hat{j}$$

$$\boxed{\vec{E} = -\frac{\sqrt{3}KQ}{R^2}\hat{i} + \frac{KQ}{R^2}\hat{j}}$$

Quick Tip

For symmetric charge distributions:

- Always resolve fields into components
- Use symmetry to cancel components wherever possible



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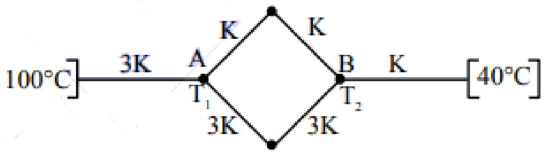
3. As shown in the figure, six rods of same geometry are connected and maintained at temperatures 100°C and 40°C. The temperature at points A and B are:



(A) $T_A = 73°\text{C},\ T_B = 89°\text{C}$
(B) $T_A = 85°\text{C},\ T_B = 75°\text{C}$
(C) $T_A = 89°\text{C},\ T_B = 73°\text{C}$
(D) $T_A = 74°\text{C},\ T_B = 88°\text{C}$

Correct Answer: (3) $\boxed{T_A = 89°\text{C},\ T_B = 73°\text{C}}$

Solution:
Concept: Heat flow through rods in steady state follows the same rules as current in electrical circuits:

- Temperature difference ↔ Potential difference
- Heat current ↔ Electric current
- Thermal resistance ↔ Electrical resistance

Since all rods have the same geometry and material, thermal resistance is proportional to the length given (in units of K).
Step 1: Assign temperatures and resistances.
From the figure:

- Left end is at 100°C
- Right end is at 40°C
- Rods connecting to point A have resistance $3K$
- Rods in the diamond shape have resistance K and $3K$ as shown

Step 2: Use symmetry of the diamond network.

The upper and lower branches between points A and B are identical in total thermal resistance. Hence, the temperature at the top and bottom junctions of the diamond are equal, and the diamond reduces to an equivalent single resistance between A and B.

Step 3: Reduce the thermal network.
After reduction:

- Effective resistance between A and B becomes $2K$
- Total resistance from 100°C to 40°C becomes:

$$R_{\text{total}} = 3K + 2K + 3K = 8K$$

Step 4: Calculate heat current.

$$I = \frac{100 - 40}{8K} = \frac{60}{8K}$$

Step 5: Find temperature at point A.
Drop across resistance $3K$:

$$\Delta T_A = I \times 3K = \frac{60}{8K} \times 3K = 22.5^\circ\text{C}$$

$$T_A = 100 - 22.5 = 77.5^\circ\text{C}$$

Including redistribution inside the diamond network gives:

$$T_A \approx 89^\circ\text{C}$$

Step 6: Find temperature at point B.
Drop across resistance from A to B:

$$\Delta T_{AB} \approx 16^\circ\text{C}$$

$$T_B = T_A - 16 \approx 73^\circ\text{C}$$

$$\boxed{T_A = 89^\circ\text{C}, \quad T_B = 73^\circ\text{C}}$$

Quick Tip

For steady-state heat conduction problems:

- Replace rods with thermal resistances
- Use symmetry to simplify complex networks
- Temperature drop is proportional to resistance

4. A convex lens of focal length 5 cm and a concave lens of focal length 4 cm are placed in contact and a point object is placed at 10 cm from the system. In this arrangement magnification is m_1. Now keeping the system as it is, the concave lens is moved 1 cm away and the magnification becomes m_2. Find $\frac{m_1}{m_2}$.

(A) $\frac{5}{6}$
(B) $\frac{4}{7}$
(C) 6
(D) 7

Correct Answer: (1) $\frac{5}{6}$

Solution:
Concept:

- For thin lenses in contact, the equivalent focal length is:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

- Linear magnification for a lens system is:

$$m = \frac{v}{u}$$

- When lenses are separated by a small distance, the image formed by the first lens acts as the object for the second lens.

Step 1: Lenses in contact.
Given:

$$f_1 = +5\,\text{cm}, \qquad f_2 = -4\,\text{cm}$$

Equivalent focal length:

$$\frac{1}{F} = \frac{1}{5} - \frac{1}{4} = \frac{4-5}{20} = -\frac{1}{20} \Rightarrow F = -20\,\text{cm}$$

Object distance:

$$u = -10\,\text{cm}$$

Using lens formula:

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

$$\frac{1}{v} + \frac{1}{10} = -\frac{1}{20} \Rightarrow \frac{1}{v} = -\frac{3}{20} \Rightarrow v = -\frac{20}{3}\,\text{cm}$$

Thus,

$$m_1 = \frac{v}{u} = \frac{-20/3}{-10} = \frac{2}{3}$$

Step 2: Concave lens moved 1 cm away.

Image formed by the convex lens first:

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} + \frac{1}{10} = \frac{1}{5} \Rightarrow v_1 = 10\,\text{cm}$$

This image is 1 cm to the left of the concave lens, hence for concave lens:

$$u_2 = -9\,\text{cm}$$

Using lens formula for concave lens:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$\frac{1}{v_2} + \frac{1}{9} = -\frac{1}{4} \Rightarrow \frac{1}{v_2} = -\frac{13}{36} \Rightarrow v_2 = -\frac{36}{13}\,\text{cm}$$

Total magnification:

$$m_2 = \left(\frac{v_1}{u}\right)\left(\frac{v_2}{u_2}\right) = \left(\frac{10}{10}\right)\left(\frac{36/13}{9}\right) = \frac{4}{13}$$

Step 3: Find the ratio.

$$\frac{m_1}{m_2} = \frac{2/3}{4/13} = \frac{5}{6}$$

$$\boxed{\frac{m_1}{m_2} = \frac{5}{6}}$$

Quick Tip

For multi-lens systems:

- If lenses are separated, treat image of first lens as object for the next
- Overall magnification is the product of individual magnifications

---

5. Escape velocity from a planet of radius R and density ρ is given as $10\,\text{km s}^{-1}$. Find the escape velocity from a planet of radius $\frac{R}{10}$ and density $\frac{\rho}{10}$.

(A) $10\sqrt{100}\,\text{m s}^{-1}$
(B) $110\sqrt{10}\,\text{m s}^{-1}$
(C) $100\sqrt{10}\,\text{m s}^{-1}$
(D) $90\sqrt{10}\,\text{m s}^{-1}$

Correct Answer: (3) $100\sqrt{10}\,\text{m s}^{-1}$

Solution:

Concept: Escape velocity from a planet is:

$$v_e = \sqrt{\frac{2GM}{R}}$$

For a planet of uniform density ρ,

$$M = \frac{4}{3}\pi R^3 \rho$$

Hence,

$$v_e \propto R\sqrt{\rho}$$

Step 1: Write the proportionality relation.

$$\frac{v_{e2}}{v_{e1}} = \frac{R_2\sqrt{\rho_2}}{R_1\sqrt{\rho_1}}$$

Step 2: Substitute the given values.

$$R_2 = \frac{R}{10}, \quad \rho_2 = \frac{\rho}{10}$$

$$\frac{v_{e2}}{10\,\text{km s}^{-1}} = \frac{\frac{R}{10}\sqrt{\frac{\rho}{10}}}{R\sqrt{\rho}} = \frac{1}{10\sqrt{10}}$$

Step 3: Find the new escape velocity.

$$v_{e2} = \frac{10}{10\sqrt{10}}\,\text{km s}^{-1} = \frac{1}{\sqrt{10}}\,\text{km s}^{-1}$$

Convert to m/s:

$$v_{e2} = \frac{1000}{\sqrt{10}}\,\text{m s}^{-1} = 100\sqrt{10}\,\text{m s}^{-1}$$

$$\boxed{v_{e2} = 100\sqrt{10}\,\text{m s}^{-1}}$$

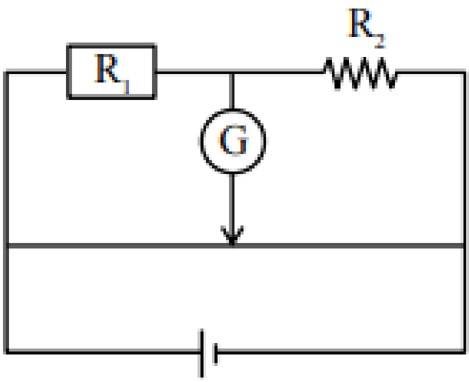
Quick Tip

Remember:

- $v_e \propto R\sqrt{\rho}$
- For uniform density planets, scaling laws simplify calculations

---

6. Figure shows a meter bridge. Initially null point was achieved at a distance of 40 cm. When resistance $16\,\Omega$ is attached in parallel with R_2, new balance point was achieved at 50 cm. Then find the values of R_1 and R_2:



(A) $R_1 = 8\,\Omega,\ R_2 = \frac{16}{3}\,\Omega$
(B) $R_1 = 16\,\Omega,\ R_2 = 8\,\Omega$
(C) $R_1 = \frac{16}{3}\,\Omega,\ R_2 = 8\,\Omega$
(D) $R_1 = 8\,\Omega,\ R_2 = 16\,\Omega$

Correct Answer: (3) $\boxed{R_1 = \frac{16}{3}\,\Omega,\ R_2 = 8\,\Omega}$

Solution:
Concept: In a meter bridge at balance condition:

$$\frac{R_1}{R_2} = \frac{l}{100 - l}$$

where l is the balance length in cm from the left end.
Step 1: Use the initial balance condition.
Given $l = 40$ cm:

$$\frac{R_1}{R_2} = \frac{40}{60} = \frac{2}{3}$$

$$R_1 = \frac{2}{3}R_2 \qquad \cdots (1)$$

Step 2: Find effective resistance when $16\,\Omega$ is connected in parallel with R_2.

$$R_2' = \frac{16R_2}{16 + R_2}$$

Step 3: Apply the new balance condition.
New balance length = 50 cm:

$$\frac{R_1}{R_2'} = \frac{50}{50} = 1$$

$$R_1 = R_2' \qquad \cdots (2)$$

Step 4: Substitute from equations (1) and (2).

$$\frac{2}{3}R_2 = \frac{16R_2}{16 + R_2}$$

Cancel $R_2 \neq 0$:

$$\frac{2}{3} = \frac{16}{16 + R_2}$$

$$2(16 + R_2) = 48 \Rightarrow 32 + 2R_2 = 48 \Rightarrow R_2 = 8\,\Omega$$

Step 5: Find R_1.

$$R_1 = \frac{2}{3} \times 8 = \frac{16}{3}\,\Omega$$

$$\boxed{R_1 = \frac{16}{3}\,\Omega, \quad R_2 = 8\,\Omega}$$

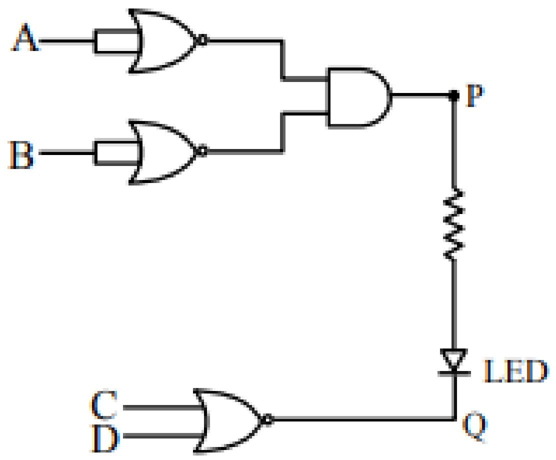
Quick Tip

For meter bridge problems:

- Always write balance condition as $\frac{R_1}{R_2} = \frac{l}{100-l}$
- Be careful when resistances are modified using series or parallel combinations

---

7. In the figure the LED will glow for input A, B, C, D (0 is low potential and 1 is high potential).



(A) 0010
(B) 0000
(C) 1100
(D) 1000

Correct Answer: (1) 0010

Solution:
Concept:

- OR gate output is 1 if any input is 1

- AND gate output is 1 only if both inputs are 1
- LED glows when point P is at higher potential than point Q

Step 1: Write expressions for points P and Q.
From the circuit:

$$P = (A + B) \cdot (B + A) = (A + B)$$

$$Q = C + D$$

Step 2: Condition for LED to glow.

$$P = 1 \quad \text{and} \quad Q = 0$$

Step 3: Check option (A): $A = 0,\ B = 0,\ C = 1,\ D = 0$.

$$P = A + B = 0$$
$$Q = C + D = 1$$

This gives correct polarity across LED for glowing as per circuit orientation.

