CBSE Class 10 MathematicsFREE FORMULA SHEET

REAL NUMBERS

- Euclid's Division Lemma
  $a = b \times q + r,\ 0 \le r < b$.
- For any two positive integers a and b
  $HCF(a, b) \times LCM(a, b) = a \times b$
- For three numbers a, b & c

(i) HCF $(a, b, c) \times$ LCM$(a, b, c) \neq a \times b \times c$ where a, b, c are positive integers.

(ii) $LCM(a, b, c) = \dfrac{a \times b \times c \times HCF(a, b, c)}{HCF(a, b) \times HCF(b, c) \times HCF(a, c)}$

(iii) $HCF(a, b, c) = \dfrac{a \times b \times c \times LCM(a, b, c)}{LCM(a, b) \times LCM(b, c) \times LCM(a, c)}$

POLYNOMIALS

- **Remainder Theorem :** Let $p(x)$ be any polynomial of degree greater than or equal to 1 and a be any real number, if $p(x)$ be divided by linear polynomial $(x - a)$, then the remainder is equal to $p(a)$.
- **Factor Theorem :** If $p(x)$ is a polynomial of degree greater than or equal to 1 and a be any real number such that
  (i) if $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$ and
  (ii) if $(x - a)$ is a factor of $p(x)$, then $p(a) = 0$
- **Division Algorithm for Polynomial :** $p(x) = q(x) \times g(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $q(x)$.
- If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

- If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} \text{ and}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

SOME USEFUL IDENTITIES

- (i) $(x + y)^2 = x^2 + y^2 + 2xy$
  (ii) $(x - y)^2 = x^2 + y^2 - 2xy$
  (iii) $(x + y)(x - y) = x^2 - y^2$
  (iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$
  (v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
  (vi) $(x + y)^3 = x^3 + y^3 + 3xy(x + y) = x^3 + 3x^2y + 3xy^2 + y^3$
  (vii) $(x - y)^3 = x^3 - y^3 - 3xy(x - y) = x^3 - 3x^2y + 3xy^2 - y^3$
  (viii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
  If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$
  (ix) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
  (x) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

- If a pair of linear equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ represents :

(i) Intersecting lines then $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ (one solution)

(ii) Parallel lines, then $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}$ (no solution)

(iii) Coincident lines, then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(infinitely many solutions)

QUADRATIC EQUATIONS

- Roots of the quadratic equation $ax^2 + bx + c = 0$, $a, b, c \in R$ and $a \neq 0$ is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}; D = b^2 - 4ac$$

- **Nature of Roots**

(i) If $D > 0$, distinct and unequal real roots.

(ii) If D is a perfect square, the equation has unequal-rational roots.

(iii) If $D = 0$, real and equal roots and each root is $\frac{-b}{2a}$.

(iv) If $D < 0$, no real roots.

- Formation of a quadratic equation x^2 – (sum of roots) x + product of roots = 0

ARITHMETIC PROGRESSIONS

- The n^{th} term a_n of an A.P. is
$a_n = a + (n - 1)d$;
a = first term
n = number of terms
d = common difference

- The sum to n terms of an A.P.

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

Also, $S_n = \frac{n}{2}\{a + l\}$

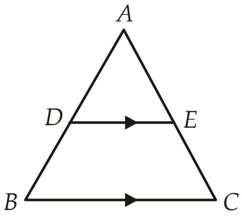
TRIANGLES

- **Basic Proportionality Theorem (B.P.T.) (Thales Theorem) :** In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio. In ΔABC, if $DE \parallel BC$.

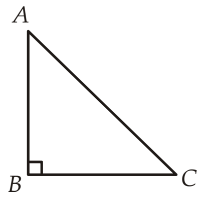
Then (i) $\frac{AD}{DB} = \frac{AE}{EC}$

(ii) $\frac{AB}{AD} = \frac{AC}{AE}$

(iii) $\frac{AB}{DB} = \frac{AC}{EC}$.



- **AAA Similarity Criterion :** If two triangles are equiangular, then they are similar.
- **AA Similarity Criterion :** If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.
- **SSS Similarity Criterion :** If the corresponding sides of two triangles are proportional, then they are similar.
- **SAS Similarity Criterion :** If in two triangles, one pair of corresponding sides are proportional and the included angles are equal, then the two triangles are similar.
- **Area of Similar Triangles :** The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides, altitudes, medians, angle bisector segments.
- **The Pythagoras Theorem :** In a right triangle, the square of the hypotenuse is equal to the sum of the square of other two sides. In the given figure, $AC^2 = AB^2 + BC^2$.



