JEE-Main-28-01-2025 (Memory Based)
[EVENING SHIFT]
Maths

Question: Evaluate $\sum_{r=1}^{13} \frac{1}{sin\left[\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right]sin\left[\frac{\pi}{4}+\frac{r\pi}{6}\right]}$

Options:

(a) $2\sqrt{3} + 2$

(b) $2\sqrt{3} - 2$

(c) $3\sqrt{2} + 2$

(d) $3\sqrt{2} - 4$

Answer: (b)

$$\sum_{r=1}^{13} \frac{1}{\sin\left[\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right]\sin\left[\frac{\pi}{4}+\frac{r\pi}{6}\right]}$$

$$\sum_{r=1}^{13} \frac{\sin\left[\frac{\pi}{4}+r\frac{\pi}{6}\right]-\left[\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right]}{\left(\sin\frac{\pi}{6}\right)\times\sin\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}$$

$$\sum_{r=1}^{13} 2\left(\cot\left(\frac{\pi}{4}+(r-1)\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{4}+\frac{r\pi}{6}\right)\right)$$

$$2\left[\cot\frac{\pi}{4}-\cot\left(\frac{\pi}{4}+\frac{13\pi}{6}\right)\right]$$

$$2\left[1-\cot\left(\frac{29\pi}{12}\right)\right]$$

$$2\left[1-\cot\left(2\pi+\frac{5\pi}{12}\right)\right]$$

$$2\left(1-\cot\frac{5\pi}{12}\right)$$

$$2\left(1-2+\sqrt{3}\right) \Rightarrow 2\left(\sqrt{3}-2\right)$$

Question: $\int \frac{dx}{x^{\frac{1}{4}}\left(x^{\frac{1}{4}}+1\right)}$ **,f(0) = -6, f(2) = ?**

Solution :

$$I = \int \frac{dx}{x^{1/4}\left(x^{1/4}+1\right)}, f(0) = -6, f(2)$$

$$I = \int \frac{4t^3 dt}{t(t+1)}$$

$$= 4\int \frac{t^2}{t+1}dt$$

$$= 4\int \frac{t^2-1+1}{t+1}$$

$$= 4\int (t-1) + 4\int \frac{1}{t+1}$$

$$\Rightarrow 4\left(\frac{t^2}{2}-t\right) + 4\ln(t+1) + c$$

$$f(x) = 2\left(x^{\frac{1}{2}}\right) - 4x^{\frac{1}{4}} + 4\ln\left(1+x^{\frac{1}{4}}\right) + c$$

$$C = -6$$

$$f(2) = 2\left(\sqrt{2}\right) - 42^{\frac{1}{4}} + 4\ln\left(1+2^{\frac{1}{2}}\right)$$

Question: Bags B_1, B_2, B_3 contains 4 Blue, 6 white balls, 5 white 5 blue balls and 6 Blue 4 white balls respectively. A bag is randomly selected and a ball is drawn. If the drawn ball is white then find the probability that B_2 bag was selected.

Solution :

$B_1,\ B_2,\ B_3$

$B_1\ =\ +4\ \ Bl\ \ \&\ 6\ w$

$B_2 = -5\ \ wl\ \ \&\ \ 5\ \ Bl$

$B_3 = 5\ \ Bl\ \ \ \&\ \ \ 4\ \ Wl$

$P(B_2/A) \Rightarrow \frac{1/3 \times 5/10}{1/3 \times 5/10 + 1/3 \times 6/10 + 1/3 \times 4/10}$

$\Rightarrow \frac{\frac{5}{3 \times 10}}{\frac{15}{3 \times 10}} \Rightarrow \frac{1}{3}$

Question: Find domain of $\sec^{-1}(2[x] + 1)$, where [.] denotes GIF.
Solution :