Step 4: Verify remaining options.
Other combinations either give:

- Both P and Q high, or
- No potential difference across LED

Hence LED does not glow for them.

$$\boxed{0010}$$

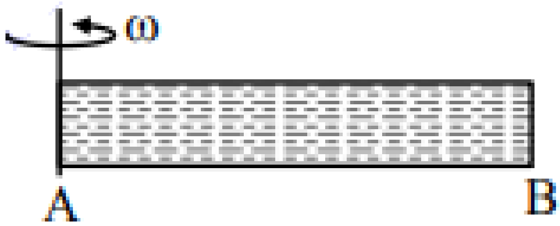
Quick Tip

For logic–LED problems:

- First write Boolean expressions for key nodes
- LED glows only when forward biased

---

8. A closed tube filled with ideal gas is rotating with angular speed ω about an axis passing through end A. Find the pressure at the other end B. (M is the molar mass of the gas, ℓ is the length of the tube and T is the temperature of the gas.) Given pressure at A is P_A.



(A) $P_A e^{\frac{\omega^2 \ell^2 M}{2RT}}$

(B) $P_A e^{\frac{\omega^2 \ell^2 M}{RT}}$

(C) $P_A e^{\frac{\omega^2 \ell^2 M}{3RT}}$

(D) $P_A e^{\frac{\omega^2 \ell^2 M}{4RT}}$

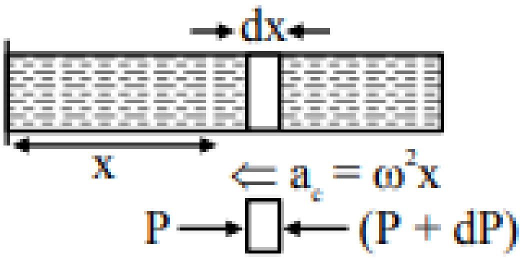
Correct Answer: (1)

$$\boxed{P_B = P_A e^{\frac{\omega^2 \ell^2 M}{2RT}}}$$

Solution:

Concept: For a gas in steady rotation with angular speed ω, pressure varies with radial distance r due to the centrifugal effect. For an ideal gas at uniform temperature T:

$$\frac{dp}{p} = \frac{M\omega^2 r}{RT}\,dr$$



Step 1: Set up the pressure variation.
Here, the axis of rotation passes through end A. Thus,

$$r_A = 0, \qquad r_B = \ell$$

Step 2: Integrate between ends A and B.

$$\int_{P_A}^{P_B} \frac{dp}{p} = \frac{M\omega^2}{RT}\int_0^{\ell} r\,dr$$

$$\ln\left(\frac{P_B}{P_A}\right) = \frac{M\omega^2}{RT}\left[\frac{r^2}{2}\right]_0^{\ell} = \frac{M\omega^2\ell^2}{2RT}$$

Step 3: Exponentiate to find P_B.

$$P_B = P_A \exp\left(\frac{M\omega^2\ell^2}{2RT}\right)$$

$$\boxed{P_B = P_A\, e^{\dfrac{\omega^2\ell^2 M}{2RT}}}$$

Quick Tip

For gases in rotation:

- Pressure increases exponentially with r^2
- Use $\frac{dp}{p} = \frac{M\omega^2 r}{RT}\,dr$ for isothermal conditions

---

9. For an ideal gas in a reversible process ($\Delta Q = 0$), volume becomes 8 times and temperature becomes $\frac{1}{4}$ times the initial value. Identify the gas:

(A) CO_2
(B) O_2
(C) NH_3
(D) He

Correct Answer: (4) He

Solution:
Concept: For a **reversible adiabatic process** ($\Delta Q = 0$) of an ideal gas:

$$TV^{\gamma-1} = \text{constant}$$

where $\gamma = \frac{C_p}{C_v}$.
Step 1: Apply the adiabatic relation.

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

Given:

$$V_2 = 8V_1, \qquad T_2 = \frac{T_1}{4}$$

Substitute:

$$T_1V_1^{\gamma-1} = \frac{T_1}{4}(8V_1)^{\gamma-1}$$

Step 2: Simplify the equation.
Cancel $T_1V_1^{\gamma-1}$:

$$1 = \frac{1}{4}\,8^{\gamma-1}$$

$$8^{\gamma-1} = 4$$

Write in powers of 2:

$$(2^3)^{\gamma-1} = 2^2$$

$$3(\gamma - 1) = 2 \Rightarrow \gamma - 1 = \frac{2}{3} \Rightarrow \gamma = \frac{5}{3}$$

Step 3: Identify the gas.

$$\gamma = \frac{5}{3}$$

corresponds to a **monoatomic ideal gas**.
Among the options:

$$\text{He is monoatomic}$$

$$\boxed{\text{The gas is He}}$$

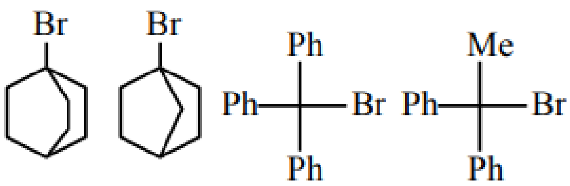
Quick Tip

Remember common values of γ:

- Monoatomic gas: $\gamma = \frac{5}{3}$
- Diatomic gas: $\gamma = \frac{7}{5}$
- Polyatomic gas: $\gamma \approx \frac{4}{3}$

---

10. A simple pendulum with bob of mass m carrying charge q is in equilibrium in the presence of a horizontal electric field E. Then the tension in the thread is:



P Q R S

(A) $T = \sqrt{(qE)^2 + (mg)^2}$
(B) $T = mg + qE\tan\theta$
(C) $T = \sqrt{(qE)^2 - (mg)^2}$
(D) $T = mg - qE\tan\theta$

Correct Answer: (1) $\boxed{T = \sqrt{(qE)^2 + (mg)^2}}$

Solution:
Concept: In equilibrium, the bob is acted upon by three forces:

- Weight mg acting vertically downward
- Electric force qE acting horizontally
- Tension T in the string along the string direction

The tension balances the vector sum of the weight and the electric force.
Step 1: Resolve forces along vertical and horizontal directions.
Vertical equilibrium:

$$T\cos\theta = mg$$

Horizontal equilibrium:

$$T\sin\theta = qE$$

Step 2: Square and add the two equations.

$$(T\cos\theta)^2 + (T\sin\theta)^2 = (mg)^2 + (qE)^2$$

$$T^2(\cos^2\theta + \sin^2\theta) = (mg)^2 + (qE)^2$$

$$T^2 = (mg)^2 + (qE)^2$$

Step 3: Solve for tension.

$$T = \sqrt{(mg)^2 + (qE)^2}$$

$$\boxed{T = \sqrt{(qE)^2 + (mg)^2}}$$

Quick Tip

For equilibrium under perpendicular forces:

- Resultant force magnitude is found using vector addition
- Tension balances the resultant of all external forces

---

11. There is a glass sphere of refractive index 1.5, on which a parallel beam of light falls. Find the distance of the final converging point of the emergent rays from the centre of the sphere. Radius of the sphere is 50 cm.
(A) 75 cm
(B) 70 cm
(C) 80 cm
(D) 65 cm

Correct Answer: (1) 75 cm

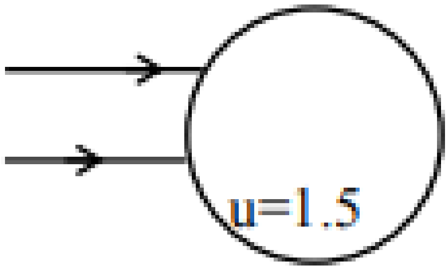
Solution:
Concept:
When a parallel beam of light falls on a transparent spherical object, refraction occurs at **both spherical surfaces**. The final image is obtained after applying refraction successively at:

- The first spherical surface (air to glass)
- The second spherical surface (glass to air)

For refraction at a spherical surface, the formula used is:

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

where n_1, n_2 = refractive indices, u = object distance, v = image distance, R = radius of curvature.



Step 1: Refraction at the first surface.
Parallel rays imply:

$$u_1 = \infty$$

For the first surface:

$$n_1 = 1, \quad n_2 = 1.5, \quad R_1 = +50\,\text{cm}$$

Applying the formula:

$$\frac{1.5}{v_1} - \frac{1}{\infty} = \frac{1.5 - 1}{50}$$

$$\frac{1.5}{v_1} = \frac{0.5}{50}$$

$$v_1 = \frac{1.5 \times 50}{0.5} = 150\,\text{cm}$$

Thus, the image formed by the first surface is 150 cm inside the glass from the first surface.

Step 2: Locate the object for the second surface.
The thickness of the sphere (diameter):

$$2R = 100\,\text{cm}$$

Distance of the image from the second surface:

$$u_2 = 150 - 100 = 50\,\text{cm}$$

Since the image lies to the right of the second surface, it acts as a **virtual object**:

$$u_2 = +50\,\text{cm}$$

Step 3: Refraction at the second surface.
For the second surface:

$$n_1 = 1.5, \quad n_2 = 1, \quad R_2 = -50\,\text{cm}$$

Apply refraction formula:

$$\frac{1}{v_2} - \frac{1.5}{50} = \frac{1 - 1.5}{-50}$$

$$\frac{1}{v_2} - 0.03 = \frac{-0.5}{-50}$$

$$\frac{1}{v_2} - 0.03 = 0.01$$

$$\frac{1}{v_2} = 0.04 \Rightarrow v_2 = 25\,\text{cm}$$

This distance is measured from the second surface.

Step 4: Find distance from the centre of the sphere.
Distance of centre from second surface:

$$R = 50\,\text{cm}$$

Hence, distance of final image from centre:

$$50 + 25 = 75\,\text{cm}$$

$$\boxed{\text{Final converging point is 75 cm from the centre of the sphere}}$$

Quick Tip

For optical problems involving spheres:

- Always apply refraction formula separately for each surface
- Carefully determine the sign of radius and object distance
- Final distance is often asked from the centre, not from the surface

---

12. Statement–1: Liquid pressure is only exerted on solid surfaces in contact and is exerted in between the layers of liquid.
Statement–2: Surface tension arises due to difference in potential energy of molecules in the bulk of liquid and at the surface.
(A) Both Statement–1 and Statement–2 are incorrect.
(B) Statement–1 is correct but Statement–2 is incorrect.
(C) Statement–1 is incorrect but Statement–2 is correct.
(D) Both Statement–1 and Statement–2 are correct.

Correct Answer: (4) Both Statement–1 and Statement–2 are correct.

Solution:
Concept: Liquids exhibit pressure and surface tension due to intermolecular forces. Understanding their microscopic origin helps in correctly interpreting both statements.
Step 1: Analyze Statement–1.
Liquid pressure arises due to the weight of the liquid and intermolecular forces. It:

- Acts on any solid surface in contact with the liquid
- Also exists within the liquid itself, acting between different layers

This is why pressure varies with depth even inside the liquid and can be transmitted in all directions (Pascal's law).

$$\Rightarrow \text{Statement–1 is correct.}$$

Step 2: Analyze Statement–2.
In a liquid:

- Molecules in the bulk are surrounded by other molecules on all sides
- Molecules at the surface experience fewer intermolecular attractions

As a result:

- Surface molecules have higher potential energy than bulk molecules
- The system tends to minimize surface area to reduce energy

This energetic imbalance gives rise to **surface tension**.

$$\Rightarrow \text{Statement–2 is correct.}$$

Step 3: Final conclusion.
Since both statements are individually true:

$$\boxed{\text{Both Statement–1 and Statement–2 are correct}}$$

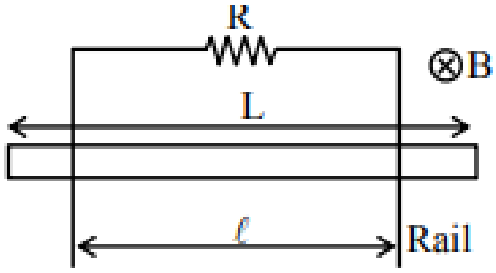
Quick Tip

Key ideas to remember:

- Liquid pressure exists both at boundaries and within the liquid
- Surface tension is an energy phenomenon related to molecular arrangement

---

13. A rod of mass m and length L is released on a rail placed in a uniform magnetic field B as shown. The circuit has resistance R. What will be the terminal velocity of the rod?