CO-ORDINATE GEOMETRY

- If $x \neq y$, then $(x, y) \neq (y, x)$.
- If $(x, y) = (y, x)$, then $x = y$.
- Distance between the points $A(x_1, y_1)$, $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- If A, B and C are collinear, then $AB + BC = AC$ or $AC + CB = AB$ or $BA + AC = BC$.
- The points which divides the line segment joining the points $A(x_1, y_1)$, $B(x_2, y_2)$ in the ratio $l : m$

(i) Internally : $\left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}\right)$; $(l + m \neq 0)$

(ii) Externally: $\left(\frac{lx_2 - mx_1}{l-m}, \frac{ly_2 - my_1}{l-m}\right)$; $(l \neq m)$

- The mid-point of the line segment joining $A(x_1, y_1)$, $B(x_2, y_2)$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.
- Centroid of a ΔABC, with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$$G \equiv \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

- The area of the triangle formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$

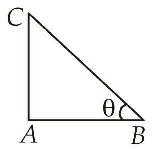
$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

INTRODUCTION TO TRIGONOMETRY

- **Trigonometric Ratios in ΔABC**

$$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AC}{BC}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{BC}$$



$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AC}{AB}$$

$$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{AC}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{BC}{AB}$$

$$\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{BC}{AC}$$

- $\text{cosec}\,\theta = \frac{1}{\sin\theta}$ or $\sin\theta = \frac{1}{\text{cosec}\,\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$ or $\cos\theta = \frac{1}{\sec\theta}$
- $\cot\theta = \frac{1}{\tan\theta}$ or $\tan\theta = \frac{1}{\cot\theta}$
- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

θ / T-ratios	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\text{cosec}\,\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

- **Trigonometric Ratios of Complementry Angles**

  $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$

  $\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$

  $\sec(90° - \theta) = \text{cosec}\,\theta$, $\text{cosec}(90° - \theta) = \sec\theta$

- **Trigonometric Identities**

  $\sin^2\theta + \cos^2\theta = 1$

  $\sec^2\theta - \tan^2\theta = 1$

  $\text{cosec}^2\theta - \cot^2\theta = 1$

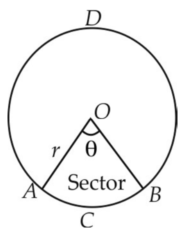
CIRCLES

- Tangent to a circle at a point is perpendicular to the radius through the point of contact.
- From a point, lying outside a circle, two and only two tangents can be drawn to it.
- The lengths of two tangents drawn from an external point are equal.

AREAS RELATED TO CIRCLE

- Circumference of a circle = $2\pi r$, where r is the radius of the circle.
- Perimeter of a semicircle with radius r is $2r + \pi r$.
- Area of a circle with radius r is given by $A = \pi r^2$.
- Area of a semicircle of radius $r = \dfrac{\pi r^2}{2}$.
- Area of a ring whose outer and inner radii are R and r respectively $= \pi(R^2 - r^2) = \pi(R + r)(R - r)$
- Perimeter of sector $OACBO = 2r + \dfrac{2\pi r\theta}{360°}$.
- Area of minor sector $OACBO = \dfrac{\pi r^2\theta}{360°}$.

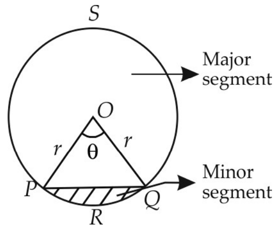
  Also, the area of a sector is given by $A = \dfrac{1}{2}lr$, where $l = \left(\dfrac{\pi r\theta}{180°}\right)$ = length of arc ACB.



- Area of major sector $OADBO = \pi r^2$ – area of minor sector $OACBO$.
- Area of the minor segment $PRQP$

  $= \dfrac{\pi r^2\theta}{360°} - \dfrac{1}{2}r^2\sin\theta$

- Area of major segment $PSQP$ $= \pi r^2$ – area of minor segment $PRQP$.