Domain of $\text{Sec}^{-1}(2[x]+1)$

$2[x]+1 \geq 1$ on $2[x]\ +1 \leq -1$

$2[x]\ \geq\ 0 \qquad [x] \leq \frac{-2}{2}$

$x \geq 0 \qquad [x]\ \leq\ -1$

$[x]\ \leq\ -1$

$\therefore x\, t(-\infty,\ -1] \cup [0, \infty]$

Question: If x^2 - (3 - 2i)x - (2i - 2) = 0 has roots α + iβ and γ + iδ find the value of $\alpha\gamma + \beta\delta$.
Solution :

$x^2\ -\ (3-2i)\,x\ -\ (2i\ -2)\ = 0$

$x = \frac{3-2i \pm \sqrt{9-4-12i+4(2i-2)}}{2}$

$x = \frac{(3-2i)\ \pm \sqrt{-4i-3}}{2}$

$= \left[\left(\frac{1}{\sqrt{2}}\right)^{10} - 2^{10}\right]\ =\ \sqrt{-4i-3}\ =\ \alpha - i\beta$

$-4i\ -\ 3\ = \alpha^2\ - \beta^2\ + i2\alpha\beta$

$x^2\ - y^2\ = -3\ \ \ 2 \times\ y = 4$

$x^2\ - y^2\ = -3\ \ \alpha y\ = -2$

$x^2\ +\ y^2\ = \sqrt{\alpha^2\ - y^2\ + 4x^2 y^2}\ = 5$

$x^2\ + y^2\ = 5$

$x^2\ - y^2 = -3$

$2x^2\ = 2 \to x^2\ = 1\ \Rightarrow\ \alpha = 1, \beta = -2$

$\alpha = -1, \beta = +2$

$n = \frac{(3-2i) \pm (1-2i)}{2}\ - 2i + 2$

$\alpha\ = 1,\ \ \beta = 0,\ \ \gamma = 2,\ \ \delta\ - 2$

$\alpha\gamma\ + \beta\delta\ = 2$

Question: $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -2 \\ 0 & 1 \end{bmatrix} P = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **$B = PAP^T$; $X = PB^{10}P^T$ Find X = ?**

Solution :

$A=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix} P=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$PP^T=I \quad B=PAP^T \quad A^2=\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}} & -2\\ 0 & 1\end{bmatrix}$

$B^2=PAP^T(PAP^T) \quad A^2=\left[\left(\frac{1}{2}\right)^2 I^2\right]$

$=PA(P^TP)AP^T$

$B^2=PA^2P^T \quad X=P^T(PA^{10}P^T)P$

$B^{10}=PA^{10}P^T \quad =\mathrm{IA}^{10}I=A^{10}$

$x=P^TB^{10}P \quad \therefore x=\mathrm{A}^{10}=\begin{bmatrix}\left(\frac{1}{\sqrt{2}}\right)^{10} & -2^{10}\\ 0 & 1^{10}\end{bmatrix}$

Question: 212, 213,..........,999 find no. of numbers in the sequence above whose sum of digits is 15.

Solution :

$212, 213, \ldots\ldots\ldots\ldots, 999$

$\because Sum=15$

$x+y+2=15$

$x\in\{2,3,\ldots\ldots.9\}$

$y\in\{0,1,2,\ldots\ldots.9\}$

Coefficient $x^{15}:\left(x^2+x^3+\ldots..+n^9\right)\left(x^{11}+x^1+\ldots..x^9\right)^2$

$\left(1+x+x^2+\ldots\ldots+x^2\right)\left(\frac{1-x^{10}}{1-x}\right)^2$

$\left(\frac{1-x^8}{1-x}\right)\left(\frac{1-x^{10}}{1-x}\right)^2$

$\left(1-x^8\right)\left(1-x^{10}\right)(1-x)^{-3}$

$\left(1-x^8\right)\left(1-2x^{10}+x^{20}\right)(1-x)^{-3}$

$\left(1-x^8-2x^{10}\right)(1-x)^{-3}$

Coefficient $x^{12}={}^{3+13-1}C_{13}-{}^{3+5-1}C_5-2^{3+3-1}C_3$

$=64$

Question: Words are formed using all letters of Word GARDEN. Find the probability in the word vowels are not together.