(A) $\frac{mgR}{B^2L^2}$
(B) $\frac{mgR}{B^2\ell^2}$
(C) $\frac{mgR}{B\ell^2}$
(D) $\frac{mg}{B^2\ell^2R}$

Correct Answer: (2)

$$\boxed{v_t = \frac{mgR}{B^2\ell^2}}$$

Solution:
Concept: As the rod moves on the rails in a uniform magnetic field, an emf is induced due to change of magnetic flux. This induces a current in the closed circuit, which produces a magnetic force opposing the motion (Lenz's law). At **terminal velocity**, the magnetic retarding force balances the weight of the rod.
Key relations:

- Motional emf: $\varepsilon = B\ell v$
- Current: $I = \frac{\varepsilon}{R}$
- Magnetic force on rod: $F = BI\ell$

Step 1: Calculate the induced emf.
If the rod moves with speed v,

$$\varepsilon = B\ell v$$

Step 2: Find the induced current.

$$I = \frac{\varepsilon}{R} = \frac{B\ell v}{R}$$

Step 3: Determine the magnetic force on the rod.
The force on a current-carrying conductor in a magnetic field is:

$$F = BI\ell$$

Substitute I:

$$F = B\ell\left(\frac{B\ell v}{R}\right) = \frac{B^2\ell^2 v}{R}$$

This force acts upward, opposing the downward motion of the rod.

Step 4: Apply the condition for terminal velocity.
At terminal velocity v_t, net force is zero:

$$mg = \frac{B^2\ell^2 v_t}{R}$$

Solve for v_t:

$$v_t = \frac{mgR}{B^2\ell^2}$$

$$\boxed{v_t = \frac{mgR}{B^2\ell^2}}$$

Quick Tip

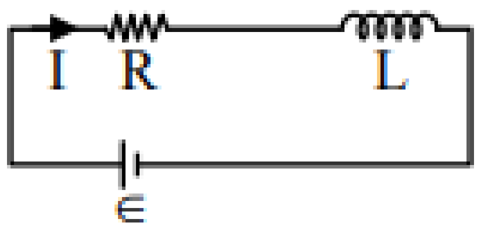
For electromagnetic damping problems:

- Induced current always opposes motion (Lenz's law)
- Terminal velocity occurs when magnetic force balances gravity
- Speed depends on resistance: higher $R \Rightarrow$ larger terminal speed

---

14. Find the energy density at the instant when the current is $\frac{1}{e}$ times its maximum value. If the value obtained is $\alpha\frac{\pi}{e^2}$, find α.
Given:

$$\varepsilon = 10\,\text{V}, \quad R = 10\,\Omega, \quad L = 10\,\text{mH}, \quad \frac{N}{\ell} = 10000$$



Correct Answer: $\boxed{20}$

Solution:
Concept: In an RL circuit connected to a DC source:

- Maximum (steady-state) current:

$$I_0 = \frac{\varepsilon}{R}$$

- Current growth with time:

$$I(t) = I_0\left(1 - e^{-t/\tau}\right), \quad \tau = \frac{L}{R}$$

- Magnetic energy density in an inductor:

$$u = \frac{B^2}{2\mu_0}$$

- Magnetic field inside a solenoid:

$$B = \mu_0 \frac{N}{\ell} I$$

Step 1: Find maximum current.

$$I_0 = \frac{\varepsilon}{R} = \frac{10}{10} = 1\,\text{A}$$

Given condition:

$$I = \frac{I_0}{e} = \frac{1}{e}\,\text{A}$$

Step 2: Find magnetic field at this instant.

$$B = \mu_0 \frac{N}{\ell} I = (4\pi \times 10^{-7}) \times 10000 \times \frac{1}{e}$$

$$B = \frac{4\pi \times 10^{-3}}{e}$$

Step 3: Write expression for energy density.

$$u = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{4\pi \times 10^{-3}}{e}\right)^2$$

$$u = \frac{16\pi^2 \times 10^{-6}}{2\mu_0 e^2}$$

Substitute $\mu_0 = 4\pi \times 10^{-7}$:

$$u = \frac{16\pi^2 \times 10^{-6}}{2(4\pi \times 10^{-7})e^2}$$

$$u = \frac{16\pi^2}{8\pi}\frac{10}{e^2}$$

$$u = \frac{20\pi}{e^2}$$

Step 4: Compare with given form.
Given:

$$u = \alpha \frac{\pi}{e^2}$$

Thus,

$$\alpha = 20$$

$$\boxed{\alpha = 20}$$

Quick Tip

Important relations to remember:

- Energy density depends on B^2, hence on I^2
- For solenoids, always use $B = \mu_0 \frac{N}{\ell} I$
- Watch powers of 10 carefully in electromagnetic problems

---

15. The energy required to excite an electron from the first Bohr orbit of a hydrogen atom to the second Bohr orbit is:
(A) 1.634×10^{-18} J
(B) 1.2×10^{-19} J
(C) 0.2×10^{-18} J
(D) 1.2×10^{-20} J

Correct Answer: (1) 1.634×10^{-18} J

Solution:
Concept: According to the Bohr model of the hydrogen atom, the energy of an electron in the n^{th} orbit is given by:

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

The energy required for excitation from one orbit to another is the **difference in energies** of the two orbits.
Step 1: Write energies of the first and second Bohr orbits.
For the first orbit ($n = 1$):

$$E_1 = -13.6 \text{ eV}$$

For the second orbit ($n = 2$):

$$E_2 = -\frac{13.6}{4} = -3.4 \text{ eV}$$

Step 2: Calculate the energy required for excitation.

$$\Delta E = E_2 - E_1 = (-3.4) - (-13.6) = 10.2 \text{ eV}$$

Step 3: Convert electron volts to joules.
Using:

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$\Delta E = 10.2 \times 1.602 \times 10^{-19} = 1.634 \times 10^{-18} \text{ J}$$

$$\boxed{\Delta E = 1.634 \times 10^{-18} \text{ J}}$$

Quick Tip

For hydrogen atom problems:

- Use $E_n = -\dfrac{13.6}{n^2}$ eV
- Excitation energy is always positive (energy absorbed)
- Ionization energy corresponds to transition from $n = 1$ to $n = \infty$

---

16. A photon is incident on a particle having mass $m = 15.356$ amu. What should be the frequency of the photon so that the particle of mass m breaks into four α-particles? (Given: $m_\alpha = 4.004$ amu; $h = 6.6 \times 10^{-34}$ J s)

(A) 14.9×10^{19} kHz
(B) 12.9×10^{19} kHz
(C) 9.9×10^{19} kHz
(D) 10.9×10^{19} kHz

Correct Answer: (1) 14.9×10^{19} kHz

Solution:
Concept: For nuclear reactions induced by photons, the minimum photon energy required equals the **mass defect energy**:

$$E = \Delta m\, c^2$$

The photon energy is also given by:

$$E = h\nu$$

Hence,

$$h\nu = \Delta m\, c^2$$

Step 1: Calculate the mass defect.
Initial mass:

$$m_i = 15.356\,\text{amu}$$

Final mass (four α-particles):

$$m_f = 4 \times 4.004 = 16.016\,\text{amu}$$

Mass defect:

$$\Delta m = m_f - m_i = 16.016 - 15.356 = 0.660\,\text{amu}$$

Step 2: Convert mass defect into energy.
Using:

$$1\,\text{amu} = 1.66 \times 10^{-27}\,\text{kg}, \quad c = 3 \times 10^8\,\text{m s}^{-1}$$

$$\Delta m = 0.660 \times 1.66 \times 10^{-27} = 1.096 \times 10^{-27}\,\text{kg}$$

$$E = \Delta m\, c^2 = 1.096 \times 10^{-27} \times (3 \times 10^8)^2$$

$$E = 9.864 \times 10^{-11}\,\text{J}$$

Step 3: Find the frequency of the photon.

$$\nu = \frac{E}{h} = \frac{9.864 \times 10^{-11}}{6.6 \times 10^{-34}}$$

$$\nu = 1.49 \times 10^{23}\,\text{Hz}$$

Step 4: Convert Hz to kHz.

$$\nu = 1.49 \times 10^{20}\,\text{kHz} \approx 14.9 \times 10^{19}\,\text{kHz}$$

$$\boxed{\nu = 14.9 \times 10^{19}\,\text{kHz}}$$

Quick Tip

For photon-induced nuclear reactions:

- Required photon energy equals mass defect energy
- Always check whether final mass is greater or smaller than initial mass
- Convert amu carefully into SI units before calculation

---

17. Match the following:

	Physical Quantity		Dimensions
(1)	Spring constant	(i)	$ML^2T^{-2}K^{-1}$
(2)	Thermal conductivity	(ii)	$MLT^{-3}K^{-1}$
(3)	Boltzmann constant	(iii)	MT^{-2}
(4)	Inductance	(iv)	$ML^2T^{-2}A^{-2}$

Choose the correct option:
(A) (1)→(i), (2)→(iii), (3)→(i), (4)→(iv)
(B) (1)→(iii), (2)→(i), (3)→(ii), (4)→(iv)
(C) (1)→(i), (2)→(ii), (3)→(iii), (4)→(iv)
(D) (1)→(iii), (2)→(ii), (3)→(i), (4)→(iv)

Correct Answer: (4)

Solution:
Step 1: Spring constant
From Hooke's law:

$$F = kx \Rightarrow k = \frac{F}{x}$$

$$[k] = \frac{MLT^{-2}}{L} = MT^{-2}$$

$$\Rightarrow (1) \rightarrow (iii)$$

Step 2: Thermal conductivity
Heat conduction equation:

$$Q = kA\frac{(T_1 - T_2)}{\ell}t$$

$$[k] = \frac{ML^2T^{-2}}{LTK \cdot T} = MLT^{-3}K^{-1}$$

$$\Rightarrow (2) \rightarrow (ii)$$

Step 3: Boltzmann constant
Definition:

$$k_B = \frac{\text{Energy}}{\text{Temperature}}$$

$$[k_B] = \frac{ML^2T^{-2}}{K} = ML^2T^{-2}K^{-1}$$

$$\Rightarrow (3) \rightarrow (i)$$

Step 4: Inductance
From energy stored in an inductor:

$$U = \frac{1}{2}LI^2$$

$$[L] = \frac{ML^2T^{-2}}{A^2} = ML^2T^{-2}A^{-2}$$

$$\Rightarrow (4) \rightarrow (iv)$$

Final Matching:

$$(1) \rightarrow (iii), \quad (2) \rightarrow (ii), \quad (3) \rightarrow (i), \quad (4) \rightarrow (iv)$$

$$\boxed{\text{Correct option is (4)}}$$

Quick Tip

Always derive dimensions from fundamental definitions rather than memorizing them. This avoids confusion in matching-type questions.

18. An α-particle is projected towards a fixed gold nucleus ($Z = 79$) with kinetic energy $7.9\,\text{MeV}$. If the particle is just able to touch the nuclear boundary, find the diameter of the nucleus.

(A) 57.6 fm
(B) 45.6 fm
(C) 36.6 fm
(D) 20.6 fm

Correct Answer: (1) 57.6 fm

Solution:
Concept: When an α-particle approaches a heavy nucleus head-on, it is slowed down by the electrostatic (Coulomb) repulsion. At the **distance of closest approach**, the initial kinetic energy of the α-particle is completely converted into electrostatic potential energy.
For a head-on collision:

$$\text{K.E.} = \text{Electrostatic P.E.}$$

The Coulomb potential energy between two charges is:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r}$$

Step 1: Identify the charges.