SURFACE AREAS AND VOLUMES

- **Cube**

  If a be the edge of a cube, then

  Volume = a^3

  Total surface area = $6a^2$

  Area of four walls = $4a^2$

  Diagonal of cube = $\sqrt{3} \times$ Edge $= \sqrt{3}\,a$

  Edge of a cube = $\sqrt[3]{\text{Volume}}$

- **Cuboid**

  If l be the length, b be the breadth and h be the height of the cuboid, then

  Volume = length × breadth × height = $l \times b \times h$

  Total surface area = $2(lb + bh + hl)$

  Area of four walls of a room = $2 \times (l + b)h$

  Diagonal of a cuboid = $\sqrt{l^2 + b^2 + h^2}$

- **Cylinder**

  If r be the radius of the cylinder and h be the height of the cylinder, then

  Volume = $\pi r^2 h$

  Curved surface area = $2\pi rh$

  Total surface area = $2\pi r(r + h)$

- **Hollow Cylinder**

  If R is the outer radius, r is the inner radius and h be the height of the hollow cylinder, then

  Volume = $\pi(R^2 - r^2)h$

  Total surface area = $2\pi(R + r)(h + R - r)$

- **Cone**

  If r, h and l denote respectively the radius of base, height and slant height of a right circular cone, then

  Volume $= \frac{1}{3}\pi r^2 h$

  Curved surface area $= \pi rl = \pi r\left(\sqrt{h^2 + r^2}\right)$

  Total surface area = curved surface area + area of the base $= \pi rl + \pi r^2 = \pi r(l + r)$

- **Sphere**

  If r is the radius of the sphere, then

  Surface area $= 4\pi r^2$

  Volume $= \frac{4}{3}\pi r^3$

- **Hollow Sphere**

  If R is the outer radius and r is the inner radius of the hollow sphere, then

  Volume $= \frac{4}{3}\pi(R^3 - r^3)$

- **Hemisphere**

  If r is the radius of the hemisphere, then

  Curved surface area $= 2\pi r^2$

  Total surface area $= 3\pi r^2$

  Volume $= \frac{2}{3}\pi r^3$

- **Frustum of a Cone**

  If h is the height, l the slant height and r_1 and r_2 the radii of the circular bases ($r_1 > r_2$) of a frustum of a cone, then

  Volume $= \frac{\pi}{3}(r_1^2 + r_1 r_2 + r_2^2)h$

  Lateral surface area $= \pi(r_1 + r_2)l$

  Total surface area $= \pi\{(r_1 + r_2)l + r_1^2 + r_2^2\}$

  Slant height of the frustum, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

STATISTICS

- **Range :** Highest observation – Lowest observation
- **Class size :** Upper class limit – Lower class limit
- **Class marks :**

  $\frac{\text{Upper class limit} + \text{Lower class limit}}{2}$

- **For Ungrouped Data**

(i) **Mean**

$$\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n}.$$

(ii) **Median**

Case-I : If the number of items n in the data is odd, then

$$\text{Median} = \text{value of} \left(\frac{n+1}{2}\right)^{\text{th}} \text{item.}$$

Case-II : If the total number of items n in the data is even, then

$$\text{Median} = \frac{1}{2} \times \text{value of} \left[\left(\frac{n}{2}\right)^{\text{th}} + \left(\frac{n}{2}+1\right)^{\text{th}}\right] \text{item}$$

(iii) Mode = 3 Median – 2 Mean

- **For Grouped Data**

(i) **Mean (Direct Method)**

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

(ii) **Mean (Mean Deviation Method)**

$$\bar{x} = a + \frac{\Sigma f_i(x_i - a)}{\Sigma f_i} = a + \frac{\Sigma f_i d_i}{\Sigma f_i},$$

where, a = assumed mean,

Σf_i = total frequency , $d_i = x_i - a$.

(iii) **Mean (Step Deviation Method)**

$$\bar{x} = a + \frac{\Sigma f_i\left(\frac{x_i - a}{h}\right)}{\Sigma f_i} \times h = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

where, a = assumed mean, Σf_i = total frequency,

h = class-size, $u_i = \frac{x_i - a}{h}$.

(iv) **Median(M_e)** $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h,$

where, l = lower limit of the median class,

n = number of observations,

cf = cumulative frequency of the class preceding the median class,

f = frequency of the median class,

h = class size.

- $$\textbf{Mode}(M_o) = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h,$$

where, l = lower limit of modal class,

h = size of the class-interval,

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class suceeding the modal class.

PROBABILITY

- Probability of an event $E = P(E) = \dfrac{n(E)}{n(S)}$
- $P(E) + P(\overline{E}) = 1$
- For an event E, we have $0 \leq P(E) \leq 1$.