Solution :

GARDEN=6!

$\times G \times R \times D \times N \times$

$= {}^5C_2 \times 2! \times 4!$

$P(\varepsilon) = \frac{{}^5C_2 \times 2 \times 4!}{6!}$

$= \frac{20 \times 4!}{6 \cdot 5 \cdot 4!}$

$= \frac{2}{3}$

Question: Area bounded between the curves C_1 : $x(1+y^2)-1=0$ and C_2 : $y^2-2x=0$ is (in sq. unit)

Options:

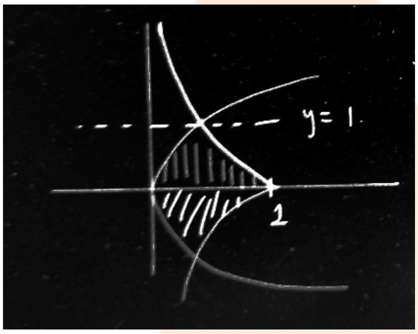
(a) $\frac{\pi}{2} - \frac{1}{3}$

(b) $\frac{\pi}{4} - \frac{1}{6}$

(c) $2\left(\frac{\pi}{2} - \frac{1}{6}\right)$

(d) $\frac{\pi}{6} + \frac{1}{2}$

Answer: (a)



$$C_x = x\left(1 + r^2\right) - 1 \text{ and}$$

$$C_y = y^2 - 2x = 0$$

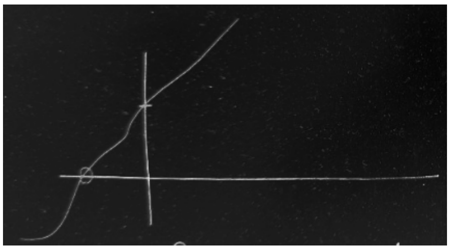
$$\int_{-1}^{1}\left(\frac{1}{1+y^2} - \frac{y^2}{2}\right)dy$$

$$\left(\tan^{-1}y - \frac{y^3}{6}\right)_{-1}^{1}$$

$$\left(\frac{\pi}{4} - \frac{1}{6}\right) - \left(-\frac{\pi}{4} + \frac{1}{6}\right)$$

$$\frac{\pi}{2} - \frac{1}{3}$$

Question: If f(x) is polynomial satisfying $f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and f(2) = 129; then find real values of "k" satisfying f(k) = -2k.
Solution :



$$f(x) \cdot f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$$

$$f(x) = \pm x^4 + 1$$

$$f(x) = \pm 2^4 + 1 = 129$$

$$x = 7$$

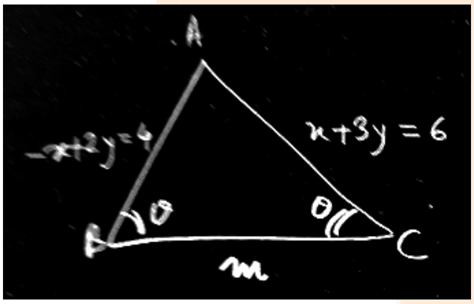
$$f(x) = x^7 + 1$$

$$f(x) = k^7 + 1 = -2k$$

$$k^7 + 2k + 1 = 0$$

$$f'(k) = 7k^6 + 2$$

Question: An isosceles triangle formed by 3 lines. Equation of equal sides of triangle are -x + 2y = 4 & x + 3y = 6. Find sum of possible values of slope of 3rd side.
Solution :



$$\tan\theta = \left|\frac{m-\frac{1}{2}}{1+\frac{m}{2}}\right| = \left|\frac{\frac{-1}{2}-m}{1+\frac{m}{3}}\right|$$

$$\frac{2m-1}{m+2} = \pm\left(\frac{-1-3m}{3-m}\right)$$

$$\frac{2m-1}{m+2} = \frac{-(1+3m)}{3-m}$$

$$\frac{2m-1}{m+2} = \frac{3m+1}{3-m}$$