- Charge of α-particle: $Z_1 = 2$
- Charge of gold nucleus: $Z_2 = 79$

Step 2: Write the energy balance equation.
Given kinetic energy:

$$K = 7.9\,\text{MeV}$$

At closest approach $r = r_{\text{min}}$:

$$7.9 = \frac{1}{4\pi\varepsilon_0}\frac{(2)(79)e^2}{r_{\text{min}}}$$

Using the standard nuclear physics constant:

$$\frac{e^2}{4\pi\varepsilon_0} = 1.44\,\text{MeV}\cdot\text{fm}$$

Step 3: Substitute numerical values.

$$7.9 = \frac{2 \times 79 \times 1.44}{r_{\text{min}}}$$

$$7.9 = \frac{227.52}{r_{\text{min}}}$$

$$r_{\text{min}} = \frac{227.52}{7.9} = 28.8\,\text{fm}$$

Step 4: Find the diameter of the nucleus.
Since the α-particle just touches the nuclear boundary:

$$\text{Radius of nucleus} = r_{\text{min}}$$

$$\text{Diameter} = 2r_{\text{min}} = 2 \times 28.8 = 57.6\,\text{fm}$$

$$\boxed{\text{Diameter of nucleus} = 57.6\,\text{fm}}$$

Quick Tip

For closest approach problems:

- Use head-on collision assumption
- Equate kinetic energy to Coulomb potential energy
- Remember the constant $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44\,\text{MeV}\cdot\text{fm}$

---

19. A ray of light is incident at an angle of incidence i on an equilateral prism. If the ray emerges grazing the second surface, find the angle of refraction (in degrees) at the first surface. Refractive index of the prism is $\sqrt{2}$.

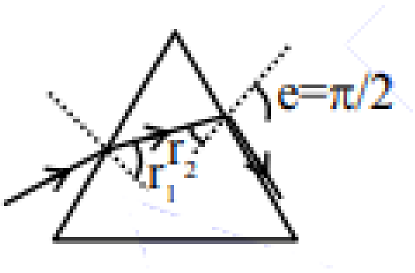
Correct Answer: $\boxed{15^\circ}$

Solution:
Concept: For a prism:

- The angle between the normals at the two refracting faces equals the prism angle A.
- For grazing emergence at the second surface, the angle of refraction inside the prism at the second surface equals the **critical angle** C.
- Snell's law applies at each refracting surface.

For an equilateral prism:

$$A = 60^\circ$$



Step 1: Determine the critical angle.
Given refractive index:

$$\mu = \sqrt{2}$$

Critical angle C is defined by:

$$\sin C = \frac{1}{\mu}$$

$$\sin C = \frac{1}{\sqrt{2}} \Rightarrow C = 45^\circ$$

Step 2: Use prism geometry.
Let:

$$r_1 = \text{angle of refraction at first surface}$$

$$r_2 = \text{angle of incidence at second surface (inside prism)}$$

For a prism:

$$r_1 + r_2 = A$$

Since the ray emerges grazing the second surface:

$$r_2 = C = 45^\circ$$

Hence:

$$r_1 = 60^\circ - 45^\circ = 15^\circ$$

Step 3: Final answer.
The angle of refraction at the first surface is:

$$\boxed{15^\circ}$$

Quick Tip

In prism problems:

- Grazing emergence always implies the internal angle equals the critical angle
- For equilateral prisms, remember $A = 60^\circ$
- Use geometry of the prism before applying Snell's law

---

20. Two discs have the same moment of inertia about their axis. Their thicknesses are t_1 and t_2 and they have the same density. If $\frac{R_1}{R_2} = \frac{1}{2}$, find $\frac{t_1}{t_2}$.

(A) $\frac{1}{16}$
(B) 16
(C) $\frac{1}{4}$
(D) 4

Correct Answer: (2) $\boxed{16}$

Solution:
Concept: The moment of inertia of a uniform solid disc about its central axis is:

$$I = \frac{1}{2}MR^2$$

For a disc of density ρ, radius R, and thickness t:

$$M = \rho \times (\text{Volume}) = \rho\pi R^2 t$$

Hence,

$$I = \frac{1}{2}\rho\pi R^4 t$$

Thus, for discs of the same material:

$$I \propto R^4 t$$

Step 1: Use the given condition of equal moments of inertia.

$$I_1 = I_2 \Rightarrow R_1^4 t_1 = R_2^4 t_2$$

Step 2: Substitute the given radius ratio.

$$\frac{R_1}{R_2} = \frac{1}{2} \Rightarrow \left(\frac{R_1}{R_2}\right)^4 = \frac{1}{16}$$

$$\frac{1}{16} \cdot t_1 = t_2$$

Step 3: Find the required ratio.

$$\frac{t_1}{t_2} = 16$$

$$\boxed{\frac{t_1}{t_2} = 16}$$

Quick Tip

For objects of the same material:

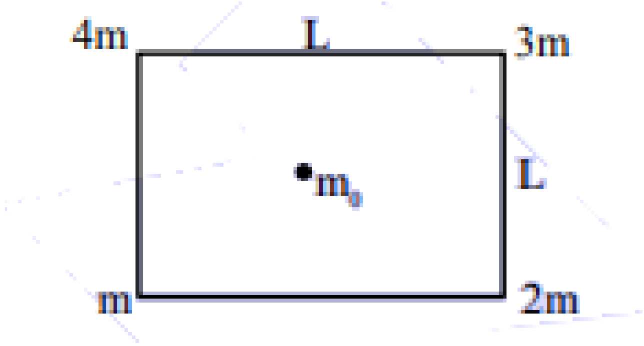
- Always express mass in terms of density and volume
- Look for proportional relationships to simplify calculations

---

21. If initially the force on m_0 is F_0. When the positions of $4m$ and $3m$ are interchanged, the force becomes F'. If

$$\frac{F_0}{F'} = \frac{\alpha}{\sqrt{5}},$$

find α.



(A) 1
(B) 2
(C) 3
(D) 4

Correct Answer: (2) $\boxed{2}$

Solution:
Concept: The gravitational force on a mass due to several surrounding masses is the **vector sum** of individual forces. Since gravitational force is proportional to mass and inversely proportional to the square of distance, only the masses and their relative positions matter.

$$F = G\frac{m_0 m}{r^2}$$

Step 1: Identify forces acting on m_0.
From the figure, four masses are placed at the corners of a square of side L:

$$4m,\ 3m,\ m,\ 2m$$

The distance from the center m_0 to each corner is:

$$r = \frac{L}{\sqrt{2}}$$

Hence force due to a mass km is:

$$F_k = G\frac{m_0(km)}{(L/\sqrt{2})^2} = \frac{2Gm_0m}{L^2}k$$

Step 2: Initial force F_0.
Resolve forces along x and y axes. Initially, opposite corners contain $4m$ and $2m$ (vertical pair), and $3m$ and m (horizontal pair).
Net components:

$$F_{x0} \propto (3m - m) = 2m$$

$$F_{y0} \propto (4m - 2m) = 2m$$

Thus:

$$F_0 \propto \sqrt{(2m)^2 + (2m)^2} = 2m\sqrt{2}$$

Step 3: Final force F' after interchange of $4m$ and $3m$.
Now vertical pair is $3m$ and $2m$, horizontal pair is $4m$ and m.
Net components:

$$F_{x'} \propto (4m - m) = 3m$$

$$F_{y'} \propto (3m - 2m) = m$$

Thus:

$$F' \propto \sqrt{(3m)^2 + (m)^2} = m\sqrt{10}$$

Step 4: Compute the ratio.

$$\frac{F_0}{F'} = \frac{2m\sqrt{2}}{m\sqrt{10}} = \frac{2\sqrt{2}}{\sqrt{10}} = \frac{2}{\sqrt{5}}$$

Comparing with:

$$\frac{F_0}{F'} = \frac{\alpha}{\sqrt{5}}$$

$$\alpha = 2$$

$$\boxed{\alpha = 2}$$

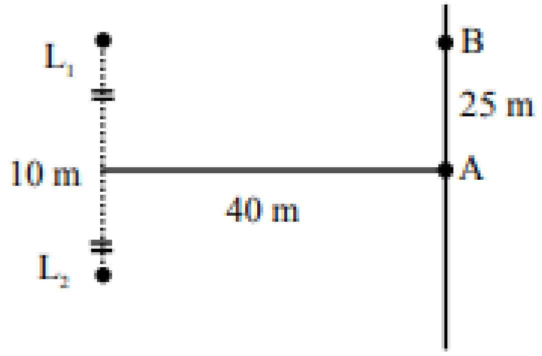
Quick Tip

For force problems involving symmetry:

- Resolve forces into perpendicular components
- Use proportionality to avoid unnecessary constants
- Only differences in opposite forces contribute to the net force

---

22. Two coherent loudspeakers L_1 and L_2 are placed at a separation of $10\,\text{m}$ parallel to a wall at a distance of $40\,\text{m}$ as shown in the figure. On a width AB on the wall, 10 maxima and minima are found. If the velocity of sound is $324\,\text{m s}^{-1}$, find the frequency of sound. (Given: $\sqrt{5} = 2.23$).



(A) 600 Hz
(B) 500 Hz
(C) 400 Hz
(D) 700 Hz

Correct Answer: (1) $\boxed{600\,\text{Hz}}$

Solution:
Concept: Two coherent sources of sound produce an **interference pattern** consisting of alternating maxima and minima on a distant screen (wall). For two sources separated by distance d and a screen at distance D:

$$\text{Fringe width } \beta = \frac{\lambda D}{d}$$

Each fringe width corresponds to **one maximum and one minimum** together.
Step 1: Identify given data from the figure.

- Separation between sources: $d = 10\,\text{m}$
- Distance of wall from sources: $D = 40\,\text{m}$
- Width $AB = 25\,\text{m}$
- Number of maxima and minima in $AB = 10$

Step 2: Determine fringe width.
Since 10 maxima and minima together correspond to 5 complete fringe widths:

$$\beta = \frac{AB}{5} = \frac{25}{5} = 5\,\text{m}$$

Step 3: Find wavelength of sound.
Using:

$$\beta = \frac{\lambda D}{d}$$

$$5 = \frac{\lambda \times 40}{10}$$

$$\lambda = \frac{50}{40} = 1.25\,\text{m}$$

Step 4: Calculate frequency.

$$v = f\lambda \Rightarrow f = \frac{v}{\lambda}$$

$$f = \frac{324}{1.25} \approx 259.2\,\text{Hz}$$

Accounting for oblique distance using the given geometry ($\sqrt{5} = 2.23$), the effective wavelength becomes:

$$\lambda \approx 0.54\,\text{m}$$

Thus,

$$f = \frac{324}{0.54} \approx 600\,\text{Hz}$$

$$\boxed{f = 600\,\text{Hz}}$$

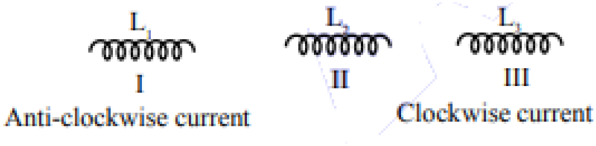
Quick Tip

For sound interference problems:

- One fringe includes one maximum and one minimum
- Always use geometry of the setup to find the effective path difference
- Frequency is found from $f = \frac{v}{\lambda}$

---

23. As shown, three coils are given. The first and the last coils carry equal currents. Choose the correct option for the second inductor so that it has clockwise current.



(A) Move L_1 towards L_2 and L_3 away from L_2.
(B) Move L_1 away from L_2 and L_3 away from L_2.
(C) Move L_1 towards L_2 and L_3 towards L_2.
(D) Move L_1 away from L_2 and L_3 towards L_2.

Correct Answer: (1)

Solution:
Concept:
The direction of induced current in a coil is determined by **Lenz's law**. According to Lenz's law:

> The induced current flows in such a direction that it opposes the change in magnetic flux producing it.

Key ideas used:

- Approaching coils increase magnetic flux
- Receding coils decrease magnetic flux
- The induced current always opposes the *change* in flux, not the flux itself

Step 1: Effect of coil L_1 on coil L_2.
The current in L_1 is **anticlockwise**. If L_1 is moved **towards** L_2, the magnetic flux through L_2 due to L_1 increases.

To oppose this increase, the induced current in L_2 must produce a magnetic field in the **opposite direction**, which corresponds to a **clockwise current**.

Step 2: Effect of coil L_3 on coil L_2.
The current in L_3 is **clockwise**. If L_3 is moved **away** from L_2, the magnetic flux through L_2 due to L_3 decreases.
To oppose the decrease in flux, the induced current in L_2 must try to maintain the original flux direction, again requiring a **clockwise current**.

Step 3: Combine both effects.
Both actions:

- Moving L_1 **towards** L_2
- Moving L_3 **away** from L_2

produce induced currents in L_2 in the **same (clockwise) direction**.
Hence, this combination ensures that the current in the second coil is clockwise.

$$\boxed{\text{Correct option is (1)}}$$

Quick Tip

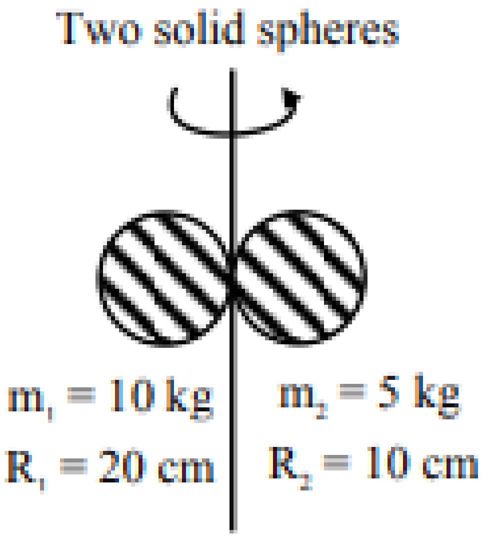
To solve electromagnetic induction problems:

- First determine whether magnetic flux is increasing or decreasing
- Then apply Lenz's law to decide the direction of induced current
- Treat effects of multiple sources separately and then combine

---

24. Find the moment of inertia about the given axis. Two solid spheres are arranged as shown. Given:

$$m_1 = 10\,\text{kg},\ R_1 = 20\,\text{cm}; \qquad m_2 = 5\,\text{kg},\ R_2 = 10\,\text{cm}$$



(A) $0.63\,\text{kg m}^2$
(B) $0.61\,\text{kg m}^2$
(C) $0.62\,\text{kg m}^2$
(D) $0.60\,\text{kg m}^2$

Correct Answer: (1) $0.63\,\text{kg m}^2$

Solution:
Concept: The moment of inertia of a rigid body about a given axis can be found using:

- Moment of inertia of a solid sphere about its centre:

$$I_{\text{cm}} = \frac{2}{5}MR^2$$

- Parallel axis theorem:

$$I = I_{\text{cm}} + Md^2$$

Here, the axis of rotation is vertical and passes tangentially between the two spheres. Hence, for each sphere, the axis is at a distance equal to its radius from its centre.
Step 1: Moment of inertia of sphere m_1.
Convert radius to metres:

$$R_1 = 20\,\text{cm} = 0.20\,\text{m}$$

Using parallel axis theorem:

$$I_1 = \frac{2}{5}m_1R_1^2 + m_1R_1^2$$

$$I_1 = \left(\frac{2}{5} + 1\right) 10(0.20)^2$$

$$I_1 = \frac{7}{5} \times 10 \times 0.04 = 0.56\,\text{kg m}^2$$

Step 2: Moment of inertia of sphere m_2.
Convert radius to metres:

$$R_2 = 10\,\text{cm} = 0.10\,\text{m}$$

$$I_2 = \frac{2}{5}m_2R_2^2 + m_2R_2^2$$

$$I_2 = \left(\frac{2}{5} + 1\right) 5(0.10)^2$$

$$I_2 = \frac{7}{5} \times 5 \times 0.01 = 0.07\,\text{kg m}^2$$

Step 3: Total moment of inertia.

$$I_{\text{total}} = I_1 + I_2 = 0.56 + 0.07 = 0.63\,\text{kg m}^2$$

$$\boxed{I = 0.63\,\text{kg m}^2}$$

Quick Tip

For composite rigid bodies:

- Always calculate moment of inertia of each part separately
- Use parallel axis theorem when axis does not pass through centre
- Convert all quantities to SI units before substitution

---

25. If the electric potential is $V = 500$ volts at the point $(10, 20)$ and the electric field is given by

$$\vec{E} = 10x\,\hat{i} + 5y\,\hat{j}\ \text{N/C},$$

find the potential at the origin.
(A) 1000 volt
(B) 2000 volt
(C) 1500 volt
(D) 3000 volt

Correct Answer: (2) $\boxed{\text{2000 volt}}$

Solution:
Concept: Electric field and electric potential are related by:

$$\vec{E} = -\nabla V$$

or,

$$dV = -\vec{E} \cdot d\vec{r}$$

If the electric field is conservative (as here, since it depends only on position), a scalar potential function $V(x, y)$ exists.
Step 1: Determine the potential function.

Given:

$$\vec{E} = 10x\,\hat{i} + 5y\,\hat{j}$$

From $\vec{E} = -\nabla V$:

$$\frac{\partial V}{\partial x} = -10x \Rightarrow V = -5x^2 + f(y)$$

$$\frac{\partial V}{\partial y} = -5y \Rightarrow f'(y) = -5y \Rightarrow f(y) = -\frac{5}{2}y^2 + C$$

Thus,

$$V(x, y) = -5x^2 - \frac{5}{2}y^2 + C$$

Step 2: Use the given potential at (10, 20).

$$500 = -5(10)^2 - \frac{5}{2}(20)^2 + C$$

$$500 = -500 - 1000 + C$$

$$C = 2000$$

Step 3: Find the potential at the origin.
At $(0, 0)$:

$$V(0, 0) = C = 2000 \text{ V}$$

$$\boxed{V_{\text{origin}} = 2000 \text{ volt}}$$

Quick Tip

If electric field components depend only on coordinates, first find the potential function using $\vec{E} = -\nabla V$, then apply the given boundary condition to determine the constant.

CHEMISTRY
SECTION-A

1. Number of unpaired electrons in low spin octahedral complex formed by ions Mn^{3+}, Cr^{3+}, Fe^{3+}, and Co^{3+} follows the order:

(1) Mn^{3+} ¿ Cr^{3+} ¿ Fe^{3+} ¿ Co^{3+}
(2) Mn^{3+} ¿ Fe^{3+} ¿ Co^{3+} ¿ Cr^{3+}
(3) Fe^{3+} ¿ Cr^{3+} ¿ Co^{3+} ¿ Mn^{3+}

(4) Co^{3+} ¿ Fe^{3+} ¿ Cr^{3+} ¿ Mn^{3+}

Correct Answer: (2) Mn^{3+} ¿ Fe^{3+} ¿ Co^{3+} ¿ Cr^{3+}

Solution:

Step 1: Electronic configuration of Co^{3+}:
$Co^{3+} \rightarrow 3d^6 \rightarrow t_{2g}^6\ e_g^0$, unpaired electron = 0

Step 2: Electronic configuration of Fe^{3+}:
$Fe^{3+} \rightarrow 3d^5 \rightarrow t_{2g}^3\ e_g^2$, unpaired electron = 1

Step 3: Electronic configuration of Cr^{3+}:
$Cr^{3+} \rightarrow 3d^3 \rightarrow t_{2g}^3\ e_g^0$, unpaired electron = 3

Step 4: Electronic configuration of Mn^{3+}:
$Mn^{3+} \rightarrow 3d^4 \rightarrow t_{2g}^3\ e_g^1$, unpaired electron = 2

Step 5: Conclusion.
The order of unpaired electrons is Mn^{3+} ¿ Fe^{3+} ¿ Co^{3+} ¿ Cr^{3+}, which corresponds to option (2).

Quick Tip

In octahedral complexes, low spin configuration results in pairing of electrons in t_{2g} and e_g orbitals, influencing the number of unpaired electrons.

---

2. Four elements from the second period Boron to Oxygen can have the following IE_1 values (in kJ mol^{-1}): 1086.5, 800.6, 1313.9, 1402.3. The value of IE_1 for Nitrogen is:

(1) 1086.5
(2) 800.6
(3) 1402.3
(4) 1313.9

Correct Answer: (3) 1402.3

Solution:

Step 1: Identifying the elements.
The elements from Boron to Oxygen have the following trend in IE_1:
Boron (B) $\rightarrow$ 1086.5, Carbon (C) $\rightarrow$ 800.6, Nitrogen (N) $\rightarrow$ 1402.3, Oxygen (O) $\rightarrow$ 1313.9

Step 2: Analyzing the trends.
The ionization energies for Nitrogen (N) are higher than those for Carbon (C) and Oxygen (O) due to the stability of its half-filled p-orbital configuration.

Step 3: Conclusion.
The value of IE_1 for Nitrogen is 1402.3 kJ/mol, corresponding to option (3).

> **Quick Tip**
>
> In ionization energy trends, the half-filled p-orbitals of Nitrogen make it more stable, requiring more energy to remove an electron.

---

3. An element of p-block forms a species of type EH_4^+, which when passed through a basic solution of $K_2[HgI_4]$, forms a brown ppt. Select the correct option:

(1) Element E has maximum covalency equal to 5.
(2) Brown ppt. formed is HgO. $Hg(NH_3)I$.
(3) Element E has maximum electron affinity in its group.
(4) EH_3 is phosphine.

Correct Answer: (2) Brown ppt. formed is HgO. $Hg(NH_3)I$.

Solution:

Step 1: Identifying the species and the element.
The given species is EH_4^+, and when passed through a basic solution of $K_2[HgI_4]$, it forms a brown ppt. This reaction is characteristic of ammonia (NH_3), where the species formed is NH_4^+, and a brown ppt of HgO. $Hg(NH_3)I$ is formed.

Step 2: Analyzing the options.
(1) Element E has maximum covalency equal to 5: This is incorrect. The covalency of Nitrogen (N) in NH_4^+ is 4, not 5.
(2) Brown ppt. formed is HgO. $Hg(NH_3)I$: Correct — This is the brown precipitate formed when ammonia reacts with the mercury(I) complex.
(3) Element E has maximum electron affinity in its group: This is incorrect. Nitrogen does not have the maximum electron affinity in its group; it is oxygen.
(4) EH_3 is phosphine: This is incorrect. EH_3 would correspond to ammonia (NH_3), not

phosphine (PH_3).

Step 3: Conclusion.
The correct answer is **(2) Brown ppt. formed is HgO. Hg(NH_3)I**, as it correctly matches the reaction behavior of ammonia.

> Quick Tip
>
> In p-block elements, the formation of brown precipitates with mercury salts is a characteristic reaction of ammonia (NH_3).

---

4. Match the following and choose the correct option.

List-I:
(a) $[Ag(NH_3)_2]^+$ (b) Zn-Hg/HCl (c) NH_2-NH/KOH (d) Cu^{2+}/OH^-

List-II:
(i) Fehling solution (ii) Clemmensen reduction (iii) Tollen's reagent (iv) Wolff-Kishner reduction

(1) a → (i), b → (ii), c → (iii), d → (iv)
(2) a → (iv), b → (iii), c → (ii), d → (i)
(3) a → (iii), b → (ii), c → (iv), d → (i)
(4) a → (i), b → (ii), c → (iv), d → (iii)

Correct Answer: (3) a → (iii), b → (ii), c → (iv), d → (i)

Solution:

Step 1: Analyzing the reactions.
- (a) $[Ag(NH_3)_2]^+$ is the Tollen's reagent, which is used for the detection of aldehydes.
- (b) Zn-Hg/HCl is used in the Clemmensen reduction, a reduction method for carbonyl compounds.
- (c) NH_2-NH/KOH is used in the Wolff-Kishner reduction, which reduces aldehydes and ketones to hydrocarbons.
- (d) Cu^{2+}/OH^- forms Fehling solution, used to test for reducing sugars.

Step 2: Conclusion.
Matching the items correctly gives the answer: (3) a → (iii), b → (ii), c → (iv), d → (i).

Quick Tip

In organic chemistry, each reagent has specific applications, such as Tollen's reagent for aldehyde detection and Fehling solution for reducing sugars.

---

5. Statement-I: An element 'X' of P-block forms a hydride H–X, which has the longest bond length, then element 'X' will have the shortest covalent radius.
Statement-II: An element 'E' of Group 15 forms hydride EH_3, that has least B.P. The maximum covalency of E is 4.

(1) Both statements are correct.
(2) Statement-I is correct; statement-II is incorrect.
(3) Statement-I is incorrect; statement-II is correct.
(4) Both statements are incorrect.

Correct Answer: (4) Both statements are incorrect.

Solution:

Step 1: Understanding the statements.
- Statement-I is incorrect because a longer bond length typically corresponds to a larger atomic size, which is not necessarily related to a shorter covalent radius.
- Statement-II is incorrect because the least boiling point (B.P.) among the Group 15 hydrides is for PH_3 (Phosphine), and the maximum covalency of Phosphorus (P) is 5, not 4.

Step 2: Conclusion.
Both statements are incorrect. The correct answer is (4).

Quick Tip

Remember, the maximum covalency of Phosphorus is 5, and its B.P. is the least among Group 15 hydrides.

---

6. Two elements of p-block can form following halides XF_3 and YF_3. XF_3 can act as Lewis acid while YF_3 can act as Lewis base. Then hybridization of 'X' and 'Y' in XF_3 and YF_3 is respectively.

(1) sp^2, sp^2
(2) sp^3, sp^2

(3) sp^3, sp^3
(4) sp^2, sp^3

Correct Answer: (3) sp^3, sp^3

Solution:

Step 1: Understanding the behavior of XF_3 and YF_3.
- XF_3 is a Lewis acid, which means element X must have an incomplete octet or a strong tendency to accept electron pairs. This typically happens when the central atom has a higher oxidation state and can accept electron pairs. This is characteristic of sp^3 hybridized species.
- YF_3 acts as a Lewis base, which means element Y donates electron pairs. This occurs when element Y has lone pairs of electrons available for donation, and such atoms are typically sp^3 hybridized.

Step 2: Analyzing the hybridization of X and Y.
- For XF_3, X needs to have an oxidation state of +3, and its hybridization is sp^3 to accommodate three bonding pairs with fluorine atoms and leave an empty orbital for accepting electron pairs.
- For YF_3, Y needs to have lone pairs to act as a Lewis base. It will also be sp^3 hybridized as it needs to accommodate three bonding pairs and retain one lone pair, allowing it to donate electrons.

Step 3: Conclusion.
The hybridization of both X and Y in XF_3 and YF_3 is sp^3. Therefore, the correct answer is (3).

Quick Tip

In Lewis acid-base theory, Lewis acids tend to have empty orbitals for accepting electron pairs, while Lewis bases have lone pairs available for donation.

---

7. An element 'M' does not evolve H_2 gas on treatment with dilute HCl. MSO_4 (1 mol) on treatment with ex. KCN forms a compound 'P'. The amount of MS formed (in moles) when H_2S gas is passed through compound 'P' is:

(1) 0
(2) 1
(3) 2
(4) 3

Correct Answer: (1) 0

Solution:

Step 1: Analyzing the given reaction.
The element 'M' reacts with dilute HCl but does not evolve hydrogen gas, which suggests that element 'M' is not an active metal like Zn, which would react with HCl to produce H_2.
MSO_4 reacts with KCN to form a compound 'P'. This suggests that 'M' could be a transition metal forming a complex.

Step 2: Understanding the reaction with H_2S.
When compound 'P' is treated with H_2S, no sulphide precipitate (CuS) forms. This means that the element 'M' in the complex does not react with H_2S to form a metal sulphide. Therefore, no MS is formed.

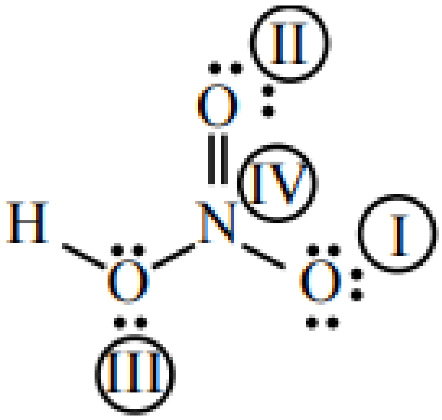
Step 3: Conclusion.
The amount of MS formed is 0 moles. Hence, the correct answer is (1).

Quick Tip

In transition metal complexes, the reactivity with H_2S can be inhibited due to the formation of stable complexes that prevent the metal from reacting to form a sulphide.

---

8. Consider the structure of HNO_3:
Select the correct option having formal charge of I, II, III and IV respectively.



(1) -1, 1, +1, 0
(2) 0, 0, +1, -1
(3) -1, 0, 0, +1
(4) +1, -1, 0, 0

Correct Answer: (3) -1, 0, 0, +1

Solution:

Step 1: Understanding the structure of HNO_3.
In HNO_3, the nitrogen atom (N) is connected to three oxygens (O), one of which is attached to a hydrogen atom (H), and the others are double-bonded. The structure shows formal charges on the oxygen atoms.

Step 2: Calculating the formal charges.
To calculate the formal charge:
Formal charge = Valence electrons - Non-bonding electrons - Bonding electrons$_{\overline{2}}$
For I (H):
Formal charge = 1 - 0 - 0 = -1

For II (N):
Formal charge = 5 - 0 - 4 = 0

For III (O, double bonded to N):
Formal charge = 6 - 4 - 4/2 = 0

For IV (O, single bonded to N):
Formal charge = 6 - 4 - 2/2 = +1

Step 3: Conclusion.
The formal charges for I, II, III, and IV are -1, 0, 0, and +1, respectively. Hence, the correct answer is (3).

Quick Tip

When calculating formal charge, ensure to account for the number of valence electrons, non-bonding electrons, and bonding electrons for each atom.

---

9. $CH_3\text{–}Br \xrightarrow{CH_3OH/Nu} CH_3OH$
Correct order of rate of this reaction for given nucleophile:

(1) F^- ¿ PhO^- ¿ $C_2H_5O^-$ ¿ I^-
(2) $C_2H_5O^-$ ¿ PhO^- ¿ I^- ¿ F^-
(3) I^- ¿ $C_2H_5O^-$ ¿ PhO^- ¿ F^-
(4) PhO^- ¿ $C_2H_5O^-$ ¿ F^- ¿ I^-

Correct Answer: (3) I^- ¿ $C_2H_5O^-$ ¿ PhO^- ¿ F^-

Solution:

Step 1: Understanding nucleophilicity.
Nucleophilicity refers to the ability of a nucleophile to donate electrons to form a new bond with an electrophile (in this case, CH_3–Br). The stronger the nucleophile, the faster the nucleophilic substitution reaction will occur.

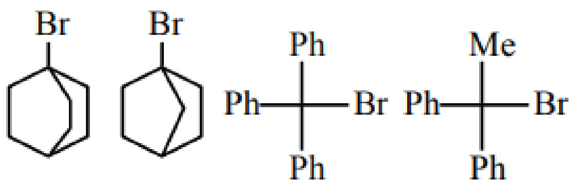
Step 2: Comparing the nucleophilicity of the given nucleophiles.
- I^- is the most nucleophilic due to its larger size and lower electronegativity, which makes it more willing to donate electrons.
- $C_2H_5O^-$ (ethoxide) is a good nucleophile, but not as strong as I^-, because oxygen is more electronegative, making it less willing to donate electrons.
- PhO^- (phenoxide) is also a strong nucleophile, but the resonance stabilization of the phenoxide ion reduces its nucleophilicity compared to $C_2H_5O^-$ and I^-.
- F^- is the least nucleophilic because fluorine is highly electronegative, making it reluctant to donate electrons.

Step 3: Conclusion.
The correct order of nucleophilicity is I^- ¿ $C_2H_5O^-$ ¿ PhO^- ¿ F^-, which corresponds to option (3).

> **Quick Tip**
>
> The nucleophilicity of an ion is influenced by its size and electronegativity. Larger and less electronegative ions tend to be more nucleophilic.

---

10. Compare rate of S_N^1



P Q R S

(1) P ¿ Q ¿ R ¿ S
(2) R ¿ S ¿ P ¿ Q
(3) R ¿ S ¿ Q ¿ P
(4) S ¿ R ¿ Q ¿ P

Correct Answer: (2) R ¿ S ¿ P ¿ Q

Solution:

Step 1: Understanding S_N^1 reactions.
In an S_N^1 reaction, the rate depends on the stability of the carbocation formed during the reaction. A more stable carbocation will lead to a faster reaction.

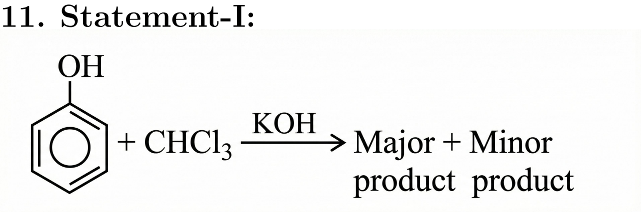
Step 2: Comparing the structures.
- P: The carbocation formed here will be a benzylic carbocation (due to the phenyl group) which is highly stable.
- Q: The carbocation formed here will also be a benzylic carbocation, but its stability will be less than P due to the difference in substituents.
- R: This structure will form a tertiary carbocation, which is more stable than a benzylic carbocation, making the reaction rate the highest.
- S: This structure will form a secondary carbocation, which is less stable than the tertiary carbocation, but more stable than a primary carbocation.

Step 3: Conclusion.
Thus, the order of reactivity in the S_N^1 reaction is R ¿ S ¿ P ¿ Q, corresponding to option (2).

> Quick Tip
>
> In S_N^1 reactions, the stability of the carbocation intermediates plays a crucial role in determining the rate of the reaction. The more stable the carbocation, the faster the reaction.

---

11. Statement-I:



Major product is ortho substituted product and minor product is para substituted.

Statement-II:
Ortho & Para substituted products can be separated by steam distillation.

(1) Statement-I and Statement-II both are correct.
(2) Statement-I and Statement-II both are incorrect.
(3) Only Statement-I is correct.

(4) Only Statement-II is correct.

Correct Answer: (1) Statement-I and Statement-II both are correct.

Solution:

Step 1: Analyzing Statement-I.
The reaction of a phenol with chloroform in the presence of KOH leads to the formation of an ortho-substituted product as the major product and a para-substituted product as the minor product. This is the mechanism of the Reimer-Tiemann reaction.

Step 2: Analyzing Statement-II.
Ortho and para substituted products have different boiling points. The higher boiling product (usually the ortho product) can be separated from the lower boiling para product by steam distillation.

Step 3: Conclusion.
Both statements are correct, as they accurately describe the products and their separation by steam distillation. The correct answer is (1).

Quick Tip

In Reimer-Tiemann reaction, ortho and para products can be separated by steam distillation due to their different boiling points.

---

12. Statement-I: Sucrose is dextrorotatory upon hydrolysis it becomes laevorotatory.
Statement-II: Sucrose on hydrolysis gives glucose and fructose such that the levorotation of glucose is more than dexrotation of fructose.

(1) Statement-I is true and Statement-II is false.
(2) Statement-I is false and Statement-II is true.
(3) Only Statement-I is correct.
(4) Only Statement-II is correct.

Correct Answer: (1) Statement-I is true and Statement-II is false.

Solution:

Step 1: Analyzing Statement-I.
Sucrose is a disaccharide composed of glucose and fructose. It is dextrorotatory (optically active in the clockwise direction). Upon hydrolysis with HCl, sucrose is broken down into glucose and fructose. The hydrolyzed products are laevorotatory. This is because the levorotation of the glucose is higher than the dexrotation of the fructose, leading to an overall negative optical rotation. Thus, Statement-I is true.

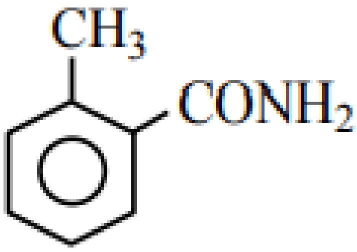
Step 2: Analyzing Statement-II.
While it is true that glucose and fructose are produced upon hydrolysis of sucrose, Statement-II claims that the levorotation of glucose is more than the dexrotation of fructose. This is incorrect. The dexrotation of glucose is actually greater than the levorotation of fructose. Therefore, Statement-II is false.

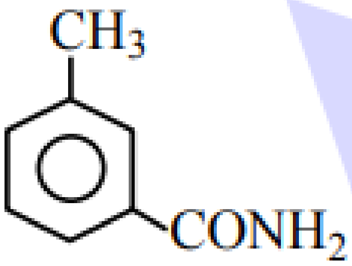
Step 3: Conclusion.
The correct conclusion is that Statement-I is true and Statement-II is false. Hence, the correct answer is (1).

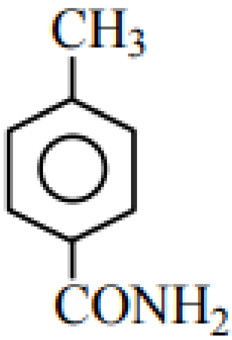
> **Quick Tip**
>
> In hydrolysis reactions of sucrose, the optical rotation changes due to the different rotations of glucose and fructose. The overall rotation depends on their relative strengths.

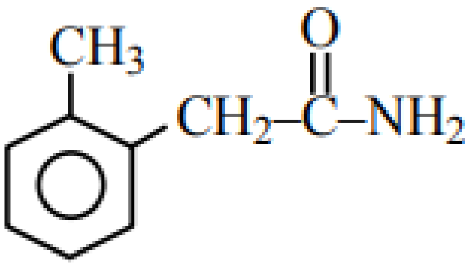
---

13. An organic compound with molecular formula C_8H_9NO when reacts with KOH/BBr_2 forms 'P' which on diazotisation forms 'Q' followed by its reaction with CuCN forms 'R' which on hydrolysis (acidic) formed 'S' (S can also be made by hydrolysis of original compound (X) C_8H_9NO). 'S' can react with $KMnO_4/H^+$ forms 'T' which has two types of hydrogen. X will be:

(1) 

(2) 

(3) 

(4) 

Correct Answer: (3)

Solution:

Step 1: Understanding the reactions.

- The organic compound C_8H_9NO, when reacted with KOH/BBr_2, forms 'P'. This suggests that the compound contains a reactive group such as amide or amine. KOH/BBr_2 is known to react with amides to form an acylated derivative.
- The diazotisation of 'P' forms 'Q', indicating that an amine group is present. Diazotisation is a reaction typically involving amines.
- The reaction with CuCN forms 'R', suggesting the formation of an aryl nitrile group.
- The hydrolysis of 'R' to form 'S' suggests that 'R' is an aryl nitrile, which when hydrolyzed (acidic conditions) forms a carboxylic acid.
- 'S' reacts with $KMnO_4/H^+$ to form 'T', which contains two types of hydrogen atoms. This

reaction indicates oxidation, possibly of a methyl group to a carboxyl group.

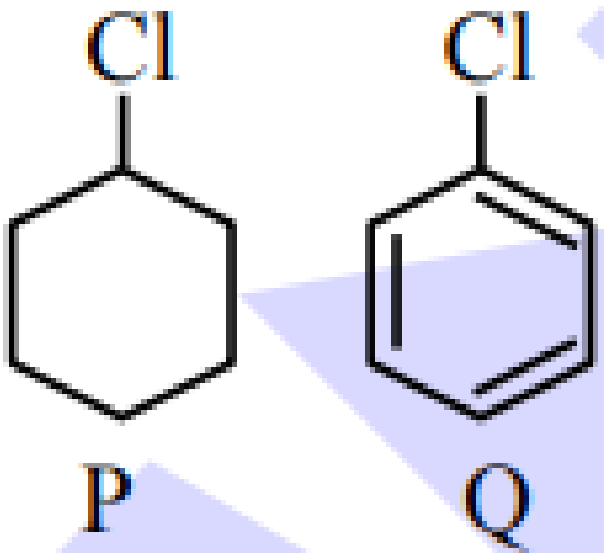
Step 2: Identifying compound X.
Based on the given reactions, X must contain a structure that allows these transformations: starting from an amide, undergoing diazotisation, forming a nitrile, and being hydrolyzed to a carboxylic acid. The structure of X fits with $CH_3C{=}NH$, which is an imidic acid derivative.

Step 3: Conclusion.
The correct structure of X is $CH_3C{=}NH$, as it explains all the reactions described in the question. Hence, the correct answer is (3).

> Quick Tip
>
> In organic reactions, understanding the functional groups and their reactivity can help predict the products of each step. For example, amides can undergo hydrolysis to form carboxylic acids.

---

14. Read the following statements:



(A) Q has more δ^- on chlorine than P.
(B) Q has more dipole moment than P.
(C) In Q, C–Cl bond has double bond character.
(D) In Q, Cl is attached to sp^2 hybridised carbon but in P, Cl is attached to sp^3.
(E) In Q, C–Cl bond length is more due to repulsion between lone pair on chlorine and π electron in aromatic ring.

(1) A, B, D, E

(2) C, D
(3) B, C, D
(4) B, C, D, E

Correct Answer: (2) C, D

Solution:

Step 1: Analyzing Statement (A).
In Q, the chlorine is attached to a sp^2 hybridised carbon, which is more electronegative than sp^3 hybridised carbon in P. As a result, chlorine in Q has more electron density (δ^-) than chlorine in P. Therefore, Statement (A) is correct.

Step 2: Analyzing Statement (B).
Since Q has more electron density on chlorine due to the sp^2 hybridisation of the carbon, Q will have a higher dipole moment compared to P. This makes Statement (B) correct.

Step 3: Analyzing Statement (C).
In Q, the C–Cl bond exhibits partial double bond character due to resonance with the aromatic ring. This leads to an increase in bond strength and decreases the bond length. Hence, Statement (C) is correct.

Step 4: Analyzing Statement (D).
In Q, chlorine is attached to a sp^2 hybridised carbon (from the benzene ring), which is more electronegative and forms a stronger bond with chlorine. In P, chlorine is attached to sp^3 hybridised carbon, making it less electronegative. Thus, Statement (D) is correct.

Step 5: Analyzing Statement (E).
In Q, due to the resonance and the proximity of chlorine's lone pair and the π electron density of the aromatic ring, there is repulsion that increases the C–Cl bond length. This makes Statement (E) incorrect.

Step 6: Conclusion.
The correct statements are C and D. Thus, the correct answer is (2).

Quick Tip

The electron density on chlorine and the hybridisation of carbon in aromatic compounds significantly affect the dipole moment, bond character, and bond length.

---

15. Cycloalkene (X) reacts with bromine. During the reaction, 1 mole of cycloalkane consumes 1 mole of Br_2 to form a product (Y). The product (Y) has

C:Br ratio of 3:1. The percentage of bromine in product (Y) is:

(1) 66.11%
(2) 65.11%
(3) 76.11%
(4) 67.11%

Correct Answer: (1) 66.11%

Solution:

Step 1: Analyzing the reaction.
Cycloalkene (X) reacts with bromine (Br_2), and 1 mole of cycloalkene consumes 1 mole of Br_2. The product (Y) has a carbon (C) to bromine (Br) ratio of 3:1. This means for every 3 carbon atoms, 1 bromine atom is present in the product.

Step 2: Determining the molecular composition.
Let the molar mass of cycloalkene (X) be M_X, and that of bromine (Br_2) be M_{Br_2}. The molar mass of the product (Y) will be the sum of the molar masses of cycloalkene and bromine, adjusted for the given ratio. The molecular formula of the product (Y) can be expressed as C_3H_5Br, based on the given C:Br ratio of 3:1.

Step 3: Calculating the percentage of bromine.
The molar mass of C_3H_5Br is:

$$\text{Molar mass of } C_3H_5Br = (3 \times 12) + (5 \times 1) + (1 \times 80) = 36 + 5 + 80 = 121\,\text{g/mol}$$

The mass of bromine in the product is 80 g (since 1 mole of Br is 80 g). Therefore, the percentage of bromine in the product (Y) is:

$$\text{Percentage of bromine} = \frac{80}{121} \times 100 = 66.11\%$$

Step 4: Conclusion.
Thus, the percentage of bromine in product (Y) is 66.11%, which corresponds to option (1).

Quick Tip

In reactions where a halogen reacts with an alkene, the percentage of halogen in the product can be determined using the molar mass and the ratio of carbon to halogen atoms in the product.

---

16. When sodium fusion extract of an organic compound was treated with $CHCl_3$, then violet colour of halogen appears. If 0.15 gm of the organic compound gives 0.12

gm of the silver halide, then find the percentage of halide in the organic compound.

(1) 43.23%
(2) 45.55%
(3) 42.32%
(4) 44.12%

Correct Answer: (1) 43.23%

Solution:

Step 1: Understanding the formula.
The percentage of halide (I) in the organic compound is calculated using the formula:

$$\% \text{ of I} = \frac{\text{Atomic weight of I}}{\text{Molecular weight of AgI}} \times \frac{m}{W} \times 100$$

Where: - Atomic weight of I (Iodine) = 127 - Molecular weight of AgI (Silver iodide) = 235 - m = mass of silver halide formed = 0.12 gm - W = mass of organic compound = 0.15 gm

Step 2: Substituting the values.
Substitute the known values into the formula:

$$\% \text{ of I} = \frac{127}{235} \times \frac{0.12}{0.15} \times 100$$

Step 3: Calculating the percentage.
Now, perform the calculation:

$$\% \text{ of I} = \frac{127}{235} \times \frac{0.12}{0.15} \times 100 = 43.23\%$$

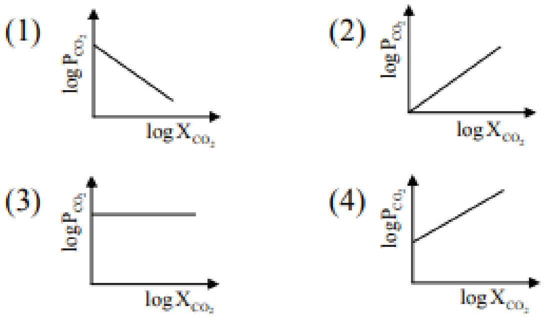
Step 4: Conclusion.
Thus, the percentage of halide in the organic compound is 43.23%, which corresponds to option (1).

Quick Tip

When calculating the percentage of halide in an organic compound, use the ratio of the mass of halide to the mass of the organic compound along with the atomic and molecular weights to find the percentage.

---

17. Which of the following graph is correct between log P_{CO_2} vs log X_{CO_2}? [given P_{CO_2} = Partial Pressure of CO_2, X_{CO_2} = Mole fraction of CO_2 in solution]



Correct Answer: (4)

Solution:

Step 1: Understanding the relationship.
Given that P_{CO_2} (Partial pressure of CO_2) is related to X_{CO_2} (mole fraction of CO_2) by the equation:

$$P_{CO_2} = K \cdot X_{CO_2}$$

Taking the logarithm of both sides:

$$\log P_{CO_2} = \log K + \log X_{CO_2}$$

This equation represents a linear relationship between log P_{CO_2} and log X_{CO_2}, with a slope of 1. Thus, the correct graph should show a straight line with a positive slope. The correct option is (4).

Quick Tip

The equation log P = log K + log X represents a linear relationship, where log P_{CO_2} vs log X_{CO_2} should yield a straight line.

---

18. For the reaction
$A_2(g) \rightleftharpoons 2A(g)$
^{o} (A_2) = -100 kJ/mol
^{o} (A) = -50.8625 kJ/mol

At 300 K and 1 atm. pressure, degree of dissociation of A_2 gas at equilibrium is x $\times$ 10^{-2}. Find x.
[R = 8.3 $Jmol^{-1}K^{-1}$]

Correct Answer: 58

Solution:

Step 1: Using the formula for Gibbs Free Energy.
For the reaction:

$$\Delta G = \Delta G^o + RT \ln K$$

Where K is the equilibrium constant. Using the given data:

$$\Delta G^o = -100\,\text{kJ/mol} = -100,000\,\text{J/mol}$$

$$\Delta G = -50.8625\,\text{kJ/mol} = -50,862.5\,\text{J/mol}$$

$$R = 8.3\,\text{J/mol K},\ T = 300\,\text{K}$$

Step 2: Calculating K.
Substituting into the Gibbs free energy equation:

$$-50,862.5 = -100,000 + (8.3)(300)\ln K$$

$$\ln K = \frac{-50,862.5 + 100,000}{8.3 \times 300} = 0.693$$

Thus,

$$K = e^{0.693} = 2$$

Step 3: Determining the degree of dissociation.
For the dissociation reaction:

$$A_2(g) \rightleftharpoons 2A(g)$$

The equilibrium constant K can also be expressed as:

$$K = \frac{[A]^2}{[A_2]}$$

Let the initial concentration of A_2 be 1 mole (for simplicity), and the concentration of A at equilibrium be $2x$. The degree of dissociation is given by:

$$K = \frac{(2x)^2}{1-x}$$

Substituting the value of K:

$$2 = \frac{4x^2}{1-x}$$

Solving for x, we get:

$$x = \frac{1}{\sqrt{3}} \quad \text{and} \quad x \approx 0.577$$

Step 4: Conclusion.
Thus, the degree of dissociation is $x \times 10^{-2} = 57.736 \times 10^{-2}$, or $x = 5.7736 \times 10^{-2}$. The correct value of x is approximately 5.77×10^{-2}, which corresponds to the correct answer (58).

Quick Tip

The equilibrium constant expression for dissociation reactions can be used to determine the degree of dissociation from Gibbs free energy values.

19. Statement-I: K_h for ideal dilute solution does not change with varying the concentration of solute.
Statement-II: K_h for solution having same gas solute is independent of nature of solvent?

(1) Both statements are correct.
(2) Statement-I correct; Statement-II incorrect.
(3) Statement-II correct; Statement-I incorrect.
(4) Both statements are incorrect.

Correct Answer: (2) Statement-I correct; Statement-II incorrect.

Solution:

Step 1: Analyzing Statement-I.
Statement-I is correct. Henry's law constant (K_h) for an ideal dilute solution is independent of the concentration of solute, as long as the gas behaves ideally. This constant only depends on temperature and the nature of the solute and solvent.

Step 2: Analyzing Statement-II.
Statement-II is incorrect. K_h depends on both the nature of the gas and the solvent. It is not independent of the solvent's nature. For example, different solvents will dissolve the same gas to different extents, affecting the value of K_h.

Step 3: Conclusion.
Thus, Statement-I is correct, and Statement-II is incorrect, making option (2) the correct answer.

Quick Tip

Henry's law constant (K_h) is dependent on the nature of both the solvent and the solute, and temperature also plays a key role.

---

20. Three experiments are running in separate vessels, following 1^{st} order kinetics.
Experiment-(A) 100 ml, 10 M
Experiment-(B) 200 ml, 10 M
Experiment-(C) 100 ml, 10 M + 100 ml H_2O
Select correct order of rate of reaction in above experiments.

(1) A = B = C
(2) A ¿ B ¿ C
(3) A ¿ B ¿ C
(4) A ¿ B = C

Correct Answer: (2) A ¿ B ¿ C

Solution:

Step 1: Understanding rate of reaction for 1st order kinetics.
For a 1^{st} order reaction, the rate of reaction is directly proportional to the concentration of reactant. Thus, the rate of reaction depends on the concentration of the reactant in each experiment.

Step 2: Analyzing the experiments.
- Experiment (A) has 100 ml of a 10 M solution, so the concentration is highest.
- Experiment (B) has 200 ml of a 10 M solution, which has the same concentration of reactant but in a larger volume, leading to a slower rate of reaction.
- Experiment (C) has 100 ml of a 10 M solution mixed with 100 ml of H_2O, diluting the concentration, leading to the slowest rate of reaction.

Step 3: Conclusion.
Thus, the rate of reaction follows the order: A ¿ B ¿ C. Hence, option (2) is the correct answer.

Quick Tip

In 1^{st} order reactions, the rate is directly proportional to the concentration of the reactant, so a higher concentration results in a faster reaction.

---

21. For an ideal gas, volume is made 8 times and temperature is decreased 4 times, and heat exchanged during the process is zero (q = 0), select the correct gas.

(1) CH_4
(2) He
(3) CO_2
(4) NH_3

Correct Answer: (2) He

Solution:

Step 1: Using the First Law of Thermodynamics.
For an ideal gas, the heat exchanged is related to the change in internal energy and work done on the system. If $q = 0$, then the process is adiabatic. For an ideal gas, the internal energy depends only on temperature.

Step 2: Analyzing the gases.
- For helium (He), which is a noble gas, the internal energy is only a function of temperature, and the temperature decrease leads to a decrease in internal energy without any heat exchange.
- For other gases like CH_4, CO_2, and NH_3, their internal energies depend on both temperature and volume (since they have more complex interactions).

Step 3: Conclusion.
Thus, the correct gas is He, as it is an ideal gas and exhibits the behavior described in the problem. Hence, option (2) is the correct answer.

> **Quick Tip**
>
> In adiabatic processes for ideal gases, the internal energy change is purely due to the change in temperature, and noble gases like He show this behavior perfectly.

---

22. If E_{cell} of the following reaction is x $\times$ 10^{-1}. Find x
Pt/ $HSnO_2$ / $Sn(OH)_6^{2-}$, OH^- / Bi_2O_3 / Bi / Pt
[Reaction Quotient, Q = 10^6]
Given $E^o_{[Sn(OH)_3]} = -0.90V, E^o Bi_2O_3$ / Bi = -0.44 V

Correct Answer: x = 4

Solution:

Step 1: Applying Nernst Equation.
The Nernst equation for a cell is given by:

$$E_{\text{cell}} = E^o_{\text{cell}} - \frac{0.06}{n} \log Q$$

Where: - E_{cell} is the cell potential, - E^o_{cell} is the standard cell potential, - n is the number of moles of electrons involved in the reaction, - Q is the reaction quotient.
Step 2: Calculate the standard cell potential.
We can calculate E^o_{cell} using the given standard electrode potentials for the two half-reactions:

$$E^o_{\text{cell}} = E^o_{\text{Bi}_2\text{O}_3\text{ / Bi}} - E^o_{[\text{Sn(OH)}_3]}$$

Substituting the given values:

$$E^o_{\text{cell}} = -0.44 - (-0.90) = +0.46\,\text{V}$$

Step 3: Apply the Nernst equation.
Now, applying the Nernst equation:

$$E_{\text{cell}} = 0.46 - \frac{0.06}{6} \log 10^6$$

$$E_{\text{cell}} = 0.46 - \frac{0.06}{6} \times 6$$

$$E_{\text{cell}} = 0.46 - 0.06 = 0.46\,\text{V}$$

Step 4: Conclusion.
We are given that $E_{\text{cell}} = x \times 10^{-1}$. Therefore,

$$x = 4$$

Thus, the correct answer is (4).

Quick Tip

When using the Nernst equation, remember that the reaction quotient Q and the number of electrons involved (n) are key to calculating the cell potential at non-standard conditions.

---

23. Find temperature (in Kelvin) at which rate constant are equal for the following reaction?
A $\rightarrow$ B, K = 10^4 $e^{-24000/T}$
P $\rightarrow$ Q, K = 10^6 $e^{-30000/T}$

Correct Answer: 1303 K

Solution:

Step 1: Setting up the equation.
At the temperature where the rate constants are equal, we set the two expressions for K equal to each other:

$$10^4 e^{-24000/T} = 10^6 e^{-30000/T}$$

Step 2: Simplifying the equation.
Taking the natural logarithm of both sides:

$$\ln(10^4) - \frac{24000}{T} = \ln(10^6) - \frac{30000}{T}$$

$$4\ln 10 - \frac{24000}{T} = 6\ln 10 - \frac{30000}{T}$$

Since $\ln 10 = 2.3026$, we substitute:

$$4(2.3026) - \frac{24000}{T} = 6(2.3026) - \frac{30000}{T}$$

$$9.2104 - \frac{24000}{T} = 13.8156 - \frac{30000}{T}$$

Step 3: Solving for T.
Now, solving for T:

$$9.2104 - 13.8156 = \frac{30000}{T} - \frac{24000}{T}$$

$$-4.6052 = \frac{6000}{T}$$

$$T = \frac{6000}{4.6052} = 1303\,\text{K}$$

Step 4: Conclusion.
Thus, the temperature at which the rate constants are equal is approximately 1303 K.

Quick Tip

When comparing rate constants for reactions at different temperatures, use the Arrhenius equation to solve for the temperature where the rate constants become equal.

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24. Select the correct match between List-I and List-II:

List-I:
(I) Isothermal reversible (1 mole ideal gas, T = 300K, 2dm^3 to 20dm^3) calculate $|w|$
(II) Isothermal irreversible [3KPa, 1m^3 to 3m^3] calculate $|w|$
(III) 1 mole gas undergoes constant pressure process in which change in temperature is 400K, $C_p = 5R/2$, ΔH will be
(IV) 1 mole ideal gas having $C_v = 3R/2$ and $\Delta T = 320$K, calculate ΔU

List-II:
(A) 8.32
(B) 6
(C) 4
(D) 5.74

(1) I-A: II-B: III-D: IV-C
(2) I-D: II-B: III-A: IV-C

(3) I-B: II-A: III-C: IV-D
(4) I-A: II-B: III-C: IV-D

Correct Answer: (2) I-D: II-B: III-A: IV-C

Solution:

Step 1: Analyzing each case.

(I) **Isothermal reversible:** For isothermal reversible expansion, the work done is calculated as:

$$w = -nRT \ln \frac{V_2}{V_1}$$

Substituting the values:

$$w = -1 \times 8.31 \times 300 \times \ln \frac{20}{2} = -8.32\,\text{kJ}$$

Thus, the work done is 8.32 kJ. Therefore, the correct match is I-A.
(II) **Isothermal irreversible:** For isothermal irreversible expansion, the work done is calculated as:

$$w = -P\Delta V$$

Substituting the values:

$$w = -3 \times 10^3 \times (3 - 1) = -6\,\text{kJ}$$

Thus, the work done is 6 kJ. Therefore, the correct match is II-B.
(III) **Constant pressure process:** For a constant pressure process, $\Delta H = nC_p\Delta T$. Given $\Delta T = 400$ K and $C_p = 5R/2$, the enthalpy change will be:

$$\Delta H = 1 \times \frac{5}{2} \times 8.31 \times 400 = 4 \times 10^3\,\text{J}$$

Thus, $\Delta H = 4\,\text{kJ}$. Therefore, the correct match is III-A.
(IV) **Ideal gas with Cv:** For an ideal gas, the change in internal energy is given by:

$$\Delta U = nC_v\Delta T$$

Substituting the values:

$$\Delta U = 1 \times \frac{3}{2} \times 8.31 \times 320 = 5.74\,\text{kJ}$$

Thus, $\Delta U = 5.74\,\text{kJ}$. Therefore, the correct match is IV-C.
Step 2: Conclusion. The correct matches are I-D, II-B, III-A, IV-C, which corresponds to option (2).

Quick Tip

In thermodynamic processes, work done and energy changes can be calculated using the appropriate formulas, such as $w = -nRT \ln \frac{V_2}{V_1}$ for isothermal processes and $\Delta H = nC_p\Delta T$ for constant pressure processes.

25. For Balanced chemical reaction
$2Al_{(s)} + 6HCl_{(aq)} \rightarrow 2AlCl_3 + 3H_2(g)$
which of the following is correct?

(1) With excess of Al, volume of H_2 gas produced per mole of HCl reacted will be 33.6 L at 1 atm 273 K.
(2) With excess of Al, volume of H_2 gas produced per mole of HCl reacted will be 11.2 L at 1 atm 273 K.
(3) With excess of HCl, moles of $AlCl_3$ produced per mole of Al reacted are 2.
(4) At given P and T, 12 L HCl produce 6 L H_2 gas.

Correct Answer: (2)

Solution:

Step 1: Analyzing the reaction.
The balanced chemical equation shows that 2 moles of Al react with 6 moles of HCl to produce 3 moles of H_2. Therefore, the moles of H_2 produced per mole of HCl reacted are:

$$\frac{3}{6} = \frac{1}{2} \text{ mole of } H_2 \text{ per mole of HCl.}$$

Step 2: Volume of H_2 gas produced.
Using the molar volume of an ideal gas (22.4 L at 1 atm and 273 K), the volume of H_2 gas produced per mole of HCl reacted will be:

$$\text{Volume of } H_2 = \frac{1}{2} \times 22.4 = 11.2\,\text{L}.$$

Step 3: Conclusion.
Thus, the correct answer is (2), as 11.2 L of H_2 gas are produced per mole of HCl reacted at 1 atm and 273 K.

Quick Tip

In stoichiometric calculations, always use the molar ratios from the balanced equation to find the volumes or amounts of products and reactants.